Asymmetric Rent-seeking Contests with Multiple Agents of Two Types*

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February, 2017

Abstract

We consider a complete-information rent-seeking contest with multiple agents of two different ability levels. A single winner is selected based on a modified lottery contest success function with constant returns to scale. Two features of variation among players are considered: type heterogeneity, measured by the lobbying effectiveness ratio between the two types; and group composition, represented by the numbers of the two types of players. We first solve for the unique Nash equilibrium of the contest game, then examine how rent dissipation ratios and total effort level depend on agents' type heterogeneity and group composition. Based on the equilibrium analysis, we further consider whether adding a preliminary stage can benefit a contest organizer who aims at maximizing total expected effort from all agents. The dominant contest structure is fully characterized by conditions on agents' type heterogeneity and group composition.

JEL Codes: C72, D72

Keywords: Rent-seeking contests; Type heterogeneity; Group composition; Contest design

*The authors would like to thank seminar participants at Tsinghua University, Sun Yat-sen University, Shandong University, and the University of International Business and Economics, as well as conference participants at the 2013 Asia Meeting of Econometric Society in Singapore and the 2015 Ecomometric Society World Congress in Canada for helpful feedback. Cai, Jiao, and Zheng gratefully acknowledge financial support from the National Natural Science Foundation of China under Grant Nos. 71403061, 71401186, 71203112, and 61661136002. The usual disclaimer applies.

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1 Introduction

Rent-seeking contests are games in which players spend costly and nonrefundable effort competing for some rent. Asymmetric rent-seeking contests with multiple agents of two different types are widely observed in the real world. For example, in R&D races, domestic and foreign firms compete for the exclusive right to commercialize their idea; and in public recruitment, both experienced and inexperienced applicants participate in the same competition for a position. In both examples, players can be classified by two groups, each of which is composed of homogeneous players. Another typical example is the college entrance examination in China—probably the largest and most important contest among high school students, for which students are divided into different classes according to their academic abilities and compete for the highest ranking in college admissions.[1] In all of these examples, agents with a high (low) type are the strong (weak) agents with a relatively high (low) ability, in terms of lobbying or learning effectiveness. It is natural to think that agents' behavior is not only affected by the ability difference of the two groups, but also depends on the size of the high-type and low-type groups of agents.

In this paper, we consider a rent-seeking contest that involves a set of contestants of two types: Agents of the same type have the same ability, while the ability of one type may differ from the other. This two-type multi-agent setup introduces a new feature to the model, compared to exisiting contest models in the literature: Such a set of agents is characterized not only by the different ability levels of the two types, but also by the number of agents of each type in the contestant population. We refer to the difference between the abilities of the two types as *type heterogeneity*, measured by the lobbying effectiveness ratio between the two types of agents, and use the terminology *group composition* to describe possible combinations of numbers of the two types. Following the mainstream contest literature (e.g., Baik, 1994; Kohli and Singn, 1999) and adopting Tullock's (1980) one-stage complete-information contest with lottery contest success function and linear cost of effort, our analysis sets out to address a classical question in the contest literature: *How much effort do agents exert in pursuit of the prize in a simultaneous move game?*

A substantial number of papers in the literature have formally investigated contests

[1] In some schools, they name the class consisting of high learning-ability students the "Rockets class" and the other class, consisting of relatively low learning-ability students, the "Enhancing class".

with asymmetric players.[2] Baik (1994, 2004) shows that in a two-player competition, the high-type player tends to bid less when the competition is less intense. This is because the high-valuation player does not need to bid as much to exceed the bid of the low-valuation player as the difference in valuation between the two players increases. Baik's theory is then extended in several directions. Stein (2002) studies the properties of a single-stage N-player rent-seeking contest with different valuations, and provides the formula to learn the number of active players. Ryvkin (2007) presents an explicit and quantitative analysis of equilibria with weak heterogeneous players. More recently, Ryvkin (2013) explores the impact of heterogeneity in players' abilities on aggregate effort in contests, showing that the sign of the effect depends on the curvature of the effort cost function.

Our analysis, however, not only echoes Stein's (2002) argument on the effects of asymmetries among players, but also contributes to the literature by exploring the effects of group composition of different types, which has not been studied before. We find that while the incentive effect of type heterogeneity is always negative for low-type players, it can be positive for high-type players (Proposition 1). An increase in the number of agents will not always decrease individual effort provision: When there is a unique high-type agent, he would like to bid harder in a moderately competitive environment, even with more low-type competitors (Proposition 2). These results improve our understanding of complete-information asymmetric contests, compared with Stein (2002) and Baik (2004), since in Stein's (2002) comparative statics studies, the number of active players will not change with players' abilities; in Baik (2004), there is no variation in population composition, because there are in total only two players of two types.

It has been widely acknowledged that contestants' incentive to exert effort crucially depends on the rules of the contest. A forward-looking contest designer may wish to implement an optimal structure to achieve a given objective. A growing strand of literature has recognized multi-stage elimination contests in which there are preliminary stages and a final stage. Following mainstream literature and based on the equilibrium results of a Tullock one-stage grand contest, we further consider one particular question pertaining to multi-stage rent-seeking contest design: *Should the contest organizer add a preliminary elimination stage to maximize the total effort of agents?*

In the second part of our paper, the contest organizer is allowed to add a preliminary stage in which agents compete against their group mates of the same type, and a finalist

[2] Reviews of studies in this area have been conducted by Nitzan (1994) and Nti (1999).

is chosen from each group. Two survivors of different types in the preliminary stage then compete against each other in the final stage. An example of such a contest is the research tournament: A government could either conduct a grand contest that combines domestic and foreign firms to compete in a single heat, or select a winning firm from each group first and then conduct a one-to-one competition. Other examples, such as election campaigns between parties or candidates, and competition for promotions in organizational hierarchies, share the same structure.[3] The question naturally asked in such scenarios is: Which organizing rule induces more effort from its contestants—the grand contest or the two-stage contest? To address this question, we fully characterize the dominant contest structure at different levels of agents' type heterogeneity and group composition.

The main findings of our analysis are summarized as follows: In general, the one-shot grand contest dominates the two-stage contest if the level of type heterogeneity is comparatively small (Theorem 1). If the number of high-type agents is less (more) than three times the number of low-type agents, adding high-type agents or removing low-type agents will make the two-stage contest more (less) preferable (Proposition 5). The grand contest is always the dominant structure when there is a unique high-type agent, and the two-stage contest will become dominant when group sizes of different types are both sufficiently large and comparable (Corollary 2).

Our study also contributes to the growing literature on multi-stage grouping contests, in which most studies focus on total effort maximization.[4] Gradstein (1998) and Gradstein and Konrad (1999) compare the total effort level between simultaneous contests and multi-stage pairwise contests. Specifically, Gradstein (1998) focuses on different contest structures with the same group size, and shows that the dominant structure depends on the contest organizer's degree of impatience and contestants' heterogeneity. Gradstein and Konrad (1999) consider the contest structure as the endogenous choice of the contest organizer, who is allowed to decide how many stages the contest can consist of, and how remaining contestants are matched at each stage. They show that the optimality of a contest design crucially depends on how discriminatory the (Tullock) contest

[3]Because of some objective factors like territory restriction, group composition is usually exogenously determined and cannot be freely designed by an outsider. This is the situation we focus on in this paper.

[4]Multi-stage contests are studied by Amegashie (1999, 2000); Baik and Kim (1997); Baik and Lee (2000); Fu and Lu (2012); Gradstein (1998); Gradstein and Konrad (1999); Katz and Tokatlidu (1996); Konrad and Kovenock (2009, 2010); Rosen (1986); Stracke (2013), etc.

is. Katz and Tokatlidu (1996) study two groups with symmetric valuation of the prize and show that asymmetry in group size acts to reduce rent dissipation. Stein and Rapoport (2004) extend the analysis of Katz and Tokatlidu (1996) to $k\ (\geq 2)$ groups, with asymmetric valuation of the rent, and compare the contest structures of Between-Group and Semi-Finals models. More recently, Stracke (2013) studies the case of four players of two types, and compares the equilibrium results between a static one-stage contest and a dynamic two-stage contest. Stracke and Sunde (2016) carefully explore the incentive effects of heterogeneity in two-stage elimination contests.[5] Our study differs from previous work by focusing on asymmetries between two types—in terms of both type heterogeneity and group composition—with any number of players, and we are able to fully characterize the dominant contest structure between the grand contest and the two-stage contest.

The rest of the paper proceeds as follows. Section 2 carries out an equilibrium analysis of the grand contest and examines how rent dissipation ratios and total effort level depend on agents' type heterogeneity and group composition. In Section 3, we set up a two-stage contest and provide a detailed analysis of the dominant contest structure for expected total effort maximization. Section 4 provides some concluding remarks.

2 Grand Contests: Model and Analysis

In this section, we first set up a model of a grand contest, and then conduct the equilibrium analysis and study the comparative statics.

2.1 Setup

Consider a rent-seeking contest among N agents, who expend costly effort to compete for a single rent $V(>0)$. Let $\mathbf{x} = (x_1, x_2, \ldots x_N)$ be the expenditures by each of the agents. Following Stein (2002), the probability that agent i wins the rent is given by

$$P_i\left(x_1, x_2, \ldots x_N\right) = \frac{\lambda_i x_i}{\sum_{j=1}^{N} \lambda_j x_j}. \tag{1}$$

[5] Hőrtnagl et al.'s (2016) experimental work further explores how heterogeneity in contestants' effort affects the competition intensity in a two-stage contest.

The parameter λ_i measures agents i's ability to convert the expenditure x_i into meaningful lobbying efforts, which represents his relative chance of winning. Thus, a strong (low) agent is one who exhibits relatively high (low) effectiveness in lobbying.

Due to the functional form in (1), the contest is imperfectly discriminating, since the winner is determined probabilistically (Tullock, 1980). An axiomatic foundation for the contest success function with this specific form is given by Skaperdas (1996) and Clark and Riis (1998). Let π_i represent the expected payoff for agent i. Then the payoff function of each agent is

$$\pi_i\left(x_1, x_2, \ldots x_N\right) = P_i\left(x_1, x_2, \ldots x_N\right) V - x_i. \tag{2}$$

We assume that there are two types of agents: n_H high-type agents with high relative lobbying effectiveness λ_H, and n_L low-type agents with low relative lobbying effectiveness λ_L, where $\lambda_H \geq \lambda_L > 0$, $n_H \geq 1$, $n_L \geq 1$, and $n_H + n_L = N$. All of the parameter values—the numbers of agents (n_H, n_L), and different levels of lobbying effectiveness (λ_H, λ_L)—are common knowledge.[6] We also assume that agents choose their effort levels simultaneously and independently.

2.2 Equilibrium

To obtain a Nash equilibrium of the game, consider an arbitrary agent i. He chooses effort level $x_i \geq 0$ to maximize his expected payoff given by equation (2), given his opponents' effort levels $x_j \geq 0$ and lobbying effectiveness λ_j.[7]

Differentiating $\pi_i\left(x_1, x_2, \ldots x_N\right)$ with respect to x_i yields

$$\frac{\partial \pi_i\left(\mathbf{x}\right)}{\partial x_i} = \frac{\lambda_i}{\sum_{j=1}^{N} \lambda_j x_j}(1 - P_i)V - 1$$

[6] Stein (2002), among many others, considers a rent-seeking model with N types of asymmetric contestants. In our paper, the level of type heterogeneity is captured by the lobbying effectiveness ratio in the contest, which is difficult to measure with more than two types. Therefore, we restrict our attention to settings with only two types.

[7] It is impossible to have all agents make zero effort deterministically in an equilibrium. When all others bid zero, a representative agent would prefer to make an infinitely small positive effort, which allows him to win the prize with probability one.

Since π_i is concave in x_i, the first-order condition

$$\frac{\partial \pi_i(\cdot)}{\partial x_i} = 0 \text{ if } x_i > 0, \text{ or } \frac{\partial \pi_i(\cdot)}{\partial x_i} \leq 0 \text{ if } x_i = 0$$

is necessary and sufficient for optimality.

A Nash equilibrium is a vector of effort levels, at which each agent's effort level is a best response to his opponents'. Due to the symmetry of the objective functions, agents of the same type choose the same effort level. Let x_H and x_L denote the equilibrium effort levels for two types of agents, respectively, and assume that the first order conditions are strictly binding. With $n_H P_H + n_L P_L = 1$, this implies that

$$P_H = \frac{n_L \lambda_H - (n_L - 1)\lambda_L}{n_H \lambda_L + n_L \lambda_H}$$
$$P_L = \frac{n_H \lambda_L - (n_H - 1)\lambda_H}{n_H \lambda_L + n_L \lambda_H}$$

and they jointly determine the equilibrium effort ratio

$$\frac{x_L^*}{x_H^*} = [\frac{n_H \lambda_L - (n_H - 1)\lambda_H}{n_L \lambda_H - (n_L - 1)\lambda_L}]\frac{\lambda_H}{\lambda_L}. \tag{3}$$

Since $\lambda_H \geq \lambda_L > 0$, we must have $\frac{x_L^*}{x_H^*} \leq 1$. And since $x_i^* \geq 0$ by assumption, $\frac{x_L^*}{x_H^*} \geq 0$ must hold in an interior Nash equilibrium. Therefore, $\frac{\lambda_L}{\lambda_H} \geq 1 - \frac{1}{n_H}$ is the necessary and sufficient condition for interior Nash equilibrium. Denote the cutoff point $\lambda_0 = 1 - \frac{1}{n_H} \in [0, 1)$, and the lobbying effectiveness ratio $\lambda = \frac{\lambda_L}{\lambda_H} \in (0, 1]$. When $\lambda < \lambda_0$, we have $\frac{x_L^*}{x_H^*} < 0$; the first order optimality condition is binding for agents with high type but not for agents with low type, which implies $\frac{\lambda_H}{\sum \lambda_j x_j}(1 - p_H)V - 1 = 0$ and $\frac{\lambda_L}{\sum \lambda_j x_j}(1 - p_L)V - 1 \leq 0$. That is, given the numbers of both types of agents, if λ is sufficiently small, agents with low type will invest zero and drop out of the competition. Thus, equilibrium effort levels for agents of both types are given by

$$x_H^* = \begin{cases} \frac{(n_H + n_L - 1)[n_L \lambda_H - (n_L - 1)\lambda_L]\lambda_L}{(n_H \lambda_L + n_L \lambda_H)^2} V & \lambda \geq \lambda_0 \\ \frac{1}{n_H}(1 - \frac{1}{n_H})V & \lambda < \lambda_0 \end{cases} \tag{4}$$

and

$$x_L^* = \begin{cases} \frac{(n_H+n_L-1)[n_H\lambda_L-(n_H-1)\lambda_H]\lambda_H}{(n_H\lambda_L+n_L\lambda_H)^2}V & \lambda \geq \lambda_0 \\ 0 & \lambda < \lambda_0 \end{cases} \tag{5}$$

It is worth noting, from (4) and (5), that individual equilibrium effort levels depend on two kinds of parameters: lobbying effectiveness (λ_L and λ_H) and group composition (n_L and n_H). We measure the level of type heterogeneity by the lobbying effectiveness ratio λ. Intuitively, contestants are extremely heterogeneous if λ is close to zero, and become homogeneous if $\lambda = 1$. A larger λ corresponds to a lower level of type heterogeneity, which implies a higher level of competitiveness. It is easy to see that if $n_L = 0$, or $n_H = 0$ or $\lambda = 1$, there is no type heterogeneity. When $n_i \geq 1$ ($i \in \{L, H\}$) and $\lambda \in (0, 1)$, each agent's equilibrium effort level depends on both type heterogeneity and group composition.

The condition for interior solution also indicates that low-type agents will drop out of the competition, except in two cases: if that the level of type heterogeneity is sufficiently low (so that $\lambda \geq \lambda_0$), or there exists only one high-type agent (implying $\lambda_0 \equiv 1 - \frac{1}{n_H} = 0$, so that $\lambda \geq \lambda_0$ is always satisfied). Stein (2002) investigates a more general N-player rent-seeking contest with different valuations and abilities among the players, identifies the condition for the number of active players, and describes the equilibrium effort. Our analysis echoes his results by assuming that n_L out of N players have the same low ability and the remaining n_H players have the same high ability. This more specific setup allows us to investigate the effect of group composition, which could not be feasibly analyzed by Stein (2002).

From equations (1) to (3) we can compute each agent's winning probability $P_i(x_L^*, x_H^*) = \frac{\lambda_i x_i^*}{n_L \cdot \lambda_L x_L^* + n_H \cdot \lambda_H x_H^*}$ and expected payoff $x_i^* \cdot \left[\frac{\lambda_i V}{n_L \cdot \lambda_L x_L^* + n_H \cdot \lambda_H x_H^*} - 1\right]$. Note that $\lambda_H \geq \lambda_L > 0$ implies $x_H^* \geq x_L^*$; agents with high ability will be more willing to expend effort, so they have a greater chance to win and a larger expected payoff in equilibrium.

We can also obtain the total effort of all agents

$$TE\left(n_L, n_H, \lambda\right) = n_L \cdot x_L^* + n_H \cdot x_H^*, \tag{6}$$

where x_L^* and x_H^* are given by (4) and (5).

In the following subsections, we will conduct comparative statics analysis to examine how the rent dissipation ratios (and individual effort levels) and total effort level respond to changes in agents' type heterogeneity and group composition.

2.3 Individual Effort

As shown in equations (4) and (5), each agent's equilibrium effort level depends on the numbers of each type of agent (n_L and n_H) and the level of type heterogeneity (λ). As pointed out by Baik (2004), the rent dissipation ratio, which is the proportion of each agent's equilibrium effort level to his valuation for the rent, measures how much effort is "dissipated" in pursuit of the rent. In our model of two types of agents, the rent dissipation ratio for high type is

$$\frac{x_H^*}{V} = \begin{cases} \frac{(n_H+n_L-1)[n_L\lambda_H-(n_L-1)\lambda_L]\lambda_L}{(n_H\lambda_L+n_L\lambda_H)^2} & \lambda \geq \lambda_0 \\ \frac{1}{n_H}(1-\frac{1}{n_H}) & \lambda < \lambda_0 \end{cases}$$

and for low type is

$$\frac{x_L^*}{V} = \begin{cases} \frac{(n_H+n_L-1)[n_H\lambda_L-(n_H-1)\lambda_H]\lambda_H}{(n_H\lambda_L+n_L\lambda_H)^2} & \lambda \geq \lambda_0 \\ 0 & \lambda < \lambda_0 \end{cases}$$

It has been shown that $\frac{x_L^*}{x_H^*} \leq 1$ according to equation (3), and thus we have the following property when comparing rent dissipation ratios, summarized in Lemma 1.

Lemma 1. *The rent dissipation ratio for a high-type agent is no less than the rent dissipation ratio for a low-type agent. They are equal if and only if $\lambda = 1$ or $n_H = n_L = 1$.*

Baik (2004) argues that in two-player asymmetric contests with a ratio-form contest success function, the equilibrium effort ratio is equal to the valuation ratio, and hence the rent dissipation ratios for the two players are the same. Lemma 1 shows that Baik's (2004) argument about rent dissipation ratios cannot be generalized to asymmetric contests with more than two players. In a contest with two types of multiple participants, a high-type agent tries harder than a low-type agent, and the rent dissipation ratio for high-type agents is always greater than the rent dissipation ratio for low-type agents.[8]

[8] We model type heterogeneity in terms of lobbying effectiveness, which is a mapping from effort to win-

The comparative statics properties of rent dissipation ratios with respect to type heterogeneity and group composition are summarized by Propositions 1 and 2, respectively.

Proposition 1. *Fix n_L and n_H.*

(a) The rent dissipation ratio for high-type agent, $\frac{x_H^*}{V}$,

(a1) if $n_H = 1$, *is strictly increasing with* λ *when* $\lambda \in (0, \frac{n_L}{2n_L-1}]$, *and strictly decreasing with* λ *when* $\lambda \in (\frac{n_L}{2n_L-1}, 1]$, *with a maximized level of* $\frac{1}{4}$;

(a2) if $n_H \geqslant 2$, *is independent of* λ *when* $\lambda \in (0, \lambda_0)$, *and strictly decreasing with* λ *when* $\lambda \in [\lambda_0, 1]$, *with a maximized level of* $\frac{1}{n_H}(1 - \frac{1}{n_H})$.

(b) The rent dissipation ratio for low-type agent, $\frac{x_L^*}{V}$, *is constant at* 0 *when* $\lambda \in (0, \lambda_0)$, *and increasing with* λ *when* $\lambda \in [\lambda_0, 1]$.

Proof. See Appendix. □

The results in Proposition 1 are consistent with Stein (2002), wherein Stein's analysis is restricted by a fixed number of active agents: The incentive effect of type heterogeneity is always nonpositive for low-type agents, while it can be positive for high-type agents. Compared to Stein's framework, our setup, though less general, allows us to more closely explore the potential positive effect of type heterogeneity on rent dissipation for high-type agents under different scenarios of group combination. For example, the inverse U-shaped relationship between type heterogeneity and high-type agents' effort level is especially true when there is only one high-type agent and more than one low-type agent (with a turning point of $\lambda = \frac{n_L}{2n_L-1} \in (0, 1)$).

An interpretation for this result is the following: When there is only one high-type agent, none of the low-type agents will drop out in equilibrium, so the competitive pressure on the high-type agent comes solely from all the low-type agents. If the level of type heterogeneity is high (such that $\lambda \in (0, \frac{n_L}{2n_L-1}]$), the competitive pressure is in the moderate range; higher pressure induces higher effort for the high type (Encouraging Effect). If the level of type heterogeneity is low (such that $\lambda \in (\frac{n_L}{2n_L-1}, 1)$), the competitive pressure from low-type agents becomes crucial to the high-type agent; the overwhelming competing pressure weakens the effort-making incentive (Discouraging Effect).

ning probabilities. Baik (2004), among many others, uses valuation or effort cost to measure heterogeneity, which are analogous. Results under different settings are available from the authors upon request.

Also, note that when there is more than one high-type agent, the incentive effect of type heterogeneity is always nonnegative for high-type agents, while when there is exactly one high-type agent and one low-type agent, such an incentive effect is always negative ($\frac{n_L}{2n_L-1} = 1$ when $n_L = 1$).

Although our two-group competition model can be viewed as a simplified setting of Stein's (2002) framework, it allows us to conduct comparative statics studies with *changeable* active participants. More importantly, we can further explore the effect of group composition on equilibrium rent dissipation ratios, which is new to the literature and summarized by Proposition 2.

Proposition 2. *Fix $\lambda \in (0,1)$.*

(a) Consider the rent dissipation ratio for a high-type agent $\frac{x_H^*}{V}$.

(a1) When $n_H = 1$,

If $\lambda \in (0, 1/2]$, $\frac{x_H^*}{V}$ is increasing with n_L.

If $\lambda \in (1/2, 1]$, $\frac{x_H^*}{V}$ is first increasing with n_L when $1 \leq n_L \leq \frac{\lambda}{2\lambda-1}$ and then decreasing with n_L when $n_L > \frac{\lambda}{2\lambda-1}$.

(a2) When $n_H \geqslant 2$, given n_L (n_H), $\frac{x_H^*}{V}$ is nonincreasing with n_H (n_L).

(b) Given n_L (n_H), the rent dissipation ratio for a low-type agent, $\frac{x_L^*}{V}$, is constant at 0 when $\lambda < \lambda_0$, and strictly decreasing with n_H (n_L) when $\lambda \geq \lambda_0$.

Proof. See Appendix. □

The results of Proposition 2 contribute to the literature by providing a first examination of study how group composition affects rent dissipation ratios while holding type heterogeneity constant. Our results show that group composition affects rent dissipation ratios in the same direction for both high-type and low-type agents, except when there is a unique high-type agent.

When there is more than one high-type agent, a marginal increase in the number of agents of any type can guarantee that there will still be multiple high-type agents after the change in group composition. This means that every agent, regardless of his type, always faces competitive pressure from high-type agents. Therefore, as more agents join the competition, the effective size of competitors is increasing and every agent is subject to higher competitive pressure. A composition trend in this direction disincentivizes both types of agents, as they anticipate a lower chance of winning and expect a smaller share of the rent, and hence decreases the equilibrium effort level.

However, when there is a unique high-type agent, he is no longer subject to competitive pressure from any other high-type agent. If the level of type heterogeneity is high or if the number of low-type agents is small, the overall competitive environment is considered to be moderate, and therefore the unique high-type agent will have incentive to bid harder when more low-type agents join the competition (Encouraging Effect). If the level of type heterogeneity is small and the number of low-type agents is large, an overwhelming competition with more agents will weaken the high-type agent's effort-making incentive (Discouraging Effect).

As seen from part (a) of Propositions 1 and 2, the number of high-type agents plays a crucial role in determining high-type agents' equilibrium effort level. With multiple high-type agents, when $\lambda \to 1$ or when n_H increases, one can think of the case in which each high-type agent faces more and more similar or identical competitors, and therefore it induces a lower level of equilibrium effort for the high type. In contrast, where there is only one high-type agent, he will slack off in a less intense competition, due to lack of competitive pressure from identical competitors.

If we consider the effect of type heterogeneity and group composition simultaneously, the optimization result of rent dissipation ratios is shown in the following Corollary.

Corollary 1. *(a) The rent dissipation ratio for high-type agents achieves the maximum value* $\frac{1}{4}$ *at* $n_L = n_H = 1, \lambda = 1$ *or* $n_H = 2, \lambda \leq \frac{1}{2}$.

(b) The rent dissipation ratio for low-type agents achieves the maximum value $\frac{1}{4}$ *at* $n_L = n_H = 1, \lambda = 1$.

2.4 Total Effort

In this section, we examine how the total effort level responds when type heterogeneity and group composition change. From expressions (4)-(6), we obtain the total effort of a one-stage grand contest

$$
\begin{aligned}
TE_{grand} &= n_L \cdot x_L^* + n_H \cdot x_H^* \\
&= \begin{cases} \frac{(n_H+n_L-1)n_L n_H[-(\lambda_H-\lambda_L)^2+\frac{1}{n_L}\lambda_L^2+\frac{1}{n_H}\lambda_H^2]}{(n_H\lambda_L+n_L\lambda_H)^2}V & \lambda \geq \lambda_0 \\ (1-\frac{1}{n_H})V & \lambda < \lambda_0 \end{cases} .
\end{aligned}
$$

For simplification, define

$$
\begin{aligned}
\overline{TE}_1\left(n_L, n_H, \lambda\right) &= \frac{TE_{grand}}{V} \\
&= \begin{cases} \frac{(n_H+n_L-1)}{[n_H\lambda+n_L]^2} n_H n_L[-(1-\lambda)^2+\frac{\lambda^2}{n_L}+\frac{1}{n_H}] & \lambda \geq \lambda_0 \\ 1-\frac{1}{n_H} & \lambda < \lambda_0 \end{cases} .
\end{aligned}
$$

We can immediately deduce the following results from the above equations.

Proposition 3. *(a) Given n_L and n_H, the total equilibrium effort from all agents is independent of λ when $\lambda \in (0, \lambda_0)$, and strictly increasing with λ when $\lambda \in [\lambda_0, 1)$.*

(b) Given $\lambda \in (0, 1]$ and n_L, the monotonicity about n_H is complicated, while total equilibrium effort is ***eventually*** *strictly increasing with n_H. To be more specific,*

(b1) when $n_H > \min\left\{\frac{1}{1-\lambda}, n_L\right\}$, total equilibrium effort is strictly increasing with n_H;

(b2) when $n_H \leq \min\left\{\frac{1}{1-\lambda}, n_L\right\}$, there exisit two cutoff values of λ, $\lambda_H^{\min}$ and $\lambda_H^{\max}$ with $\lambda_0 < \lambda_H^{\min} < \lambda_H^{\max} < 1$, such that: (b2.1) If $\lambda_0 \leq \lambda < \lambda_H^{\min}$, total equilibrium effort is strictly decreasing with n_H; (b2.2) if $\lambda_H^{\min} \leq \lambda < \lambda_H^{\max}$, total equilibrium effort is first increasing then decreasing with n_H; (b2.3) if $\lambda_H^{\max} \leq \lambda < 1$, total equilibrium effort is strictly increasing with n_H

(c) Given $\lambda \in (0, 1]$ and n_H,

(c1) if $\lambda < \lambda_0$, total equilibrium effort is independent of n_L;

(c2) if $\lambda \geq \lambda_0$, total equilibrium effort is strictly increasing with n_L.

Proof. See Appendix. □

Proposition 3(a) describes the effect of type heterogeneity (λ) on equilibrium total effort level. Given group composition, only high-type agents are involved in the competition if λ is small enough such that $\lambda < \lambda_0$, and the total equilibrium effort from these active competitors is constant, regardless of λ. However, when λ is large enough such that $\lambda \geq \lambda_0$, although the equilibrium effort levels of the two types do not always move in the same direction (as shown in Proposition 1), total effort increases with λ because contestants' "strength" becomes more similar, and hence the competition becomes more fierce.

Propositions 3(b) and 3(c) characterize how the equilibrium total effort level depends on the number of high-type agents (n_H) and the number of low-type agents (n_L), respectively. Given type heterogeneity, the relationship between total effort level and n_L is weakly monotonic, while the relationship between total effort level and n_H is not necessarily so. When λ is relatively small ($\lambda \in (0, \lambda_0)$) or sufficiently large ($\lambda \in (\lambda_H^{\max}, 1)$), having more high-type agents always increases the total effort level. However, when $\lambda \in [\lambda_0, \lambda_H^{\max}]$ (cases b2.1 and b2.2), the tradeoff between the increase in the number of high-type agents and the decrease in the effort levels (of both high-type agents and low-type agents) is complicated, and the overall effect on total effort level is ambiguous.

Having carefully conducted the equilibrium analysis for the grand contest, we are now ready to move to the next section, in which the contest designer is allowed to construct a two-stage contest.

3 Contest Design

The equilibrium behavior of the bidders depends critically on the institutional elements of the contest. Central to the contest literature is the question of how the contest rules affect equilibrium bidding. As Gradstein and Konrad (1999) point out, "Contest structures result from the careful consideration of a variety of objectives, one of which is to maximize the effort of contenders." Following this literature and based on the equilibrium analysis, we consider the structural design of the contest to maximize the total effort of all agents. Specifically, we examine which one of the rules, grand contest or two-stage contest, should be selected to maximize expected total effort.

3.1 A Two-Stage Rent-seeking Contest

In this section, we set up a two-stage contest for two types of agents to compete for one monopoly rent. Agents of the same type are rigged into two competing groups.[9] In the first stage, an agent competes against contestants within his own group and a finalist is chosen from each group. In the second stage, the two finalists from different groups compete for the unique prize. The subgame perfect pure-strategy Nash equilibrium is obtained through backward induction. The effort of agent i in stage s is x_{is} and the corresponding expected payoff is π_{is}, where $s \in \{1, 2\}$.

- Stage Two

The pair-wise competition between agents i and j in the second stage is a standard Tullock two-agent contest with different lobbying effectiveness levels. Each agent chooses effort x_{i2} to maximize his expected payoff

$$\pi_{i2}\left(x_{i2}, x_{j2}\right) = \frac{\lambda_i x_{i2}}{\lambda_i x_{i2} + \lambda_j x_{j2}} V - x_{i2}.$$

The subgame perfect equilibrium efforts are $x^*_{H2} = x^*_{L2} = \frac{\lambda_H \lambda_L}{(\lambda_H + \lambda_L)^2} V$. Aggregate rent-seeking effort in the second stage is given by $x^*_{H2} + x^*_{L2}$. The equilibrium expected payoffs are $\pi_{H2} = (\frac{\lambda_H}{\lambda_H + \lambda_L})^2 V$ and $\pi_{L2} = (\frac{\lambda_L}{\lambda_H + \lambda_L})^2 V$, which are nonnegative.

- Stage One

In the first stage, given the outlay of the other agents in his own group, high-type agent i chooses $x^{(i)}_{H1}$ and low-type agent j chooses $x^{(j)}_{L1}$ to maximize their expected payoffs separately. Note that the "prize value" in the first stage is simply the expected payoff in the second stage. Given group composition, the general maximization problem for the high-type agent is

$$\pi^{(i)}_{H1}\left(x^{(1)}_{H1}, x^{(2)}_{H1}, \dots x^{(n_H)}_{H1}\right) = \frac{\lambda_H x^{(i)}_{H1}}{\sum_{k=1}^{n_H} \lambda_H x^{(k)}_{H1}} \pi_{H2} - x^{(i)}_{H1},$$

[9] We focus on the case in which types are separated in the first stage. A full characterization of the optimal two-stage contest would investigate how the preliminary stage should be designed, i.e., optimal grouping, which is disregarded here and left for future work.

and for the low-type agent is

$$\pi_{L1}^{(j)}\left(x_{L1}^{(1)},x_{L1}^{(2)},\dots x_{L1}^{(n_L)}\right)=\frac{\lambda_L x_{L1}^{(j)}}{\sum_{k=1}^{n_L}\lambda_L x_{L1}^{(k)}}\pi_{L2}-x_{L1}^{(j)}.$$

The symmetric equilibrium efforts are $x_{H1}^{*}=\frac{1}{n_H}(1-\frac{1}{n_H})(\frac{\lambda_H}{\lambda_H+\lambda_L})^2V$ and $x_{L1}^{*}=\frac{1}{n_L}(1-\frac{1}{n_L})(\frac{\lambda_L}{\lambda_H+\lambda_L})^2V$. The corresponding expected payoffs are $\pi_{H1}=\frac{1}{n_H^2}(\frac{\lambda_H}{\lambda_H+\lambda_L})^2V$ and $\pi_{L1}=\frac{1}{n_L^2}(\frac{\lambda_L}{\lambda_H+\lambda_L})^2V$, which are nonnegative.

The total rent-seeking effort of the two-stage contest is

$$\begin{aligned}TE_{two-stage}&=n_Hx_{H1}^{*}+n_Lx_{L1}^{*}+x_{H2}^{*}+x_{L2}^{*}\\&=[1-\frac{(\frac{\lambda_H^2}{n_H}+\frac{\lambda_L^2}{n_L})}{(\lambda_H+\lambda_L)^2}]V.\end{aligned}$$

Define

$$\begin{aligned}\overline{TE}_2\left(n_L,n_H,\lambda\right)&=\frac{TE_{two-stage}}{V}\\&=1-\frac{1}{(1+\lambda)^2}(\frac{1}{n_H}+\frac{\lambda^2}{n_L}).\end{aligned}\tag{7}$$

We can get the following properties with regard to type heterogeneity and group composition.

Proposition 4. *(a) Given n_L and n_H, the total equilibrium effort from all agents in a two-stage contest is*

(a1) first increasing with $\lambda\in(0,\frac{n_L}{n_H}]$, then decreasing with $\lambda\in(\frac{n_L}{n_H},1]$, if $n_L\le n_H$;

(a2) strictly increasing with $\lambda\in(0,1]$, if $n_L>n_H$.

(b) Given λ and $n_L\left(n_H\right)$, the total equilibrium effort from all agents in a two-stage contest is strictly increasing with $n_H\left(n_L\right)$.

Proof. See Appendix. □

Total effort of the two-stage contest is strictly increasing with group size, while the effect of type heterogeneity depends on the features of group composition: When there are more high-type agents than low-type agents, total effort of the two-stage contest first increases and then decreases with type heterogeneity, exhibiting an inverted U-shape relation; When there are fewer high-type agents than low-type agents, type heterogeneity reduces total effort of the two-stage contest.

Intuitively, the two-stage contest is *less accurate* in selecting the winner, as it guarantees that a weak agent will enter the second stage. Heterogeneity reduces the stage-2 effort of strong agents while increasing the effort provision from stage 1, since heterogeneity in stage 2 will increase the continuation value for which strong agents compete in stage 1. Overall, the net effect of type heterogeneity on total effort is nonmonotonic.

3.2 Total Effort Maximization

For ease of comparison, we have normalized total effort by a factor of V, defined by $\overline{TE}_1(n_L, n_H, \lambda)$ and $\overline{TE}_2(n_L, n_H, \lambda)$. Note that low-type agents may drop out in a grand contest when the level of type heterogeneity is high (i.e., $\lambda < \lambda_0$), but this is not the case in a two-stage contest. In the next two subsections, we explore the dominant contest structure when the level of type heterogeneity is above or below the critical value λ_0.

3.2.1 Dominant Contest when $\lambda \geq \lambda_0$

When λ exceeds the critical value $\lambda_0 \equiv 1 - \frac{1}{n_H}$, all agents participate in a grand contest. Then we have

$$\begin{aligned}\overline{TE}_1 - \overline{TE}_2 = &-\frac{1}{(n_H\lambda + n_L)^2(1+\lambda)^2} \\ &[\frac{n_H}{n_L}(n_L - 1)(n_L^2 + n_H n_L + n_H - n_L)\lambda^4 \\ &+ 2n_H n_L(2 - n_H - n_L)\lambda^2 \\ &+ \frac{n_L}{n_H}(n_H - 1)(n_H^2 + n_L n_H + n_L - n_H)].\end{aligned} \tag{8}$$

The roots of the equation $\overline{TE}_1 - \overline{TE}_2 = 0$ are

$$\lambda_1 = \sqrt{\frac{1-\frac{1}{n_H}}{1-\frac{1}{n_L}}}, \tag{9}$$

$$\lambda_2 = \sqrt{\frac{1+\frac{1}{n_L+n_H-1}\frac{n_L}{n_H}}{1+\frac{1}{n_L+n_H-1}\frac{n_H}{n_L}}}. \tag{10}$$

Note that only one of the roots is smaller than 1, and λ_1 is no less than λ_0. If $n_H > n_L$, λ_2 is chosen; otherwise, λ_1 is chosen. Considering all possible cases, we have the following result.

Lemma 2. *(a) If $n_H < n_L$, then $0 < \lambda_0 < \lambda_1 < 1 < \lambda_2$. The grand contest is inferior to the two-stage contest when $\lambda \in [\lambda_0, \lambda_1)$; equivalent when $\lambda = \lambda_1$; and superior when $\lambda \in (\lambda_1, 1]$.*

(b) If $n_H = n_L$, then $\lambda_1 = \lambda_2 = 1$. The grand contest is inferior to the two-stage contest when $\lambda \in [\lambda_0, 1)$ and equivalent when $\lambda = 1$.

(c) If $n_H > n_L$, there are two possibilities.

(c1) If $n_L < n_H \leq 3n_L$, then $0 < \lambda_0 < \lambda_2 < 1 < \lambda_1$. The grand contest is inferior to the two-stage contest when $\lambda \in [\lambda_0, \lambda_2)$; equivalent when $\lambda = \lambda_2$; and superior when $\lambda \in (\lambda_2, 1]$.

(c2) If $n_H \geq 3n_L + 1$, then $0 < \lambda_2 < \lambda_0 < 1 < \lambda_1$. The grand contest is superior to the two-stage contest when $\lambda \in [\lambda_0, 1]$.

Proof. See Appendix □

Lemma 2 summarizes the comparison of results for total effort between a grand contest and a two-stage contest under the condition of a low level of type heterogeneity. It indicates that in a pool of contestants with two types of agents, a grand contest dominates a two-stage contest when the number of high types is more than three times the number of low types or when asymmetry of abilities is small. However, when the proportion of low-type agents and the degree of asymmetry increase, a two-stage contest could elicit more effort.

3.2.2 Dominant Contest when $\lambda < \lambda_0$

When λ falls below the critical value $\lambda_0 \equiv 1 - \frac{1}{n_H}$, only high-type agents participate in a grand contest. Does this mean that a two-stage contest, which has more active participants and more stages, can elicit higher total expected effort? We write

$$\overline{TE}_1 - \overline{TE}_2 = \frac{(\frac{n_H}{n_L} - 1)\lambda^2 - 2\lambda}{n_H(1+\lambda)^2}. \tag{11}$$

The roots of the equation $\overline{TE}_1 - \overline{TE}_2 = 0$ are

$$\lambda_3 = \frac{2}{\frac{n_H}{n_L} - 1}, \tag{12}$$

$$\lambda_4 = 0. \tag{13}$$

If $n_H > n_L$, λ_3 is chosen, otherwise λ_4 is chosen. Considering all possible cases, we have the following result.

Lemma 3. *(a) If $n_H < n_L$, then $\lambda_3 < 0$. Thus the grand contest is inferior to the two-stage contest when $\lambda \in (0, \lambda_0)$.*

(b) If $n_H = n_L$, then the grand contest is inferior to the two-stage contest when $\lambda \in (0, \lambda_0)$.

(c) If $n_H > n_L$, there are two possibilities.

(c1) If $n_L < n_H \leq 3n_L$, then $\lambda_3 > \lambda_0$, the grand contest is inferior to the two-stage contest when $\lambda \in (0, \lambda_0)$.

(c2) If $n_H \geq 3n_L + 1$, then $\lambda_3 < \lambda_0$, the grand contest is inferior to the two-stage contest when $\lambda \in (0, \lambda_3)$; equivalent when $\lambda = \lambda_3$; and superior when $\lambda \in (\lambda_3, \lambda_0)$.

Proof. See Appendix. □

Lemma 3 summarizes the comparison of results for total effort between a grand contest and a two-stage contest under the condition of a high level of type heterogeneity. In such a case, low-type agents drop out of the grand contest. However, this does not mean that a two-stage contest, which has more active participants and more stages, can always elicit higher total expected effort. As shown in Proposition 3, the net effect of type heterogeneity on total effort is nonmonotonic. The grand contest can be dominant if $n_H \geq 3n_L + 1$, and $\lambda \in (\lambda_3, \lambda_0)$. Therefore, when the number of high types is more than three times the number of low types and the level of type heterogeneity is insignificant, a two-stage contest may induce less total effort than a grand contest, in which only high-type agents compete among themselves.

Combining the results of Lemma 2 and 3, we are able to rank the cutoffs of λ accordingly: (a) if $n_H < n_L$, then $\lambda_3 < 0 < \lambda_0 < \lambda_1 < 1 < \lambda_2$; (b) if $n_H = n_L$, then $\lambda_1 = \lambda_2 = 1$; (c) If $n_L < n_H \leq 3n_L$, then $0 < \lambda_0 < \lambda_2 < 1 < \min\{\lambda_1, \lambda_3\}$; (d) If $n_H \geq 3n_L + 1$, then $0 < \lambda_3 < \lambda_2 < \lambda_0 < 1 < \lambda_1$.[10]

Define

$$\lambda^* \equiv \begin{cases} \lambda_1 & n_H < n_L \\ \lambda_2 & n_L \leq n_H \leq 3n_L \\ \lambda_3 & n_H \geq 3n_L + 1 \end{cases} .$$

The dominant total-effort-maximizing contest structure between the grand contest and the two-stage contest can be stated as follows:

Theorem 1. *Given group composition (n_L, n_H) and type heterogeneity λ, the grand contest is inferior to the two-stage contest when $\lambda \in (0, \lambda^*)$; they are equivalent when $\lambda = \lambda^*$; and the grand contest is superior to the two-stage contest when $\lambda \in (\lambda^*, 1]$.*

Amegashie (1999) and Gradstein and Konrad (1999) carefully examine the optimal contest structure in Tullock contests with the same group size and show that different contest structures are all equivalent in a lottery contest with homogeneous agents. Gradstein (1998) further demonstrates the superiority of two-stage contests with heterogeneous agents. Given that the group composition is exogenously separated in the first stage, our result in Theorem 1 not only echoes their arguments, but further illustrates the dominant contest structure with unequal group size.

With equal group size, $\lambda^* = 1$, our analysis justifies Gradstein's (1998) results:[11] The two-stage contest elicits higher total effort than the grand contest. Furthermore, type heterogeneity makes the two-stage contest more advantageous in the case of $\lambda \geq \lambda_0$.

[10] Detailed proofs are available from the authors upon request.

[11] Gradstein (1998) assumes that the organizer is less patient than the contestants; in that case, the organizer's degree of impatience is among $(0, 1)$. Our paper assumes that the organizer is sufficiently patient, and therefore his discount factor is 1.

With unequal group size, Theorem 1 states that in general, neither the grand contest nor the two-stage contest is superior to the other in all cases. The dominant contest structure mainly depends on two factors: type heterogeneity and group composition. A grand contest outperforms a two-stage contest when type heterogeneity is relatively small: $\lambda > \lambda^*$, where the value of λ^* is determined by group composition (n_L, n_H). When the type heterogeneity is relatively large ($\lambda < \lambda^*$), the two-stage contest will become dominant. Figure 1 summarizes the main results of the contest design problem.

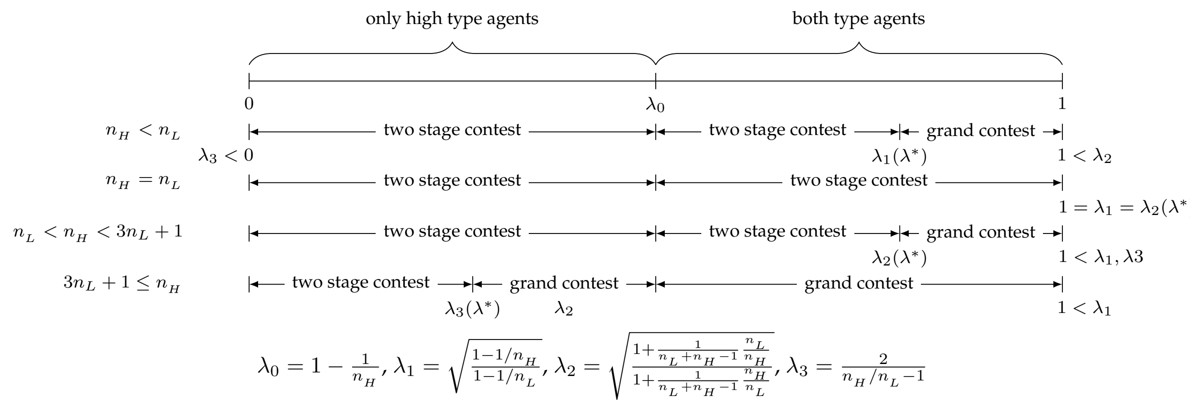

Figure 1: Dominant contest structure

Proposition 5. *When $n_H \leq 3n_L$, λ^* is increasing with n_H and decreasing with n_L; When $n_H \geq 3n_L + 1$, λ^* is decreasing with n_H and increasing with n_L.*

Proof. See Appendix. □

Proposition 5 describes the comparative statics results of how the cutoff value λ^* depends on the change in group composition. If the size of the high-type group is less (more) than three times the size of the low-type group, expanding the high-type group or shrinking the low-type group will make the two-stage contest more (less) preferable.

Note that when $n_H = 1 < n_L$, we have $\lambda^* = \lambda_0 = \lambda_1 = 0$. Moreover, when n_H and n_L go to infinity and $n_H \leq 3n_L$, cutoff values λ_0, λ_1, and λ_2 will approach 1, and we can obtain the following corollary immediately.

Corollary 2. *(a) When $n_H = 1$, the total effort elicited from grand contest is no less than in the two-stage contest; they are equivalent if and only if $n_H = n_L = 1$.*

(b) When n_H and n_L are sufficiently large, and the number of high types is not overwhelming $(n_H \leq 3n_L)$, the two-stage contest is always superior.

Findings in Corollary 2 have many economic applications: In the design of a job-promotion mechanism, if one employee's ability is significantly ahead of others, it is preferable for the manager to choose a grand contest to incentivize all employees to work hard. This partially explains why the heads of departments always thirst after talent in recruiting.

On the other hand, in R&D races with a large number of fresh PhDs and senior researchers competing for a outstanding achievement reward, it is probably better to divide contestants into two groups and conduct a two-stage contest. For example, in a research competition, such as the funding competition organized by National Natural Science Foundation of China, junior scholars and more senior scholars are divided into two groups by seniority and selected separately in a preliminary selection, after which the shortlisted candidates compete for the high distinction.

4 Conclusion and Discussion

This paper investigates the properties of a modified lottery rent-seeking model with multiple agents of two types, where both the heterogeneity of types and the combination of types play important roles. First, we derive the Nash equilibrium solution for this simultaneous move game, and examine how rent dissipation ratios and total effort level respond when agents' type heterogeneity or group composition changes. Our analysis extends the previous results of Stein (2002), in which the number of active agents is assumed to be fixed in the comparative statics analysis. We find that type heterogeneity and group composition do not necessarily negatively affect agents' effort levels. The number of high-type agents plays a crucial role in their effort provision. High-type agents tend to work harder in a moderate competition and bid less in an intense competition. These results enhance our understanding of the relationship between competitiveness and effort level in a contest environment.

Subsequently, we investigate whether organizing a two-stage contest—in which agents first compete within their own type, then the winners from each group compete in the second stage—can benefit a contest organizer who would like to maximize expected total effort. Given all possible conditions on type heterogeneity and group composition, the dominant contest structure for the contest organizer is fully characterized. Our work contributes to the literature on two-stage contest design, extending the results of Gradstein (1998) and Stracke (2013), among others. Our results suggest that the dominant contest structure in our analysis depend on both type heterogeneity and group composition. In general, if the ability gap between two types of agents is relatively significant, the two-stage contest dominates the grand contest. If the ability gap is relatively insignificant, the grand contest will outperform the two-stage contest.

This paper can be extended in several respects. A natural extension would be to consider a more general framework that allows for more than two groups and more than two stages. Regarding optimal contest design with a preliminary stage, a full characterization would investigate how this preliminary stage should be designed. Important questions could be: When is it optimal to separate high-type and low-type agents, and when should they be mixed? How many agents from each group should be selected as finalists? Our analysis relies on several key assumptions. We have discussed the restrictions of our model, and these simplifications are made mainly for ease of analysis, without compromising our main results and insights. Relaxation of these assumptions to allow for a more general analysis of rent-seeking models is left to future research.

Appendix

Proof of Proposition 1

Proof. Define the rent dissipation ratio for high type $\bar{x}_H = x_H/V$, and for low type $\bar{x}_L = x_L/V$. We have

$$\frac{\partial \bar{x}_H}{\partial \lambda} = \begin{cases} \frac{n_L(n_H+n_L-1)[n_L-(n_H+2n_L-2)\lambda]}{(n_H\lambda+n_L)^3} & \lambda \geq \lambda_0 \\ 0 & \lambda < \lambda_0 \end{cases}$$

and

$$\frac{\partial \bar{x}_L}{\partial \lambda} = \begin{cases} \frac{n_H(n_H+n_L-1)[(1-\lambda)n_H+(n_H+n_L-2)]}{(n_H\lambda+n_L)^3} & \lambda \geq \lambda_0 \\ 0 & \lambda < \lambda_0 \end{cases}$$

Note that $\partial\bar{x}_L/\partial\lambda \geqslant 0$ for all $\lambda \in (0,1]$. The critical value of λ for $\frac{\partial\bar{x}_H}{\partial\lambda} = 0$ is $\lambda_{x_H} = \frac{n_L}{n_H+2n_L-2}$; let $\lambda_{x_H} > \lambda_0$ and we get

$$(n_H - 2)(n_H + n_L - 1) < 0,$$

which is satisfied only if $n_H = 1$.

Therefore, if $n_H = 1$, $\frac{\partial\bar{x}_H}{\partial\lambda} > 0$ when $\lambda \in [\lambda_0, \frac{n_L}{2n_L-1}]$, and $\frac{\partial\bar{x}_H}{\partial\lambda} < 0$ when $\lambda \in (\frac{n_L}{2n_L-1}, 1]$, with a maximized level of $\frac{1}{4}$ at $\lambda = \frac{n_L}{2n_L-1}$; if $n_H \geqslant 2$, $\frac{\partial\bar{x}_H}{\partial\lambda} < 0$ when $\lambda \in [\lambda_0, 1]$, with a maximized level of $\frac{1}{n_H}(1 - \frac{1}{n_H})$ at $\lambda = \lambda_0$.

□

Proof of Proposition 2

Proof. First take derivative w.r.t n_H

$$\frac{\partial\bar{x}_H}{\partial n_H} = \begin{cases} \frac{[n_L-(n_L-1)\lambda]\lambda}{(n_H\lambda+n_L)^3}\left[n_L - (n_H + 2n_L - 2)\lambda\right] & \lambda \geq \lambda_0 \\ \frac{1}{n_H^3}(2 - n_H) & \lambda < \lambda_0 \end{cases}$$

and

$$\frac{\partial\bar{x}_L}{\partial n_H} = \begin{cases} \frac{n_L-(n_L-1)\lambda}{(n_H\lambda+n_L)^3}\left[n_H\lambda - (2n_H + n_L - 2)\right] & \lambda \geq \lambda_0 \\ 0 & \lambda < \lambda_0 \end{cases}$$

Note that the critical value of λ for $\frac{\partial\bar{x}_H}{\partial n_H} = 0$ is $\lambda_{x_H} = \frac{n_L}{n_H+2n_L-2}$. As shown in the proof of Proposition 1, $\lambda_{x_H} \leqslant \lambda_0$ if and only if $n_H \geqslant 2$. Therefore, we have $\frac{\partial\bar{x}_H}{\partial n_H} < 0$ for any $n_H \geqslant 2$. Moreover, $\frac{\partial\bar{x}_L}{\partial n_H} < 0$ always holds, since $n_H\lambda - (2n_H + n_L - 2) < 0$ for any $n_H \geq 1, n_L \geq 1$.

Then take derivative w.r.t n_L:

$$\frac{\partial\bar{x}_H}{\partial n_L} = \begin{cases} \frac{[n_H\lambda-(n_H-1)]\lambda}{(n_H\lambda+n_L)^3}\left[n_L - (n_H + 2n_L - 2)\lambda\right] & \lambda \geq \lambda_0 \\ 0 & \lambda < \lambda_0 \end{cases}$$

and

$$\frac{\partial\bar{x}_L}{\partial n_L} = \begin{cases} \frac{n_H\lambda-(n_H-1)}{(n_H\lambda+n_L)^3}\left[n_H\lambda - (2n_H + n_L - 2)\right] & \lambda \geq \lambda_0 \\ 0 & \lambda < \lambda_0 \end{cases}$$

Note that $n_H\lambda - (n_H - 1) \geq 0$, since $\lambda \geq \lambda_0$. Analysis of n_H applies here again. To be more specific, $\bar{x}_H$ and $\bar{x}_L$ will both decrease with n_L given $n_H \geqslant 2$.

However, if $n_H = 1$, $\frac{\partial \bar{x}_H}{\partial n_L} = \frac{\lambda^2}{(\lambda+n_L)^3}\left[(1-2\lambda)\, n_L + \lambda\right]$. $\frac{\partial \bar{x}_H}{\partial n_L} \geq 0$ when $\lambda \in (0, \frac{1}{2}]$ for any $n_L \geq 1$ or $\lambda \in (\frac{1}{2}, 1]$ and $1 \leq n_L \leq \frac{\lambda}{2\lambda-1}$; $\frac{\partial \bar{x}_H}{\partial \lambda} < 0$ when $\lambda \in (\frac{1}{2}, 1]$ and $n_L > \frac{\lambda}{2\lambda-1}$.

□

Proof of Proposition 3

Proof. Take $\overline{TE}_1$ first derivatives with respect to λ and n_H, n_L:

$$\frac{\partial \overline{TE}_1}{\partial \lambda} = \begin{cases} \frac{2(n_H+n_L-1)^2 n_H n_L}{[n_H\lambda+n_L]^3}(1-\lambda) & \lambda \geq \lambda_0 \\ 0 & \lambda < \lambda_0 \end{cases} \tag{14}$$

$$\frac{\partial \overline{TE}_1}{\partial n_H} = \begin{cases} -\frac{[n_L-(n_L-1)\lambda]}{[n_H\lambda+n_L]^3} G_{n_L,\lambda}(n_H) & \lambda \geq \lambda_0 \\ \frac{1}{n_H^2} & \lambda < \lambda_0 \end{cases} \tag{15}$$

$$\frac{\partial \overline{TE}_1}{\partial n_L} = \begin{cases} -\frac{[n_H\lambda-n_H+1]}{[n_H\lambda+n_L]^3} G_{n_H,\lambda}(n_L) & \lambda \geq \lambda_0 \\ 0 & \lambda < \lambda_0 \end{cases} \tag{16}$$

where

$$\begin{aligned} G_{n_L,\lambda}(n_H) &= n_H\left[(n_L-1)\lambda^2 - 3n_L\lambda + 2n_L\right] \qquad (17a)\\ &\quad - n_L\left[(n_L-1)\lambda - n_L + 2\right] \\ G_{n_H,\lambda}(n_L) &= n_L\left[2n_H\lambda^2 - 3n_H\lambda + n_H - 1\right] \qquad (17b)\\ &\quad + n_H\lambda\left(n_H\lambda - n_H - 2\lambda + 1\right) \end{aligned}$$

(a) Since Equation (14) is nonnegative for $\lambda \in (0, 1]$, $\overline{TE}_1$ is increasing with λ.

(b) When $\lambda < \lambda_0 (n_H > \frac{1}{1-\lambda})$, the property in (b1) can be obtained from Equation (15) directly.

When $\lambda \geq \lambda_0 (n_H \leq \frac{1}{1-\lambda})$, consider Equation (17a) as a quadratic function of λ; note that the quadratic function is convex and has the axis of symmetry

$$\lambda_s = \frac{3n_L n_H + n_L(n_L - 1)}{2n_H(n_L - 1)} > 1.$$

Thus the function is monotonically decreasing in $\lambda \in (\lambda_0, 1]$. The function values at the two end points $\lambda = \{\lambda_0, 1\}$ are

$$G_{n_L,\lambda=1}(n_H) = -(n_H + n_L) < 0,$$

and

$$G_{n_L,\lambda=\lambda_0}(n_H) = -\frac{1}{n_H}(n_H + n_L - 1)(n_H - n_L - 1),$$

respectively. We consider the following two cases:

(i) If $n_H \geq n_L + 1$, we have

$$G_{n_L,\lambda=\lambda_0}(n_H) \leq 0,$$

Since neither $G_{n_L,\lambda=\lambda_0}(n_H)$ nor $G_{n_L,\lambda=1}(n_H)$ is greater than 0, we conclude that $G_{n_L,\lambda}(n_H) \leq 0$ for any $\lambda \in (\lambda_0, 1)$, and $\overline{TE}_1$ is increasing with n_H.

(ii) If $n_H \leq n_L$, we have

$$G_{n_L,\lambda=\lambda_0}(n_H) > 0,$$

Since $G_{n_L,\lambda=\lambda_0}(n_H)$ and $G_{n_L,\lambda=1}(n_H)$ have opposite signs, the function must have a zero point λ_H in the region $(\lambda_0, 1]$, where

$$\lambda_H = \frac{(3n_L n_H + n_L(n_L - 1) - \sqrt{(n_L^2 + 8n_L)n_H^2 + (2n_L^3 + 6n_L^2 - 8n_L)n_H + n_L^2(n_L - 1)^2}}{2(n_L - 1)n_H}.$$

It is easy to verify that $\frac{\partial \lambda_H}{\partial n_H} > 0$. Thus we have $G_{n_L,\lambda}(n_H) \leq 0$ when $\lambda_H \leq \lambda \leq 1$; $G_{n_L,\lambda}(n_H) > 0$ when $\lambda_0 < \lambda < \lambda_H$. Thus, $\overline{TE}_1$ is increasing with n_H when $\lambda_H \leq \lambda$ and decreasing with n_H when $\lambda_H \geq \lambda$.

When $n_H = 1$, we have $\lambda_H = \frac{(n_L + 2 - \sqrt{n_L^2 + 8})n_L}{2(n_L - 1)} \equiv \lambda_H^{\min}$. When $n_H = n_L$, we have $\lambda_H = \frac{4n_L - 1 - \sqrt{4n_L^2 + 12n_L - 7}}{2(n_L - 1)} \equiv \lambda_H^{\max}$.

Therefore, If $\lambda_H^{\max} < \lambda$, $\overline{TE}_1$ is increasing with n_H for $n_H \leq n_L$; If $\lambda_H^{\min} > \lambda$, $\overline{TE}_1$ is decreasing with n_H for $n_H \leq n_L$; If $\lambda_H^{\min} < \lambda < \lambda_H^{\max}$, $\overline{TE}_1$ is first increasing then decreasing with n_H.

(c) When $\lambda < \lambda_0$, it is easy to see that $\overline{TE}_1$ is independent of n_L.

When $\lambda \geq \lambda_0$, considering Equation (17b) as a quadratic convex function of λ, we have

$$G_{n_H,\lambda=1}(n_L) = -(n_H + n_L) < 0,$$

and

$$G_{n_H,\lambda=\lambda_0}(n_L) = -2(n_H - 1)(\frac{n_L}{n_H} + 1 - \frac{1}{n_H}) \leq 0.$$

This function must not be greater than 0 for any $\lambda \in (\lambda_0, 1]$. Therefore, $\overline{TE}_1$ is increasing with n_L.

□

Proof of Proposition 4

Proof. Take $\overline{TE}_2$ first derivatives with respect to λ and n_H, n_L:

$$\frac{\partial \overline{TE}_2}{\partial \lambda} = -\frac{2\left[n_H\lambda - n_L\right]}{n_H n_L\left(1+\lambda\right)^3} \tag{18}$$

$$\frac{\partial \overline{TE}_2}{\partial n_H} = \frac{1}{n_H^2\left(1+\lambda\right)^2} \tag{19}$$

$$\frac{\partial \overline{TE}_2}{\partial n_L} = \frac{\lambda^2}{n_L^2\left(1+\lambda\right)^2} \tag{20}$$

Note that Equation (19) and Equation (20) are positive for any $\lambda \in (0, 1]$.

Moreover, $\overline{TE}_2$ is increasing with λ if $n_L > n_H$. Otherwise, it would be a reversed U-shaped function with respect to λ. □

Proof of Lemma 2

Proof. The sign of $\overline{TE}_1 - \overline{TE}_2$ is determined by the term in the square bracket in equation (8); define

$$f\left(\lambda^2\right) = \frac{n_H}{n_L}(n_L-1)(n_L^2+n_Hn_L+n_H-n_L)\lambda^4 \\ +2n_Hn_L(2-n_H-n_L)\lambda^2 + \frac{n_L}{n_H}(n_H-1)(n_H^2+n_Ln_H+n_L-n_H)$$

which can be considered as a quadratic function of λ^2. The quadratic function opens up and has two roots, λ_1^2 and λ_2^2. Thus $f\left(\lambda^2\right) < 0$ for $\lambda^2 \in \left(\lambda_1^2, \lambda_2^2\right)$, and $f\left(\lambda^2\right) > 0$ for $\lambda^2 < \lambda_1^2$ or $\lambda^2 > \lambda_2^2$. $\overline{TE}_1 - \overline{TE}_2$ has the opposite sign with $f\left(\lambda^2\right)$.

(a) If $n_H < n_L$, we can find that $\lambda_1 < 1 < \lambda_2$ and

$$\lambda_1 = \sqrt{\frac{1-\frac{1}{n_H}}{1-\frac{1}{n_L}}} > \sqrt{1-\frac{1}{n_H}} = \lambda_0.$$

Then we have $\overline{TE}_1 - \overline{TE}_2 < 0$ when $\lambda \in [\lambda_0, \lambda_1)$; $\overline{TE}_1 - \overline{TE}_2 = 0$ when $\lambda = \lambda_1$; and $\overline{TE}_1 - \overline{TE}_2 > 0$ when $\lambda \in (\lambda_1, 1]$.

(b) If $n_H = n_L$, then $\lambda_1 = \lambda_2 = 1$. We have $\overline{TE}_1 - \overline{TE}_2 < 0$ for any $\lambda \in [\lambda_0, 1)$, $\overline{TE}_1 - \overline{TE}_2 = 0$ when $\lambda = 1$.

(c) If $n_H > n_L$, we can find that $\lambda_2 < 1 < \lambda_1$. Thus, we have

$$\lambda_2^2 - \lambda_0^2 = \frac{1}{[(n_H+n_L-1)n_L+n_H]n_H^2} \\ \{n_H[(n_H+n_L-1)n_Hn_L+n_L^2] - [(n_H+n_L-1)n_L+n_H](n_H-1)^2\} \\ = -\frac{1}{[(n_H+n_L-1)n_L+n_H]n_H^2} \\ \{n_H^3 - 2(n_L+1)n_H^2 + (-3n_L^2+3n_L+1)n_H + n_L(n_L-1)\} \\ = -\frac{(n_H+n_L-1)}{[(n_H+n_L-1)n_L+n_H]n_H^2} \\ [n_H - \frac{1}{2}(3n_L+1+\sqrt{9n_L^2+2n_L+1})][n_H - \frac{1}{2}(3n_L+1-\sqrt{9n_L^2+2n_L+1})]$$

Since $\frac{1}{2}(3n_L + 1 + \sqrt{9n_L^2 + 2n_L + 1}) \in (3n_L, 3n_L + 1)$ and $\frac{1}{2}(3n_L + 1 - \sqrt{9n_L^2 + 2n_L + 1}) \in (0, 1)$, we obtain

(c1) If $n_L < n_H \leq 3n_L$, then $0 < \lambda_0 < \lambda_2 < 1 < \lambda_1$. We have $\overline{TE}_1 - \overline{TE}_2 < 0$ when $\lambda \in [\lambda_0, \lambda_2)$; $\overline{TE}_1 - \overline{TE}_2 = 0$ when $\lambda = \lambda_2$; and $\overline{TE}_1 - \overline{TE}_2 > 0$ when $\lambda \in (\lambda_2, 1]$.

(c2) If $n_H \geq 3n_L + 1$, then $0 < \lambda_2 \leq \lambda_0 < 1 < \lambda_1$. We have $\overline{TE}_1 - \overline{TE}_2 > 0$ for any $\lambda \in [\lambda_0, 1]$.

□

Proof of Lemma 3

Proof. The sign of $\overline{TE}_1 - \overline{TE}_2$ is determined by the numerator in equation (11), define

$$g(\lambda) = (\frac{n_H}{n_L} - 1)\lambda^2 - 2\lambda,$$

which can be considered as a quadratic function of λ when $n_H \neq n_L$. The quadratic function opens up when $n_H > n_L$ and down when $n_H < n_L$. The two roots of the quadratic function are λ_3 and $\lambda_4 = 0$.

(a) If $n_H < n_L$, then $g(\lambda)$ opens down and $\lambda_3 < 0$. Therefore $g(\lambda) < 0$ for any $\lambda \in (0, \lambda_0)$.

(b) If $n_H = n_L$, then $g(\lambda) = -2\lambda < 0$ for any $\lambda \in (0, \lambda_0)$.

(c) If $n_H > n_L$, then $g(\lambda)$ opens up. Since

$$\begin{aligned}
\lambda_3 - \lambda_0 &= \frac{2n_H - (\frac{n_H}{n_L} - 1)(n_H - 1)}{(\frac{n_H}{n_L} - 1)n_H} \\
&= \frac{2n_H n_L - (n_H - n_L)(n_H - 1)}{(n_H - n_L)n_H} \\
&= \frac{-n_H^2 + (3n_L + 1)n_H - n_L}{(n_H - n_L)n_H} \\
&= -\frac{1}{(n_H - n_L)n_H} \\
&\quad [n_H - \frac{1}{2}(3n_L + 1 + \sqrt{9n_L^2 + 2n_L + 1})][n_H - \frac{1}{2}(3n_L + 1 - \sqrt{9n_L^2 + 2n_L + 1})]
\end{aligned}$$

we have

(c1) If $n_L < n_H \leq 3n_L$, then $\lambda_3 > \lambda_0$. Therefore $g(\lambda) < 0$ for any $\lambda \in (0, \lambda_0)$.

(c2) If $n_H \geq 3n_L + 1$, then $\lambda_3 \leq \lambda_0$. Therefore $g(\lambda) < 0$ for $\lambda \in (0, \lambda_3)$, $g(\lambda) = 0$ for $\lambda = \lambda_3$, $g(\lambda) > 0$ for $\lambda \in (\lambda_3, \lambda_0)$.

□

Proof of Proposition 5

Proof. Since $\lambda^* = \begin{cases} \lambda_1 & n_H \leq n_L \\ \lambda_2 & n_L < n_H \leq 3n_L, \\ \lambda_3 & n_H \geq 3n_L + 1 \end{cases}$ it suffices to show that $\frac{\partial \lambda_1}{\partial n_H} > 0$, $\frac{\partial \lambda_1}{\partial n_L} < 0$; $\frac{\partial \lambda_2}{\partial n_H} > 0$, $\frac{\partial \lambda_2}{\partial n_L} < 0$; $\frac{\partial \lambda_3}{\partial n_H} < 0$, $\frac{\partial \lambda_3}{\partial n_L} > 0$.

Note that $\lambda_1 = \sqrt{\frac{(1-\frac{1}{n_H})}{(1-\frac{1}{n_L})}} > 0$. Since $\frac{\partial \lambda_1^2}{\partial n_H} = \frac{1}{(1-\frac{1}{n_L})n_H^2} > 0$ and $\frac{\partial \lambda_1^2}{\partial n_L} = -\frac{(1-\frac{1}{n_H})}{(1-\frac{1}{n_L})^2 n_L^2} < 0$, we have $\frac{\partial \lambda_1}{\partial n_H} > 0$, $\frac{\partial \lambda_1}{\partial n_L} < 0$.

Note that $\lambda_2 = \sqrt{\frac{1+\frac{1}{n_L+n_H-1}\frac{n_L}{n_H}}{1+\frac{1}{n_L+n_H-1}\frac{n_H}{n_L}}} > 0$. Thus we have

$$\frac{\partial \lambda_2^2}{\partial n_H} = \frac{n_H^2 n_L + n_H^2 - 2n_H n_L + 2n_H n_L^2 + n_L^3 - 3n_L^2}{(n_L^2 + n_H n_L - n_L + n_H)^2}$$

Since $1 \leq n_L < n_H$, we have $n_H^2 n_L + n_H^2 - 2n_H n_L > n_H^2 n_L + n_H^2 - 2n_H^2 = n_H^2 n_L - n_H^2 \geq 0$ and $2n_H n_L^2 + n_L^3 - 3n_L^2 \geq 2n_H n_L^2 - 2n_L^2 > 0$. Thus $\frac{\partial \lambda_2^2}{\partial n_H} > 0$, which implies $\frac{\partial \lambda_2}{\partial n_H} > 0$.

Similarly, we have

$$\frac{\partial \lambda_2^2}{\partial n_L} = -\frac{n_H^3 + 2n_H^2 n_L + n_H n_L^2 + n_L^2 - 2n_H n_L - 3n_H^2}{(n_L^2 + n_H n_L - n_L + n_H)^2}$$

Since $1 \leq n_L < n_H$, we have $n_H \geq 2$. When $n_H = 2$, $n_H^3 + 2n_H^2 n_L + n_H n_L^2 + n_L^2 - 2n_H n_L - 3n_H^2 = 3n_L^2 + 4n_L - 4 > 0$. When $n_H \geq 3$, $n_H^3 + 2n_H^2 n_L + n_H n_L^2 + n_L^2 - 2n_H n_L - 3n_H^2 \geq 3n_H^2 + 6n_H n_L + n_H n_L^2 + n_L^2 - 2n_H n_L - 3n_H^2 = 4n_H n_L + n_H n_L^2 + n_L^2 > 0$. Thus $\frac{\partial \lambda_2^2}{\partial n_L} < 0$, which implies $\frac{\partial \lambda_2}{\partial n_L} < 0$.

Note that $\lambda_3 = \frac{2}{\frac{n_H}{n_L} - 1}$, and it is easy to see that $\frac{\partial \lambda_3}{\partial n_H} = -\frac{2n_L}{(n_H - n_L)^2} < 0$ and $\frac{\partial \lambda_3}{\partial n_L} = \frac{2n_H}{n_L^2 (n_H - n_L)^2} > 0$. $\square$

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