Competition between Virtual Operators in a Two-sided Telecommunication Market

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Abstract

Consider a market where two large companies provide services to the population through "cloud" virtual operators which buy companies' services and sell them to clients. Each of the large companies determines a price, which it obtains through selling its services to virtual operators. The number of its clients and its resource (a characteristic of the company's attractiveness for clients), are known. The game process is a repetition of a two-step game in which virtual operators choose companies and prices for services. Each of the virtual operators needs to choose a company whose services he is going to sell and to define a price for the services to sell to clients. Each virtual operator chooses the probability of choosing the company and the price for services which they sell to clients, taking into account that clients' distribution among operators is given by the Hotelling specification. Each virtual operator at each step tries to maximize his

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payoff. We find the virtual operators' optimal strategies and consider the question: Can the system achieve some stationary state under the repeated two-step game, or does it form a repeating cycle of states?

Keywords: cloud operators, repeated two-step game, Hotelling specification, Nash equilibrium, stationary state

1 Introduction

A virtual operator is a service provider which does not have its own infrastructure and needs to enter into a business agreement with a real operator (e.g. network operator, internet or hosting provider, data center). It obtains access to an operator's resources at wholesale rates and provides services to customers at its own independent retail prices. Examples of such virtual operators are virtual network operators (e.g. mobile), or different cloud services using resource bases belonging to large internet/hosting providers.

We consider a market where two large telecommunications companies lend their resources (network, computing, communication). Each company i allocates some resource r_i and grants an access to it with a wholesale rate p_i per unit of the resource. The company i has its own client base (subscribers) of size n_i. Virtual operators lease companies' resources and provide services to their clients. Virtual operator j enters into a business agreement with company i with the probability x_{ij}. In particular $x_{ij} = 1$ means that a virtual operator j contracts with a company i and $x_{ij} = 0$ otherwise. Each virtual operator leasing resources obtains a utility C_j from using them and also obtains a payoff by selling services to clients choosing it with price q_j. Companies' clients choose operators to buy services from according to their own preferences.

We consider the market as a two-step repeated game. Players of the game are virtual operators. For each virtual operator j its profit is $C_j + u_j(x, q)$ where C_j is large enough to provide a non-negativity of profit and payoff $u_j(x, q)$ depends on strategies x and q we consider in a form of *revenue_per_client* * *clients_count* – *leasing_costs*. Companys' clients choose operators to buy service from by comparing their utilities $h_{ij} = S + r_i x_{ij} - q_j$. The game is a repetition of two steps. At step 1 each player j chooses a company to contract with, maximizing $u_j(x, q)$ by x_{ij}, $i = 1, 2$ with fixed prices q. At step 2 each player j assigns a price for its service, maximizing $u_j(x, q)$ by q_j with fixed x.

2 The game for 2 operators

The partition of company i's clients choosing operator j is given by the Hotelling specification [1]

$$\frac{1}{2} + \frac{h_{ij} - h_{i(3-j)}}{2k_{ij}},$$

where $h_{ij} = S + r_i x_{ij} - q_j$ is the utility of company i's clients by choosing operator j, where $k_{ij} > 0$ measures the degree of dissatisfaction which company i's clients feel towards operator j and reflects a subjective difference between companies and virtual operators.[1]

Then the payoff for the virtual operator j is

$$u_j(x, q) = q_j \sum_i n_i \left(\frac{1}{2} + \frac{h_{ij} - h_{i(3-j)}}{2k_{ij}} \right) - \sum_i \frac{x_{ij}}{x_{i1} + x_{i2}} p_i r_i.^2$$

We use the following notation to simplify the expressions. $x_j := x_{1j}$ is the probability to contract with the first company for operator j and the opposite probability $1 - x_j := x_{2j}$ is the probability that operator j chooses the second company. With such notations payoffs will be transformed as follows

$$u_1(x, q) = \frac{q_1}{2} \left((n_1 + n_2) - (q_1 - q_2) \left(\frac{n_1}{k_{11}} + \frac{n_2}{k_{21}} \right) + \left(\frac{n_1 r_1}{k_{11}} - \frac{n_2 r_2}{k_{21}} \right) (x_1 - x_2) \right) - \frac{x_1}{x_1 + x_2} p_1 r_1 - \frac{1 - x_1}{2 - x_1 - x_2} p_2 r_2,$$

$$u_2(x, q) = \frac{q_2}{2} \left((n_1 + n_2) + (q_1 - q_2) \left(\frac{n_1}{k_{12}} + \frac{n_2}{k_{22}} \right) - \left(\frac{n_1 r_1}{k_{12}} - \frac{n_2 r_2}{k_{22}} \right) (x_1 - x_2) \right) - \frac{x_2}{x_1 + x_2} p_1 r_1 - \frac{1 - x_2}{2 - x_1 - x_2} p_2 r_2.$$

[1] We assume that $(k_{ij} - k_{i(3-j)})(h_{ij} - h_{i(3-j)}) \geq 0$ to ensure that the number of clients of company i using operator 1 or operator 2's services add up to no greater than n_i.

[2] We assume that a company which has no operator contracting with it has its costs compensated by each of the operator, which can be understood as an operational or maintenance cost with efficiency loss in order to provide basic service to that company's client base. Mathematically, we require $\frac{x_{ij}}{x_{i1}+x_{i2}} = 1$ if $x_{i1} = x_{i2} = 0$.

Consider the first step of the game where operators define their distribution between companies. Payoffs $u_j(x,q)$ are convex in x_j, optimal values are $x_j \in \{0,1\}$, so optimal strategies at this step are pure strategies. We find the equilibrium for x where the strategy for each player is the best response to his opponent's strategy. Such situations are described by

$$x^*(q) = (x_1^*(q), x_2^*(q)) = \begin{cases} (0,0), \text{ if } \begin{cases} q_1 R_1 \le -p_2 r_2, \\ q_2 R_2 \le -p_2 r_2, \end{cases} \\ (0,1), \text{ if } \begin{cases} q_1 R_1 \le p_1 r_1, \\ q_2 R_2 \ge -p_2 r_2, \end{cases} \\ (1,0), \text{ if } \begin{cases} q_1 R_1 \ge -p_2 r_2, \\ q_2 R_2 \le p_1 r_1, \end{cases} \\ (1,1), \text{ if } \begin{cases} q_1 R_1 \ge p_1 r_1, \\ q_2 R_2 \ge p_1 r_1, \end{cases} \end{cases}$$

where $R_j := \frac{n_1 r_1}{k_{1j}} - \frac{n_2 r_2}{k_{2j}}$, $j = 1, 2$. Each $x_j^*(q)$ is the best response to the opponent's $x_{3-j}^*(q)$. Note that an equilibrium at the first step does not always exist. If R_1 and R_2 are both more or less than zero simultaneously, a pure-strategy equilibrium exists (for some cases there are two or three equlibria), but if they have different signs then there are areas for q values for which there is no pure-strategy equilibrium. For example, if $R_1 < 0$ and $R_2 > 0$ there is no pure-strategy equilibrium where $\frac{p_1 r_1}{R_1} < q_1 < -\frac{p_2 r_2}{R_1}$ and $-\frac{p_2 r_2}{R_2} < q_2 < \frac{p_1 r_1}{R_2}$.

Consider the second step in which players determine prices. We allow for negative prices to also be feasible and interpret this scenario as a situation where an operator invests money towards some promotion actions in order to minimize costs.

The payoff $u_j(x,q)$ is parabolic and concave in q_j. The optimal q_j is

$\arg\max_{q_j} u_j(x,q)$ and equilibrium for q is

$$q^*(x) = (q_1^*(x), q_2^*(x)) =$$
$$\left(\frac{n_1+n_2}{3}\left(\frac{2}{K_1}+\frac{1}{K_2}\right)+\frac{x_1-x_2}{3}\left(\frac{2R_1}{K_1}-\frac{R_2}{K_2}\right),\right.$$
$$\left.\frac{n_1+n_2}{3}\left(\frac{2}{K_2}+\frac{1}{K_1}\right)-\frac{x_1-x_2}{3}\left(\frac{2R_2}{K_2}-\frac{R_1}{K_1}\right)\right) =$$
$$\left(\frac{(n_1+n_2)(K_1+2K_2)+(x_1-x_2)(2R_1K_2-R_2K_1)}{3K_1K_2},\right.$$
$$\left.\frac{(n_1+n_2)(2K_1+K_2)-(x_1-x_2)(2R_2K_1-R_1K_2)}{3K_1K_2}\right),$$

where $K_j = \frac{n_1}{k_{1j}} + \frac{n_2}{k_{2j}}$, $j = 1, 2$.

Then the system changes its states in the following way. First, operators enter into the game with initial prices q^0. At the first transition they define their equilibrium distribution between companies with respect to their initial prices and change their prices with respect to the new distribution between companies. Then, at the second transition they change their distribution to equilibrium with respect to new prices and update prices, and so on. The process can be represented as follows: (Initial state with initial prices q^0) $\xrightarrow{1}$ $(x^1 := x^*(q^0),\ q^1 := q^*(x^1)) \xrightarrow{2} (x^2 := x^*(q^1),\ q^2 := q^*(x^2)) \xrightarrow{3} \dots$

After any transition of the system, if a pure-strategy equilibrium for the first step exists, the following states are possible:
1) $(0,0)$ with prices $\left(\frac{(n_1+n_2)(K_1+2K_2)}{3K_1K_2}, \frac{(n_1+n_2)(2K_1+K_2)}{3K_1K_2}\right)$,
2) $(0,1)$ with prices $\left(\frac{(n_1+n_2)(K_1+2K_2)-(2R_1K_2-R_2K_1)}{3K_1K_2}, \frac{(n_1+n_2)(2K_1+K_2)+(2R_2K_1-R_1K_2)}{3K_1K_2}\right)$,
3) $(1,0)$ with prices $\left(\frac{(n_1+n_2)(K_1+2K_2)+(2R_1K_2-R_2K_1)}{3K_1K_2}, \frac{(n_1+n_2)(2K_1+K_2)-(2R_2K_1-R_1K_2)}{3K_1K_2}\right)$,
4) $(1,1)$ with prices $\left(\frac{(n_1+n_2)(K_1+2K_2)}{3K_1K_2}, \frac{(n_1+n_2)(2K_1+K_2)}{3K_1K_2}\right)$.

In the general case, the pure-strategy equilibrium for the first step where players define their distribution between companies does not always exist. Now we consider two special cases where a pure-strategy equilibrium always exists.

2.1 Company-dependent client preferences

Consider the case where client preferences are company-dependent, that is $k_{ij} = k_i$ for $j = 1, 2$.[3]

In this case, payoff functions are

$$u_1(x,q) = \frac{q_1}{2}\left((n_1+n_2) - (q_1-q_2)\left(\frac{n_1}{k_1}+\frac{n_2}{k_2}\right) + (x_1-x_2)\left(\frac{n_1 r_1}{k_1}-\frac{n_2 r_2}{k_2}\right)\right) - \frac{x_1}{x_1+x_2}p_1 r_1 - \frac{(1-x_1)}{2-x_1-x_2}p_2 r_2,$$

$$u_2(x,q) = \frac{q_2}{2}\left((n_1+n_2) + (q_1-q_2)\left(\frac{n_1}{k_1}+\frac{n_2}{k_2}\right) - (x_1-x_2)\left(\frac{n_1 r_1}{k_1}-\frac{n_2 r_2}{k_2}\right)\right) - \frac{x_2}{x_1+x_2}p_1 r_1 - \frac{(1-x_2)}{2-x_1-x_2}p_2 r_2.$$

Without loss of generality we assume that $R := \frac{n_1 r_1}{k_1} - \frac{n_2 r_2}{k_2} > 0$, otherwise we can re-enumerate companies. Also we do not consider trivial case where $R = 0$ and companies are in some sense identical. Equilibria for both steps in this case are

$$x^*(q) = (x_1^*(q), x_2^*(q)) = \begin{cases} (0,0), \text{ if } \begin{cases} q_1 R \le -p_2 r_2, \\ q_2 R \le -p_2 r_2, \end{cases} \\ (0,1), \text{ if } \begin{cases} q_1 R \le p_1 r_1, \\ q_2 R \ge -p_2 r_2, \end{cases} \\ (1,0), \text{ if } \begin{cases} q_1 R \ge -p_2 r_2, \\ q_2 R \le p_1 r_1, \end{cases} \\ (1,1), \text{ if } \begin{cases} q_1 R \ge p_1 r_1, \\ q_2 R \ge p_1 r_1. \end{cases} \end{cases}$$

$$q^*(x) = (q_1^*(x), q_2^*(x)) = (Q + P(x), Q - P(x))$$
$$\text{where } Q = \frac{k_1 k_2 (n_1+n_2)}{n_1 k_2 + n_2 k_1}, P(x) = \frac{(x_1-x_2)(n_1 r_1 k_2 - n_2 r_2 k_1)}{3(n_1 k_2 + n_2 k_1)}$$

[3] Note that when $k_{i1} = k_{i2}$ the number of clients of company i using operator 1 or operator 2's services add up to exactly n_i, which is a reasonable outcome.

In the case of company-dependent client preferences, pure-strategy equilibrium x exists for any prices q, so after any transition system arrives at one of the states $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$ new equilibrium prices are updated depending on the distribution between companies and equal to $q(0,0) = q(1,1) = (Q,Q)$, $q(0,1) = (Q-P, Q+P)$, $q(1,0) = (Q+P, Q-P)$, where $Q = \frac{k_1k_2(n_1+n_2)}{n_1k_2+n_2k_1}$ and $P = \frac{(n_1r_1k_2-n_2r_2k_1)}{3(n_1k_2+n_2k_1)}$.

Now there is an important question: Does the system come to a stationary state or to a cycle of states? For the case of $R = \frac{n_1r_1}{k_1} - \frac{n_2r_2}{k_2} > 0$ we obtain the following results:

- The system comes to a stationary state after ≤ 3 transitions.
- The system can come to stationary states $(0,1)$ and $(1,0)$, if

$$\frac{k_1k_2R\left(n_1+n_2-\frac{R}{3}\right)}{n_1k_2+n_2k_1} \leq p_1r_1.$$

- The system can come to stationary state $(1,1)$, if

$$\frac{k_1k_2R(n_1+n_2)}{n_1k_2+n_2k_1} \geq p_1r_1.$$

- If $\frac{k_1k_2R\left(n_1+n_2-\frac{R}{3}\right)}{n_1k_2+n_2k_1} \leq p_1r_1 \leq \frac{k_1k_2R(n_1+n_2)}{n_1k_2+n_2k_1}$ any of states $(0,1)$, $(1,0)$, $(1,1)$ is stationary.
- Equilibrium and stationary states do not depend on p_2.

2.2 Operator-dependent client preferences

Consider now the case where client preferences are operator-dependent, that is $k_{ij} = k_j$ for $i = 1,2$.

The payoff functions are

$$u_1(x,q) = \frac{q_1}{2k_1}\left((n_1+n_2)(k_1-q_1+q_2)+(n_1r_1-n_2r_2)(x_1-x_2)\right) - \frac{x_1}{x_1+x_2}p_1r_1 - \frac{(1-x_1)}{2-x_1-x_2}p_2r_2,$$

$$u_2(x,q) = \frac{q_2}{2k_2}\left((n_1+n_2)(k_2+q_1-q_2) - (n_1r_1-n_2r_2)(x_1-x_2)\right) - \frac{x_2}{x_1+x_2}p_1r_1 - \frac{(1-x_2)}{2-x_1-x_2}p_2r_2.$$

As in the previous case, without loss of generality, we assume that $R := n_1r_1 - n_2r_2 > 0$. Equilibria for both steps are as follows:

$$x^*(q) = (x_1^*(q), x_2^*(q)) = \begin{cases} (0,0), \text{ if } \begin{cases} q_1R \le -p_2r_2k_1, \\ q_2R \le -p_2r_2k_2, \end{cases} \\ (0,1), \text{ if } \begin{cases} q_1R \le p_1r_1k_1, \\ q_2R \ge -p_2r_2k_2, \end{cases} \\ (1,0), \text{ if } \begin{cases} q_1R \ge -p_2r_2k_1, \\ q_2R \le p_1r_1k_2, \end{cases} \\ (1,1), \text{ if } \begin{cases} q_1R \ge p_1r_1k_1, \\ q_2R \ge p_1r_1k_2. \end{cases} \end{cases}$$

$$q^*(x) = (q_1^*(x), q_2^*(x)) = \left(\frac{2k_1+k_2}{3} + \frac{(x_1-x_2)R}{3(n_1+n_2)}, \frac{2k_2+k_1}{3} - \frac{(x_1-x_2)R}{3(n_1+n_2)}\right).$$

As in the previous case, equilibrium x exists for any prices q, and after any transition the system is in one of four states with corresponding equilibrium prices: $q(0,0) = q(1,1) = (\frac{2k_1+k_2}{3}, \frac{2k_2+k_1}{3})$, $q(0,1) = (\frac{2k_1+k_2}{3} - P, \frac{2k_2+k_1}{3} + P)$, $q(1,0) = (\frac{2k_1+k_2}{3} + P, \frac{2k_2+k_1}{3} - P)$, where $P = \frac{R}{3(n_1+n_2)}$.

We investigate a similar question: Does the system come to a stationary state or to a cycle of states? As in previous case, we obtain the following results for $R = n_1r_1 - n_2r_2 > 0$.

- The system comes to a stationary state after ≤ 3 transitions.
- The system can come to stationary states

$$\begin{aligned} &(0,1), \text{ if } \left(\tfrac{2k_1+k_2}{3} - \tfrac{R}{3(n_1+n_2)}\right)R \le p_1r_1k_1, \\ &(1,0), \text{ if } \left(\tfrac{2k_2+k_1}{3} - \tfrac{R}{3(n_1+n_2)}\right)R \le p_1r_1k_2, \\ &(1,1), \text{ if } \begin{cases} \tfrac{2k_1+k_2}{3}R \ge p_1r_1k_1, \\ \tfrac{2k_2+k_1}{3}R \ge p_1r_1k_2. \end{cases} \end{aligned}$$

- Equilibrium and stationary states do not depend on p_2.

3 $M \geq 3$ operators and company-dependent client preferences

Now consider the market where $M \geq 3$ operators distribute themselves between two companies. In this case a partition of company i's clients choosing operator $j = 1, \ldots, M$ is defined as

$$\frac{1}{M} + \sum_{l \neq j} \frac{h_{ij} - h_{il}}{Mk_i}.$$

The payoff for operator j is

$$u_j(x, q) = q_j \sum_i n_i \left(\frac{1}{M} + \sum_{l \neq j} \frac{h_{ij} - h_{il}}{Mk_i} \right) - \sum_i \frac{x_{ij}}{\sum\limits_l x_{il}} p_i r_i.^4$$

As in the case of two operators we use the notation x_j to represent to the probability that an operator j contracts with the first company. With this notation a payoff function for an operator $j = 1, \ldots, M$ is

$$u_j(x, q) = \frac{q_j}{M} \Bigg((n_1 + n_2) - ((M-1)q_j - \sum_{l \neq j} q_l) \left(\frac{n_1}{k_1} + \frac{n_2}{k_2} \right) + ((M-1)x_j - \sum_{l \neq j} x_l) \left(\frac{n_1 r_1}{k_1} - \frac{n_2 r_2}{k_2} \right) \Bigg) - \frac{x_j}{\sum\limits_l x_l} p_1 r_1 - \frac{(1 - x_j)}{M - \sum\limits_l x_l} p_2 r_2$$

We also assume here that $R = \frac{n_1 r_1}{k_1} - \frac{n_2 r_2}{k_2} > 0$.

Equilibria for the first step are $x^*(q) = \{x_j^*(q) \in \{0, 1\}\}_{j=1}^M$, where $K = \sum\limits_l x_l^*(q)$, and

$$\begin{cases} \frac{(M-1)Rq_j}{M} \geq \frac{p_1 r_1 \mathbb{I}_{K>1}}{K} - \frac{p_2 r_2 \mathbb{I}_{K<M}}{M-K+1} \text{ for } x_j^*(q) = 1, \\ \frac{(M-1)Rq_j}{M} \leq \frac{p_1 r_1 \mathbb{I}_{K>0}}{K+1} - \frac{p_2 r_2 \mathbb{I}_{K<M-1}}{M-K} \text{ for } x_j^*(q) = 0. \end{cases}$$

[4] As in the case of two operators, here we also assume that a company which has no operator contracting with it has its costs compensated by each of the operator. Mathematically, we require $\frac{x_{ij}}{\sum\limits_l x_{il}} = 1$ if $x_{ij} = 0$ for all j.

Equilibrium prices at the second step are

$$q_j^*(x) = \frac{k_1k_2(n_1+n_2)}{(M-1)(n_1k_2+n_2k_1)} + \frac{k_1k_2R\left(Mx_j - \sum\limits_l x_l\right)}{(2M-1)(n_1k_2+n_2k_1)}.$$

Note that operators are symmetric and can vary only in strategies. After any second step, operators choosing the same company have the same price. We divide operators into groups

$$A = \{j \in \{1,\dots,M\} | x_j = 1\}, B = \{j \in \{1,\dots,M\} | x_j = 0\},$$

where the group A consists of operators contracting with the first company and the group B contains remaining operators choosing the second company. Then any system state is described by group A size $K = |A| = \sum\limits_j x_j$ with equilibrium prices $q_A(K) = Q + \frac{k_1k_2R(M-K)}{(2M-1)(n_1k_2+n_2k_1)}$ and $q_B(K) = Q - \frac{k_1k_2RK}{(2M-1)(n_1k_2+n_2k_1)}$, where $Q = \frac{k_1k_2(n_1+n_2)}{(M-1)(n_1k_2+n_2k_1)}$.

Then state $0 \le K \le M$ is an equilibrium if

$$\begin{cases} \frac{(M-1)Rq_A(K)}{M} \ge \frac{p_1r_1\mathbb{I}_{K>1}}{K} - \frac{p_2r_2\mathbb{I}_{K<M}}{M-K+1} \text{ for } K > 0, \\ \frac{(M-1)Rq_B(K)}{M} \le \frac{p_1r_1\mathbb{I}_{K>0}}{K+1} - \frac{p_2r_2\mathbb{I}_{K<M-1}}{M-K} \text{ for } K < M. \end{cases}$$

In this case we obtain the following results for the system where $R = \frac{n_1r_1}{k_1} - \frac{n_2r_2}{k_2} > 0$.

- The system comes to a stationary state after ≤ 3 transitions.
- There is a set of states $\{1, M-1, M\}$ such that at least one of them is a stationary state and can be achieved after ≤ 3 transitions.
- Other stationary states from the remaining set $\{2,\dots,M-2\}$ are not excluded but can be achieved after 1 transition if the system by chance has suitable initial prices q.

An Example. Consider the system with two companies. The first company has been in the market for a long time and has more subscribers ($n_1 = 25$, $n_2 = 10$), but its resources are more expensive ($p_1 = 10$, $p_2 = 5$). The second company trying to catch market opportunities, assigns low prices, implements an aggressive advertising policy ($k_1 = 1$, $k_2 = 2$) and allocates more resources for accessibility ($r_1 = 0.5$, $r_2 = 0.6$). There are $M = 10$ virtual operators. Depending on initial prices q, the system arrives at one of states presented in the table and then changes state until it arrives at one of stationary states (see bold font in the table). Note that in states $K = 8$ and $K = 9$ operators choosing the second company have negative prices, but it does not interfere with the fact that the state $K = 9$ is stationary. Note also that the stationary states $K = 2, 3$ are isolated and that the system can arrive at them only by chance with suitable initial prices.

K	q_A	q_B	Possible transitions
0	–	0.13	1, 10
1	0.28	0.112	
2	0.26	0.096	
3	0.246	0.08	
4	0.23	0.063	1, 10
5	0.213	0.046	1, 9
6	0.196	0.03	1, 9
7	0.18	0.013	1, 9
8	0.163	-0.004	1, 9
9	0.146	-0.02	
10	0.13	–	

4 Conclusion

We consider a market in which two virtual operators lease wholesale resources from two possible companies and set prices for companies' clients. The strategic interaction between virtual operators is modeled as a two-step repeated game. In the first step, virtual operators jointly choose the probability distribution of contracting with each company given the prices operators charge, and in the second step, virtual operators jointly determine the retail prices given the probability distribution of contracting with each company. We solve for the optimal strategies for operators in each step and discuss about conditions under which a pure-strategy equilibrium exists in the first step.For the cases in which clients have company-dependent or operator-dependent preferences, we fully characterize the conditions for obtaining a stationary equilibrium. Furthermore, we extend the analysis to the case of more than two operators under company-dependent client preferences and obtain the analogous stationary equilibrium conditions.

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