Multi-period complete-information games with self-control: a dual-self approach*

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Abstract The existing literature on dual-self has focused on individual decision-making problems. We adopt the theoretical framework established by Fudenberg and Levine (American Economic Review, 2006, 96(5): 1449-1476) in 2006 to propose a new dual-self model that expands individual decision problems to multi-player strategic situations, better reflecting human interactions in reality. For the example of two-period complete-information game with multiple equilibria provided in this paper, we analyze it using our proposed dual-self model and compare the results we found with those of economic models without dual-self. Consistent with the Fudenberg-Levine axioms, our model defines a more general concept of self-control cost that takes into account multi-player interactions.

Keywords self-control, dual-self, complete information, game theory

Chinese Library Classification F224.32

2010 Mathematics Subject Classification 91A10, 91A20, 91A40, 91A80

多期完全信息博弈中的自我控制现象：一个基于双重自我模型的分析框架*

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摘要 现有文献对双重自我的研究仅限于对单人决策问题的探讨. 基于Fudenberg和Levine (American Economic Review, 2006, 96(5): 1449-1476) in 2006 的理论框架，建立将单人决策拓展至多人博弈的双重自我模型，以更准确地刻画和研究人们在现实生活中的互动行为. 利用该模型对某一完全信息下的两期多重均衡博弈展开分析，并与其他无双重自我个体的经济学模型进行对比. 双重自我模型在与Fudenberg-Levine模型的公理假设保持一致的基础上，提出了更具一般性的体现多人互动特征的自我控制成本概念.

关键词 自我控制，双重自我，完全信息，博弈论

中图分类号 F224.32

数学分类号(2010) 91A10, 91A20, 91A40, 91A80

收稿日期：2012年11月20日

* Supported by National Natural Science Foundation of China Young Scholar Research Grant (No. 71203112) and Tsinghua University Independent Research Project Grant (No. 2012Z02181).

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0 Introduction

Studies on the topic of dual-self, which refers to the game between a long-run patient self and a sequence of short-run impulsive selves within an individual, have attracted much attention in a wide range of contexts. However, the existing literature on dual-self models has focused only on individual decision-making problems[1−14], rather than games involving strategic situations among multiple players. This paper makes a first attempt to analyze the dual-self model in the context of games.

Our model depicts games played along two dimensions. The first dimension refers to a game played within an individual between his long-run self and a sequence of short-run selves. The second dimension refers to the aspect of the game played between individual players. We adopt the dual-self model (DSM) established by Fudenberg and Levine[1] and consider the features of interactions among players. Our adoption is mainly for the following two reasons.

First, the concept of short-run self and long-run self used in this DSM is consistent with those in the previous related literature[1−14]. The long-run patient self of Fudenberg and Levine[1] is similar to the planner in Shefrin and Thaler [2]; the impartial spectator in Adam Smith[3]; the cold state in Bernheim and Rangel[4]; and the deliberative system in various work done by Loewenstein and O'Donoghue[5−6]. Correspondingly, the concept of short-run impulsive selves in Fudenberg and Levine[1] is coherent to the doer in Shefrin and Thaler[2]; the passion-driven in Adam Smith[3]; the hot state in Bernheim and Rangel[4]; and the affective system in Loewenstein and O'Donoghue's work[5−6].

Second, this DSM provides advancements to the model with quasi-hyperbolic utility (QHM) by O'Donoghue and Rabin[7]. DSM provides a unique equilibrium in some cases where QHM provides multiple equilibria. Thus the DSM approach can perhaps give more precise predictions and better explain empirical facts than the QHM approach. Furthermore, DSM provides an explanation for a broad range of behavioral anomalies and thus a better fit for the modular structure of the brain[15].

Another key difference between DSM and QHM is worth noting: the DSM established by Fudenberg and Levine[1] emphasizes that the long-run self and a sequence of short-run selves share the same preferences over stage-game outcomes; they differ only in how they regard the future. In particular, they assume that the short-run self only cares about the outcome in the current period. QHM assumes otherwise. We adopt Fudenberg and Levine's[1] assumption because we believe that modeling individuals as having both long-run and short-run selves is a more reasonable and realistic approach.

Moreover, according to Fudenberg and Levine[1], each individual has two selves: the long-run patient self tries to maximize his lifetime utility; however, each of the short-run impulsive selves lives for only one period, each thus cares only about his immediate payoffs. Therefore self-control on the short-run selves must be imposed by the long-run self in order for the short-run selves to make decisions that are optimal for the long-run self.

One may refer to the repeated game with history-dependent strategy (HDSM) in discussing multi-period games[16−17]. Notably two key differences set our theory apart from it. First, HDSM assumes that every player is one single-self whereas our model assumes that players are dual-self. Second, in HDSM the decisions made by players at every stage game are based on past incidences, whereas in our model the short-run selves' strategies at each stage game are not necessarily dependent on previous outcomes[18]. However, in this paper, since we only consider the complete information case, we focus on the first difference and assume that both short-run and long-run selves' strategies are history-dependent.

This paper is organized as follows: Section 1 provides the basic model; Section 2 provides an example of two-period complete-information game with multiple equilibria to illustrate the practicality of our model; Section 3 is the conclusion.

1 The model

We follow Fudenberg and Levine's[1] assumptions on self-control and extend their framework to the multi-player case.

There are I ($\geqslant 2$) players, $i = 1, 2, \cdots, I$, and T ($\geqslant 1$) periods, $t = 1, \cdots, T$, where I is finite and T can be potentially infinite. Player i's discount factor between any two consecutive periods is constant and denoted by δ_i, where $\delta_i \in [0, 1]$.

Each player i is considered a dual-self agent: a sequence of short-run selves $\{SRS_i^t\}_{t=1}^T$, each of whom lives only for one period, and a long-run self LRS_i, who lives forever. A short-run self plays a one-period strategy to maximize his short-run payoff while alive, and the long-run self plays a series of self-control strategies over time to maximize his long-run payoff.

SRS_i^t's choice is denoted by s_i^t, where $s_i^t \in S_i^t \subseteq \mathbb{R}$. We write $s^t = (s_1^t, s_2^t, \cdots, s_I^t)$, and $\mathbf{s} = \left(s^1, s^2, \cdots, s^T\right)$. For simplicity, we assume $S_i^t = S_i^{t'}$ $\forall i$ $\forall t, t'$. LRS_i's self-control action in period t is denoted by r_i^t, where $r_i^t \in R_i^t \subseteq \mathbb{R}$. Similarly, we have $r^t = (r_1^t, r_2^t, \cdots, r_I^t)$, and $\mathbf{r} = \left(r^1, r^2, \cdots, r^T\right)$. We also assume $R_i^t = R_i^{t'}$ $\forall i$ $\forall t, t'$. These restrictions $R_i^t = R_i^{t'}$ $\forall i$ $\forall t, t'$ and $S_i^t = S_i^{t'}$ $\forall i$ $\forall t, t'$ are for modeling convenience only, and they can be relaxed without significant change of the results.

A finite history of play in period t, denoted by h_t, consists of all the players' past actions. $h_t = \left(r^1, s^1, r^2, s^2, \cdots, r^{t-1}, s^{t-1}\right)$ if $t \geqslant 2$, and $h_t = \emptyset$ if $t = 1$.

In general, player i's period-t payoff (i.e. the payoff for SRS_i^t) depends on both the past actions and the current actions. For simplicity, we assume in this paper that a player's per-period payoff only depends on the current actions of all the players, that is,

$$u_i^t\left(r^t, s^t\right) : \prod_{i=1}^{I} R_i^t \times \prod_{i=1}^{I} S_i^t \longrightarrow \mathbb{R}. \tag{1.1}$$

Under the dual-self framework, it is natural to assume that the lifetime payoff is time-additive. Thus, player i's lifetime payoff (i.e. the payoff for LRS_i) is

$$U_i\left(\mathbf{r}, \mathbf{s}\right) = \sum_{t=1}^{T} \left[\delta_i\right]^{t-1} u_i^t\left(r^t, s^t\right). \tag{1.2}$$

Now we impose assumptions regarding self-control actions. For simplicity, we omit the superscript t for all the notations.

Assumption 1.1[Costly Self-Control] For any player i, for any $s_i, \mathbf{r}_{-i}, \mathbf{s}_{-i}$, if $r_i \neq 0$, then $u_i\left(r_i, \mathbf{r}_{-i}, s_i, \mathbf{s}_{-i}\right) < u_i\left(0, \mathbf{r}_{-i}, s_i, \mathbf{s}_{-i}\right)$.

Assumption 1.2[Unlimited Self-Control] For any player i, for any $s_i, \mathbf{r}_{-i}, \mathbf{s}_{-i}$, there exists r_i, such that for any s_i', $u_i(r_i, \mathbf{r}_{-i}, s_i', \mathbf{s}_{-i}) \leqslant u_i(r_i, \mathbf{r}_{-i}, s_i, \mathbf{s}_{-i})$.

Assumption 1.3[Independent Self-Control] For any player i, for any $r_i, s_i, \mathbf{s}_{-i}$, for any $\mathbf{r}_{-i}$, $u_i(r_i, \mathbf{r}_{-i}, s_i, \mathbf{s}_{-i}) = u_i(r_i, \mathbf{0}, s_i, \mathbf{s}_{-i})$.

Assumption 1.3 is the key for our analysis of interactions among dual-self players. It means that a player's payoff only depends on all players' actions and his own self-control action, and does not depend on other players' self-control actions. This is a reasonable characterization of many situations in reality. Given this assumption, we can simplify a lot the definition for self-control cost, since other players' self-control decisions do not matter.

Definition 1.1[Self-Control Cost] Given any short-run selves' strategy choosing profile $\mathbf{s}$, let $R_i(\mathbf{s}) = \{r_i \in R_i | u_i(r_i, \mathbf{0}, s_i, \mathbf{s}_{-i}) \geqslant u_i(r_i, \mathbf{0}, \cdot, \mathbf{s}_{-i})\}$, then player i's self-control cost is defined as

$$C_i(\mathbf{s}) = C_i(s_i, \mathbf{s}_{-i}) \equiv u_i(\mathbf{0}, \mathbf{s}) - \sup_{r_i \in R_i(\mathbf{s})} u_i(r_i, \mathbf{0}, \mathbf{s}).$$

It is important to mention: (1) Assumption 1.1 guarantees that self-control cost is non-negative; (2) Assumption 1.2 guarantees that the set $\{r_i | u_i(r_i, \mathbf{0}, s_i, \mathbf{s}_{-i}) \geqslant u_i(r_i, \mathbf{0}, \cdot, \mathbf{s}_{-i})\}$ is nonempty, and hence self-control cost is well-defined; (3) Assumption 1.3 guarantees that a player's self-control cost is independent of other players' self-control actions.

Assumption 1.4[Continuity] For any player i, $u_i(r_i, \mathbf{r}_{-i}, s_i, \mathbf{s}_{-i})$ is continuous in r_i, s_i.

We have the following property regarding self control cost.

Property 1.1[Strict Cost of Self-Control] Under Assumptions 1.1-1.4,

$$s_i \in \arg\max_{s_i' \in S_i} u_i(0, \mathbf{0}, s_i', \mathbf{s}_{-i}) \Leftrightarrow C_i(s_i, \mathbf{s}_{-i}) = 0.$$

Proof By Definition 1.1, $C_i(s_i, \mathbf{s}_{-i}) = 0$ if and only if $u_i(\mathbf{0}, \mathbf{s}) = \sup_{r_i \in R_i(\mathbf{s})} u_i(r_i, \mathbf{0}, \mathbf{s})$.

By Assumption 1.4, $\sup_{r_i \in R_i(\mathbf{s})} u_i(r_i, \mathbf{0}, \mathbf{s}) = \max_{r_i \in R_i(\mathbf{s})} u_i(r_i, \mathbf{0}, \mathbf{s})$.

By Assumption 1.1, $u_i(\mathbf{0}, \mathbf{s}) = \max_{r_i \in R_i(\mathbf{s})} u_i(r_i, \mathbf{0}, \mathbf{s})$ if and only if $0 \in R_i(\mathbf{s})$.

By the definition of $R_i(\mathbf{s})$, $0 \in R_i(\mathbf{s})$ if and only if $u_i(0, \mathbf{0}, s_i, \mathbf{s}_{-i}) = \max_{s_i' \in S_i} u_i(0, \mathbf{0}, s_i', \mathbf{s}_{-i})$, or $s_i \in \arg\max_{s_i' \in S_i} u_i(0, \mathbf{0}, s_i', \mathbf{s}_{-i})$.

2 A 2-period game with multiple equilibria

Suppose two players, A and B, are playing a 2-period normal-form game. In each period, each player has two actions available: C(Cooperate) and D(Defect). Player i's payoff function in period t is denoted by $u_i^t(s_A^t, s_B^t) : \{C, D\} \times \{C, D\} \longrightarrow \mathbb{R}$, where $i = A, B$, $t = 1, 2$, and s_i^t is the player i's strategy in period t. Player i's discount factor between the two periods is denoted by δ_i, where $\delta_i \in [0, 1]$, $i = A, B$. Hence player i's total payoff is $u_i^1(s_A^1, s_B^1) + \delta_i u_i^2(s_A^2, s_B^2)$. The payoff matrices for period 1 and period 2 are shown below, respectively, and we assume $2a > c > a > 0$, $b > 0$. As we can see clearly, the only difference between these two matrices is that when both players play C, their single-period payoffs are (a, a) in period 1, and $(2a, 2a)$ in period 2.

period 1

		B: C	D
A	C	(a, a)	$(-b, c)$
	D	$(c, -b)$	$(0, 0)$

period 2

		B: C	D
A	C	$(2a, 2a)$	$(-b, c)$
	D	$(c, -b)$	$(0, 0)$

We consider the following 3 scenarios and compare the results under different scenarios.

2.1 Scenario 1: History-independent strategy

In Scenario 1, we assume that the game at each period is considered independent. In other words, the players' decisions in period 2 are independent of the outcome in period 1.

It is easy to know that in period 1, the equilibrium strategy profile is (D, D) and the equilibrium payoff profile is $(0, 0)$. In period 2, the possible highest equilibrium payoffs are $(2a, 2a)$ with the equilibrium strategy profile (C, C). So the total maximum equilibrium payoffs are $(2\delta_A a, 2\delta_B a)$.

2.2 Scenario 2: History-dependent strategy

In Scenario 2, we assume that players' strategies can depend on the history of the outcomes. In this case, with multiple Nash Equilibria in period 2, it is possible for players to cooperate in period 1 in order to achieve higher payoffs. We are interested in the following equilibrium strategy profile, which achieves the highest equilibrium payoffs for both players.

Player i's strategy is as follows:

(1) In period 1 he plays C;

(2) In period 2 he plays $\begin{cases} C & \text{if the previous outcome is } (C, C); \\ D & \text{otherwise,} \end{cases}$ $i = A, B$.

It is easy to verify that player i has no incentive to deviate if and only if $\delta_i \in \left[\frac{c-a}{2a}, 1\right]$, $i = A, B$. When $\min\{\delta_A, \delta_B\} \in \left[\frac{c-a}{2a}, 1\right]$, player i can achieve his highest payoff $a + 2\delta_i a$.

2.3 Scenario 3: Dual-self approach

In Scenario 3, instead of adopting the single-self decision making model which is used in the first 2 scenarios, we apply the dual-self approach to our example. Suppose each player has a long-run self and 2 short-run selves (who live only in period 1 and period 2, respectively). The long-run self of a player i can impose a costly self-control action $r_i^t \in R_i^t \subseteq \mathbb{R}$ in period t, which may vary across different periods and will affect his own payoff (but not the other player's payoff) at the current period. Under the above settings, Player i's payoff function in period t is denoted by $u_i^t(r_i^t, s_A^t, s_B^t) : R_i^t \times \{C, D\} \times \{C, D\} \longrightarrow \mathbb{R}$, where $i = A, B$, $t = 1, 2$, r_i^t is LRS_i's self-control action in period t, and s_i^t is SRS_i^t's strategy in period t. For simplicity, we took the redundent term $\mathbf{r}_{-i}$ out of the expression of the payoff function, by Assumption 1.3. Player i's total payoff is thus $u_i^1\left(r_i^1, s_A^1, s_B^1\right) + \delta_i u_i^2\left(r_i^1, s_A^2, s_B^2\right)$.

Let $C_i^t(s_A^t, s_B^t)$ be the self-control cost of player i in period t, when players' strategy profile in that period is (s_A^t, s_B^t). In the following analysis, we need the concept of optimal self-control actions, which is defined below.

Definition 2.1[Optimal self-control] Given $i \in \{A, B\}$ and any strategy choice profile (s_A^t, s_B^t) by SRS_A^t and SRS_B^t, an optimal self-control action $r^{i,t}_{s_A^t, s_B^t}$ imposed by LRS_i in period t satisfies the following two conditions:

(1) $C_i^t(s_A^t, s_B^t) = u_i^t(0, s_A^t, s_B^t) - u_i^t\left(r^{i,t}_{s_A^t, s_B^t}, s_A^t, s_B^t\right)$;

(2) $s_i^t \in \arg\max_{s \in S_i^t} u_i^t\left(r^{i,t}_{s_A^t, s_B^t}, s, s_{-i}^t\right)$.

It is easy to see from the above definition that $r^{i,t}_{s_A^t, s_B^t}$ is such a self-control action for LRS_i in period t that can ensure that SRS_i^t has no incentive to unilaterally deviate from (s_A^t, s_B^t) at the lowest possible cost. In this sense we call $r^{i,t}_{s_A^t, s_B^t}$ an "optimal" self-control.

2.3.1 The structure of self-control cost in period 1

By Assumption 1.1 and Property 1.1, the structure of the payoff matrix in period 1 gives:

$$\begin{array}{ll} C_A^1(C,C) > 0 = C_A^1(D,C), & C_A^1(C,D) > 0 = C_A^1(D,D), \\ C_B^1(C,C) > 0 = C_B^1(C,D), & C_B^1(D,C) > 0 = C_B^1(D,D). \end{array}$$

By Definition 2.1, in period 1, we have

$$a > u_A^1\left(r_{CC}^{A1}, C, C\right) \geqslant u_A^1\left(r_{CC}^{A1}, D, C\right), \tag{2.1}$$

$$a > u_B^1\left(r_{CC}^{B1}, C, C\right) \geqslant u_B^1\left(r_{CC}^{B1}, C, D\right). \tag{2.2}$$

(2.1) means that in period 1 there exists a nonzero optimal self-control r_{CC}^{A1} so that SRS_A^1 chooses C over D when SRS_B^1 chooses C. Similarly, (2.2) means that there is a nonzero optimal self-control r_{CC}^{B1} so that SRS_B^1 chooses C over D when SRS_A^1 chooses C.

Note that when the self-control is too costly in period 1 ($C_i^1(C,C) \geqslant a$, $i = A, B$), the long-run selves have no incentive to cooperate, because in that case each player's period-1 payoff is non-positive if they choose to be cooperative while their payoff is zero if they defect. In order to focus on the interesting case, we make the following assumption.

Assumption 2.1[Gains from Cooperation] For $i = A, B$, player i's self-control cost for strategy choice profile (C, C) at period 1 must satisfy the following condition:

$$C_i^1(C,C) < a.$$

2.3.2 The structure of self-control cost in period 2

Similarly in period 2, we have

$$\begin{array}{ll} C_A^2(D,C) > 0 = C_A^2(C,C), & C_A^2(C,D) > 0 = C_A^2(D,D), \\ C_B^2(C,D) > 0 = C_B^2(C,C), & C_B^2(D,C) > 0 = C_B^2(D,D). \end{array}$$

$$2a = u_A^2(0, C, C) \geqslant u_A^2(0, D, C), \tag{2.3}$$

$$2a = u_B^2(0, C, C) \geqslant u_B^2(0, C, D), \tag{2.4}$$

$$0 = u_A^2(0, D, D) \geqslant u_A^2(0, C, D), \tag{2.5}$$

$$0 = u_B^2(0, D, D) \geqslant u_B^2(0, D, C). \tag{2.6}$$

2.3.3 An equilibrium strategy profile

We are interested in the following equilibrium strategy profile, which achieves the highest equilibrium payoffs for both players.

Player A's strategy is as follows:

A1 In period 1, LRS_A imposes a self-control $r_A^1 = r_{CC}^{A1}$;

$$SRS_A^1 \text{ chooses } s_A^1\left(r_A^1\right) = \begin{cases} C & \text{if } u_A^1\left(r_A^1, C,C\right) \geqslant u_A^1\left(r_A^1, D, C\right), \\ D & \text{otherwise.} \end{cases}$$

A2 In period 2, LRS_A imposes a zero self-control $r_A^2 = 0$; SRS_A^2 chooses

$$s_A^2\left(h_2, r_A^2\right) = \begin{cases} C & \text{if} \begin{cases} u_A^2\left(r_A^2, C,C\right) \geqslant u_A^2\left(r_A^2, D, C\right) \text{ and } \left(s_A^1, s_B^1\right) = (C,C), \\ \text{or} \\ u_A^2\left(r_A^2, D,D\right) < u_A^2\left(r_A^2, C, D\right) \text{ and } \left(s_A^1, s_B^1\right) \neq (C,C); \end{cases} \\ D & \text{otherwise.} \end{cases}$$

Similarly, player B's strategy is as follows:

B1 In period 1, LRS_B imposes a self-control $r_B^1 = r_{CC}^{B1}$;

$$SRS_B^1 \text{ chooses } s_B^1\left(r_B^1\right) = \begin{cases} C & \text{if } u_B^1\left(r_B^1, C,C\right) \geqslant u_B^1\left(r_B^1, C, D\right), \\ D & \text{otherwise.} \end{cases}$$

B2 In period 2, LRS_B imposes a zero self-control $r_B^2 = 0$; SRS_B^2 chooses

$$s_B^2\left(h_2, r_B^2\right) = \begin{cases} C & \text{if} \begin{cases} u_B^2\left(r_B^2, C,C\right) \geqslant u_B^2\left(r_B^2, C, D\right) \text{ and } \left(s_A^1, s_B^1\right) = (C,C), \\ \text{or} \\ u_B^2\left(r_B^2, D,D\right) < u_B^2\left(r_B^2, D, C\right) \text{ and } \left(s_A^1, s_B^1\right) \neq (C,C); \end{cases} \\ D & \text{otherwise.} \end{cases}$$

2.3.4 Analysis

To see why the strategy profile described above forms a Subgame Perfect Nash Equilibrium, let us check whether players have incentive to deviate or not. It suffices to only consider player A since the game is symmetric.

2.3.4.1 Consider SRS_A^2

Assuming SRS_B^2, LRS_B in period 2, and LRS_A in period 2 are playing the strategies described above, LRS_B will impose a zero self-control $r_B^2 = 0$ in period 2. By (2.4) and (2.6), SRS_B^2 plays C if the previous outcome is (C, C) and plays D if the previous outcome is (D, D). Since there is no self-control imposed by LRS_A in period 2, by (2.3) and (2.5), SRS_A^2 will naturally play the equilibrium strategy to maximize his payoff.

2.3.4.2 Consider SRS_A^1

Assuming player B, SRS_A^2 and LRS_A are playing the strategies described above, LRS_B will impose self-control $r_B^1 = r_{CC}^{B1}$ in period 1. By (2.2), SRS_B^1 plays C. Given that LRS_A will impose self-control $r_A^1 = r_{CC}^{A1}$ in period 1, the payoff of SRS_A^1 is $u_A^1\left(r_{CC}^{A1}, C,C\right)$ by playing C and his payoff is $u_A^1\left(r_{CC}^{A1}, D,C\right)$ by playing D. By (2.1), $u_A^1\left(r_{CC}^{A1}, C,C\right) \geqslant u_A^1\left(r_{CC}^{A1}, D,C\right)$, so SRS_A^1 has no incentive to deviate from playing C.

2.3.4.3 Consider LRS_A in perird 2

Assuming SRS_A^2, SRS_B^2 and LRS_B in period 2are playing the strategies described above, LRS_B will impose a zero self-control $r_B^2 = 0$ in period 2. By (2.4) and (2.6), SRS_B^2

plays C if the previous outcome is (C,C) and plays D if the previous outcome is (D,D). Since (C,C) and (D,D) are the Nash Equilibria for the stage game in period 2, LRS_A has no incentive to impose any non-zero self-control, which would reduce his payoff by Assumption 1.1.

2.3.4.4 Consider LRS_A in period 1

Assuming player B, SRS_A^1, SRS_A^2 and LRS_A in period 2 are playing the strategies described above, LRS_B will impose self-control $r_B^1 = r_{CC}^{B1}$ in period 1. By (2.2), SRS_B^1 plays C. If LRS_A follows the strategy described above, his total payoff should be $u_A^1\left(r_{CC}^{A1},C,C\right)+\delta_A u_A^2(0,C,C) = u_A^1\left(r_{CC}^{A1},C,C\right)+2\delta_A a$. However, if he deviates in period 1 by imposing a different self-control r_A^{1*}, his total payoff would be

$$\begin{cases} u_A^1\left(r_A^{1*},C,C\right)+2\delta_A a & \text{if } u_A^1\left(r_A^{1*},C,C\right) \geqslant u_A^1\left(r_A^{1*},D,C\right), \\ u_A^1\left(r_A^{1*},D,C\right) & \text{otherwise.} \end{cases}$$

Let $R_A^1(C,C) = \left\{r \in R_A^1 | u_A^1(r,C,C) \geqslant u_A^1(r,D,C)\right\}$. In order to make sure that LRS_A has no incentive to deviate in period 1, we must have

$$u_A^1\left(r_{CC}^{A1},C,C\right)+2\delta_A a \geqslant \max_{r_A^{1*}\neq r_{CC}^{A1},\, r_A^{1*}\in R_A^1(C,C)} \left[u_A^1\left(r_A^{1*},C,C\right)+2\delta_A a\right], \tag{2.7}$$

$$u_A^1\left(r_{CC}^{A1},C,C\right)+2\delta_A a \geqslant \max_{r_A^{1*}\neq r_{CC}^{A1},\, r_A^{1*}\notin R_A^1(C,C)} \left[u_A^1\left(r_A^{1*},D,C\right)\right]. \tag{2.8}$$

By Definition 2.1, we know

$$u_A^1(0,C,C)-u_A^1\left(r_{CC}^{A1},C,C\right) = C_A^1(C,C) \leqslant u_A^1(0,C,C) - \max_{\substack{r_A^{1*}\neq r_{CC}^{A1},\\ r_A^{1*}\in R_A^1(C,C)}} u_A\left(r_A^{1*},C,C\right). \tag{2.9}$$

Hence $u_A^1\left(r_{CC}^{A1},C,C\right) \geqslant \max_{r_A^{1*}\neq r_{CC}^{A1},\, r_A^{1*}\in R_A^1(C,C)} u_A^1\left(r_A^{1*},C,C\right)$, which implies (2.7) always holds. Thus, in order to make sure that LRS_A has no incentive to deviate in period 1, it suffices to have (2.8) hold. Since by Property 1.1,

$$\max_{r_A^{1*}\neq r_{CC}^{A1},\, r_A^{1*}\notin R_A^1(C,C)} \left[u_A^1\left(r_A^{1*},D,C\right)\right] = u_A^1(0,D,C) = c, \tag{2.10}$$

it suffices to have $u_A^1\left(r_{CC}^{A1},C,C\right)+2\delta_A a \geqslant c$.

Solving for δ_A, we obtain the following result:

$$\delta_A \geqslant \frac{c-u_A^1\left(r_{CC}^{A1},C,C\right)}{2a} > \frac{c-a}{2a}. \tag{2.11}$$

By Definition 2.1 and Assumption 2.1, we have $a - u_A^1\left(r_{CC}^{A1},C,C\right) = C_A^1(C,C) < a$, which implies $u_A^1\left(r_{CC}^{A1},C,C\right) > 0$. This guarantees $\frac{c-u_A^1\left(r_{CC}^{A1},C,C\right)}{2a} < 1$, hence the solution set to inequality (2.11) is nonempty.

Similarly, we can have an expression for δ_B:

$$\delta_B \geqslant \frac{c-u_B^1\left(r_{CC}^{B1},C,C\right)}{2a} > \frac{c-a}{2a}. \tag{2.12}$$

Therefore, player i $(i=A,B)$ has no incentive to deviate if and only if $\delta_i \geqslant \frac{c-u_i^1\left(r_{CC}^{i1},C,C\right)}{2a}$.

We conclude our analysis with the following proposition.

Proposition 2.1 In the 2-period complete-information game between 2 dual-self players, described in Scenario 3, the strategy profile specified in Section 2.3.3 forms a Subgame Perfect Nash Equilibrium if and only if

$$\delta_i \in \left[\frac{c-u_i^1\left(r_{CC}^{i1}, C, C\right)}{2a}, 1\right], \ i=A, B.$$

2.3.5 Remarks

(1) In Scenario 2 the range for δ_i is the same for both players, whereas under the dual-self setting of Scenario 3, in equilibrium, the range for δ_i depends on player i's self-control cost structure. Note that $\delta_i \in \left[\frac{c-u_i^1\left(r_{CC}^{i1}, C, C\right)}{2a}, 1\right] \subset \left[\frac{c-a}{2a}, 1\right]$ and $\frac{c-u_i^1\left(r_{CC}^{i1}, C, C\right)}{2a} \longrightarrow \frac{c-a}{2a}$ as $C_i^1\left(C, C\right) \longrightarrow 0$. This implies that when the self-control cost is small, the equilibrium range of discount factors in Scenario 3 is close to that in Scenario 2.

(2) In Scenario 2 player i's highest equilibrium payoff is $a+2\delta_i a$, which is greater than $2\delta_i a$, player i's highest equilibrium payoff in Scenario 1. In Scenario 3, player i's highest equilibrium payoff is $u_i^1\left(r_{CC}^{i1}, C, C\right)+2\delta_i a$, and it is greater than $2\delta_i a$ under Assumption 2.1. The difference in equilibrium payoffs between Scenarios 2 and 3 is exactly the self-control cost $C_i^1\left(C, C\right)=a-u_i^1\left(r_{CC}^{i1}, C, C\right)$, $i=A, B$. When the self-control cost goes to zero, the gap in equilibrium payoffs between Scenarios 2 and 3 will disappear.

3 Conclusion

The current literature on dual-self has focused on individual decision-making problems. We propose a dual-self model that adopts the theoretical framework established by Fudenberg and Levine[1] and expands to strategic interactions among multiple players. In this paper, we use an example of two-period cooperation game with multiple equilibria to illustrate the application of our dual-self model in games of complete information. We analyze the example under each of the three scenarios: history-independent strategy, history-dependent strategy, and dual-self approach, and compare the results.

We would like to point out that our current work on dual self has neither expanded to games of incomplete information nor considered the impact of social preferences, which could be directions for future work.

Aknowledgements

We are indebted to David K. Levine for his generous advice and continuous support. We would like to thank Jaimie W. Lien for the constructive suggestions and beneficial discussions. We are grateful to Jason Shachat, Yongchao Zhang and other session participants at the China International Conference on Game Theory and Applications for their helpful comments. All errors are our own.

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