Bayesian Persuasion in Sequential Tullock Contests

Microeconomics III

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Agenda of Presentation

- Background and Literature
- Model and Results
  - Contest Setting
  - Bayesian Persuasion
  - Discrete Type
  - Binary Type
- Summary and Discussion

Before Starting: An Example for Bayesian Persuasion

Example (Prosecution)

A prosecutor tries to convince a judge that a defendant is guilty. Suppose the judge must choose one of two actions: to *acquit* or *convict* a defendant. The defendant is either *guilty* or *innocent*. The judge gets 1 for choosing the right action and 0 otherwise. The prosecutor gets 1 if the judge convicts and 0 otherwise. The prosecutor and the judge share a prior belief Pr(*guilty*)= 0.3. We formalize an investigation as distributions $\pi(\cdot|guilty)$ and $\pi(\cdot|innocent)$ on some set of signal realizations, shown as follows. If persuaded, the judge convicts with probability 60 percent.

$$\begin{aligned} \pi(i|innocent) &= \frac{4}{7} \qquad & \pi(g|innocent) &= \frac{3}{7} \\ \pi(i|guilty) &= 0 \qquad & \pi(g|guilty) &= 1 \end{aligned} \tag{1}$$

Background

- Organizations are often interested in how to motivate individuals by using information disclosure policy.
- Real World Example:
  - A sales manager wants to maximize his employees' total efforts in a sales contest, and needs to decide under what conditions he will disclose the information about employees' abilities.
- In a competing environment with heterogeneous competitors, information disclosure plays a great role and often known as one of the the most popular and effective policies to increase total effort.
- Bayesian persuasion, an approach of information disclosure, is studied extensively after Kamenica and Gentzkow (2011).

Most Related Literature

Sequential (Tullock) Contests:

- Leininger (1993): **Binary** type distribution, complete information, contestants choose whether they move 'Early' (E) or 'Late'(L)
- Linster (1993): **Binary** type distribution, complete information (also solving for unique PBE for incomplete information setting)
- Morgan (2003): **Binary** type distribution, complete information
- Kuang and Zheng (2017): **Binary** type distribution, three different scenarios with incomplete information

Most Related Literature

Information Disclosure in Simultaneous (Tullock) Contests:

- Zhang and Zhou (2016): Asymmetric setting, discrete type distribution, type-dependent optimal **Bayesian Persuasion** (interior solution only)
- Serena (2016): Symmetric setting, binary distribution, designer's type-dependent symmetric disclosure policy

Most Related Literature

Information Disclosure in Sequential (Tullock) Contests:

- Chen, Kuang and Zheng (2017): Optimal Joint Design of Timing and Information Disclosure in Tullock Contests
- ...

What We Do in this Project

- The problem we consider in this paper:
  - One competitor with private information on binary distributed types will compete against the other competitor in a sequential Tullock contest game to maximize their own payoff
  - The designer, who can pre-commit to a Bayesian persuasion signal, aims to maximize the total effort of competitors.
- Questions to answer:
  - Does Bayesian persuasion improve the result under binary type distributions in simultaneous Tullock contests when considering boundary solutions?
  - How do competitors rationally behave under different signals and posterior distributions in asymmetric-information sequential Tullock contests?
  - How to find the optimal Bayesian persuasion signal in sequential Tullock contests?

Contributions to the Literature

- We show the **Uniqueness of Perfect Bayesian Equilibrium** for 2-player asymmetric-information sequential Tullock contests with discrete types.
- We strengthen the result by Zhang and Zhou (2016) that the **Optimal Bayesian Persuasion strategy** with a binary follower is **degenerated** for **simultaneous Tullock contests**.
- Most importantly, we characterize the **Optimal Bayesian Persuasion strategy** for **sequential Tullock contests**, and we show that even with binary type distributions, Bayesian persuasion can potentially generate higher total effort than degenerated strategies.

Contest Settings with Binary Type

- 2 risk neutral players ($i = A, B$) participate in a single-prize sequential Tullock contest, player A moves first and player B moves next.
- Player A's value of winning v_A is public information
- Player B's value of winning $v_B \in \{v_l, v_h\}$ is private information, with prior $\Pr(v_B = v_l) = q$ and $\Pr(v_B = v_h) = 1 - q$
- Player i chooses his bid $b_i \geq 0$, and his success function is $p_i(b_1, b_2) = \frac{b_i}{b_1+b_2}$.
- Player i's payoff is $u_i(b_1, b_2|v_i) = \frac{b_i}{b_1+b_2} v_i - b_i$.

Contest Settings with Discrete Type

- V_B is a discrete random variable on Ω with $N \geq 2$ values $v_{B1} < \cdots < v_{BN}$.
- $\Delta^{N-1} = \{\mu \in \mathbb{R}^N : \mu_n \geq 0, \sum_{n=1}^{N} \mu_n = 1\}$ denote the standard $(N-1)$-simplex in $\mathbb{R}^N$ and $\text{int}(\Delta^{N-1})$ denote the interior of Δ^{N-1}.
- For each point $\mu \in \text{int}(\Delta^{N-1})$, the probability for each valuation is strictly positive.
- Denote the prior distribution of V_B as $\mu^0 = \{\mu_1^0, \cdots, \mu_N^0\}$. Assume $\mu^0 \in \text{int}(\Delta^{N-1})$.

Bayesian Persuasion (with Discrete Type)

- The contest designer can pre-commit to a signal before contest starts in order to maximize expected total effort Υ.
- A signal π consists of a realization space S and a family of likelihood distributions $\pi = \{\pi(\cdot|v_{Bn})\}_{n=1}^{N}$ over S.
- For each v_{Bn}, the signal generates a distribution over the signal space S, and the signal π can be represented by an $N \times |S|$ matrix.
- Potential instruments for contest designer are quite rich, such as no disclosure and full disclosure.
- When a signal $s \in S$ is realized, contestant A needs to update his belief about contestant B by applying Bayes' rule. Denote this posterior belief as $\mu \in \Delta^{N-1}$, which may lie on the boundary of Δ^{N-1}.

Timing of the game

- The contest designer chooses and pre-commits to a signal π.
- Nature moves and draws a valuation for contestant B, say v_{Bn}.
- The contestant designer carries out his commitment and a signal realization $s \in S$ is generated according to $\pi(s|v_{Bn})$.
- The signal realization s is observable by the public and leads to a posterior belief of contestant B, denoted as μ.
- The contest takes place, contestant A chooses his effort, then contestant B chooses his effort afterwards.

Bayesian Plausible

- Denote a distribution of posteriors as $\tau \in \Delta(\Delta^{N-1})$.
- τ is a random variable that takes value in the simplex Δ^{N-1}.
- We call τ *Bayesian-plausible* if the expected posterior probability equals the prior.
- Kamenica and Gentzkow (2011) show that finding optimal signal π is equivalent to searching over **Bayesian-plausible** distribution of posteriors τ, which maximize the expected value of the posterior expected total effort $\mathbb{E}(\Upsilon)$.

Theorem

The optimal signal always exists and achieves an expected total effort equal to $\mathbf{cav}\Upsilon(\mu^0)$.

Uniqueness of Perfect Bayesian Equilibrium

Theorem

In incomplete information sequential Tullock contest with two contestants, where A's valuation is commonly known as v_A and B's valuation is discrete distributed, there exists a unique perfect Bayesian equilibrium in which contestant A chooses effort x_A^* such that

$$x_A^* \in \{\min x_A \in \mathbb{R} : \frac{\partial \Pi_A}{\partial x_A}^+ \leq 0\} \tag{2}$$

where

$$\frac{\partial \Pi_A}{\partial x_A}^+ = \sum_{n=1}^{N} \frac{\mu_n v_A}{2\sqrt{x_A v_{Bn}}} \mathbb{I}(x_A < v_{Bn}) - 1$$

and contestant B chooses effort accordingly, then

$$\Upsilon(\mu) = \mathbb{E}_\mu(\max(\sqrt{v_{Bn} x_A^*}, x_A^*)) \tag{3}$$

Now, the Binary Type Case

Optimal Bayesian Persuasion in Simultaneous Contest

Theorem

In simultaneous Tullock contest with binary distribution, either no disclosure or full disclosure is optimal among all possible signals. In other words, Bayesian persuasion cannot yield higher effort.

Proof Sketch.

Bayesian persuasion cannot yield higher effort when solution is interior by Zhang and Zhou (2016). What we need to do is to analyze the boundary case, namely when

$$v_A > \frac{v_H\sqrt{v_L}}{\sqrt{v_H} - \sqrt{v_L}} \tag{4}$$

We claim that **full disclosure** is optimal in such a case. □

Optimal Bayesian Persuasion in Simultaneous Contest

Proof Sketch (cont.)

In other words, $\phi(q)$ lies under the line segment between $(0, \phi(0))$ and $(1, \phi(1))$. Mathematically, for any $q \in [0, 1]$

$$\phi(q) \leq \phi(0) + q(\phi(1) - \phi(0)) = (1 - q)\phi(0) + q\phi(1) \quad (5)$$

we call it **full disclosure condition**. By complex algebra techniques, we show that for any $q \in [0, 1]$, **full disclosure condition** holds. □

Optimal Bayesian Persuasion in Sequential Contest

Theorem

The optimal Bayesian persuasion is shown in the following table.

range of v_A	q	*optimal*
$v_A < 2v_L$	*[0,1]*	*no disclosure*
$v_A \in [2v_L, 2\sqrt{v_H v_L}]$	*[0,1]*	*no disclosure*
$v_A \in (2\sqrt{v_H v_L}, 2v_H]$	*[0,1]*	*full disclosure*
$v_A > 2v_H$	$q \in (1 - \frac{2v_H}{v_A}, 1)$	*BP*
	$q \in [0, 1 - \frac{2v_H}{v_A}] \cup \{1\}$	*no disclosure*

Contest designer benefits from Bayesian persuasion if and only if $v_A > 2v_H$ *and* $q \in (1 - \frac{2v_H}{v_A}, 1)$.

Proof Details (1) $v_A < 2v_L$

Lemma

With binary distribution such that $v_A < 2v_L$, no disclosure is optimal among all possible signals.

Proof.

Solutions are interior since $v_L = v_{B1} > \frac{v_A}{2}$ condition is satisfied.

The optimal strategy for contestant A is $x_A^* = \frac{v_A^2}{4}(\frac{q}{\sqrt{v_L}} + \frac{1-q}{\sqrt{v_H}})^2$.

We can re-write the expected total effort as $\phi_1(q)$

$$\phi_1(q) = \frac{v_A}{2}\Big((2q^2 - 2q + 1) + (q - q^2)(\frac{\sqrt{v_H}}{\sqrt{v_L}} + \frac{\sqrt{v_L}}{\sqrt{v_H}})\Big) \tag{6}$$

Second order derivative $\phi_1''(q) = v_A(2 - \frac{\sqrt{v_H}}{\sqrt{v_L}} - \frac{\sqrt{v_L}}{\sqrt{v_H}}) < 0$. □

Proof Details (2) $v_A \in [2v_L, 2\sqrt{v_H v_L}]$

Lemma

When $v_A \in [2v_L, 2\sqrt{v_H v_L}]$, no disclosure is optimal.

Proof Sketch.

If we extend the left side of $\phi(q)$, quadratic function $\phi_1(q)$, into the interval $[q_1, 1]$. We can observe that linear function $\phi_2(q)$ is located below concave $\phi_1(q)$ among interval $q \in [q_1, 1]$ both function intersect at $q = q_1$ and

$$\phi_2(1) = v_L < \frac{v_A}{2} = \phi_1(1) \tag{7}$$

Hence, at turning point q_1, the left-hand side derivative is obviously higher than right-hand side derivative. In conclusion, the concave closure of $\phi(q)$ is itself. □

Proof Details (3) $v_A \in (2\sqrt{v_H v_L}, 2v_H]$

Lemma

When $v_A \in (2\sqrt{v_H v_L}, 2v_H]$, full disclosure is optimal.

Proof Sketch.

The total expected effort $\phi(q)$ is a two-piecewise function. In order to prove that full disclosure is optimal, we need to prove that for any $q \in [0, 1]$, **full disclosure condition** holds. The slope of line segment between $(0, \phi(0))$ and $(1, \phi(1))$ is $\phi(1) - \phi(0) = v_L - \frac{v_A}{2}$ while when $q \in [q_2, 1]$, the derivative $\phi_2'(q) = v_L - \sqrt{v_H v_L} > \phi(1) - \phi(0)$ and $\phi(q_2) < \phi(0) + q_2(\phi(1) - \phi(0))$. Hence, **full disclosure condition** holds when $q \in [q_2, 1]$. Then can prove that when $q \in [0, q_2]$, **full disclosure condition** holds. □

Proof Details (4) $v_A > 2v_H$

Theorem

Under the assumption of binary distribution, the contest designer benefits from Bayesian persuasion if and only if $v_A > 2v_H$ and $q \in (q_3, 1)$. The optimal signal induces beliefs $q_h = q_3$("HIGH") and $q_l = 1$("LOW"). Then the expected total effort after Bayesian persuasion, $\mathbf{cav}\phi(q)$ is a piecewise linear function,

$$\mathbf{cav}\phi(q) = \begin{cases} v_H & q \leq q_3 \\ v_L + (1-q)\frac{v_A(v_H - v_L)}{2v_H} & q > q_3 \end{cases} \tag{8}$$

An Example of Bayesian Persuasion in a Sequential Tullock Contest

Example

In a sequential Tullock contest, winning valuation of contestant A $v_A = 4.8$ and prior belief about v_B is

$$v_B = \begin{cases} 1 & \text{w.p.} \quad 0.5 \\ 2 & \text{w.p.} \quad 0.5 \end{cases}$$

The expected total effort by three different kinds of strategy are listed in the following table.

Strategy	Expected total effort
No disclosure	$\frac{1+\sqrt{2}}{2} \approx 1.2071$
Full disclosure	1.5
Bayesian Persuasion	1.6

Summary

- We fully characterize the optimal Bayesian persuasion signal in a sequential Tullock contest of 2 players, and show that even with binary type distributions, Bayesian persuasion can potentially generate higher total effort than degenerated strategies.
- Our work extends the Bayesian persuasion results of simultaneous Tullock contests in Zhang and Zhou (2016).

Directions for Future Work

- Correlated Signal When Both Players Have Private Values in Simultaneous Tullock Contests
- Optimal Joint Design for Bayesian Persuasion and Timing Policy
- Choice of Players to Be Persuaded

Thank You!