Bayesian Persuasion in All-pay Auction Contests*

Jian Chen (Tsinghua University)†
Zhonghong Kuang (Tsinghua University)‡
Jie Zheng (Tsinghua University)§

First Version August 31th, 2017; Updated November 8, 2019

Abstract

We study the optimal information disclosure policy via Bayesian persuasion approach (Kamenica and Gentzkow, 2011) in a two-player all-pay auction contest with one-sided asymmetric information in both simultaneous move setup and sequential move setup. The designer can pre-commit to a signal device that generates a type-dependent distribution, signaling the type of the player with private information to the uninformed player. We completely characterize the optimal Bayesian persuasion signal in both simultaneous contests and sequential contests, and divide the effective Bayesian persuasion strategies into three categories (*Threat, Fluke, and Harmony*). As extensions of the model, we analyze the scenario where the informed contestant has the opportunity to reveal information after the designer's disclosure policy is implemented, and the scenario where the designer can decide on the order of move for the contestants in addition to the information disclosure decision. Our results provide insights for the optimal design of information structure for situations where two information-asymmetric parties are competing against each other.

JEL Classification: C72, D72, D82.

Keywords: All Pay Auction; Mixed Strategy; Bayes Nash Equilibrium; Information Disclosure; Bayesian Persuasion

---

*We are especially grateful to Kai Konrad, Dan Kovenock, and Jingfeng Lu for detailed comments which helped us improve the manuscript. For helpful comments and insights, we thank Bo Chen, Jaimie W. Lien, seminar participants at the xxxxx, and conference participants at the 2017 NWPU Interdisciplinary Forum on Operation Research, Economics and Management and International Conference on Game Theory & 4th Microeconomics Workshop. This research is funded by the National Natural Science Foundation of China (No.61661136002 and No.71873074). All errors are our own.

†Department of Management Science and Engineering, School of Economics and Management, Tsinghua University, Beijing, China, 100084 (e-mail: chenj@sem.tsinghua.edu.cn).

‡Department of Management Science and Engineering, School of Economics and Management, Tsinghua University, Beijing, China, 100084 (e-mail: kuangzhh.15@sem.tsinghua.edu.cn).

§Department of Economics, School of Economics and Management, Tsinghua University, Beijing, China, 100084 (e-mail: jie.academic@gmail.com).

1 Introduction

Contests are prevalent and essential in social and economic activities. Examples of contests include patent races among firms as well as promotion tournaments in organizations, lobbying and election campaigns among politicians, and sports competition. In such contests, agents expend irreversible resources to win a reward. The main objective of the contest organizer is to maximize the expected total effort of contestants. In a patent race, the industry organizer hopes to maximize total investment on some specific technology. The employer who organizes a promotion contest wishes to boost employees' efforts.

As mentioned in the literature and occured in reality, agents (or called contestants) have private information about their own capabilities or the valuation of the prize, which highly influence the strategy of contestants. Meanwhile, contest organizer can set game rules for information disclosure, which naturally raises the question of information disclosure in contests. The contest designer may apply different information disclosure policies to influence the beliefs of contestants about private information of each other. In the US, the government agency can require lobbyists to provide financial information; Federal Election Campaign Act requires candidates in political campaigning to disclose sources of campaign contributions and campaign expenditure quarterly; as for competition among listed companies, the Securities and Exchange Commission (SEC) and the Federal Accounting Standards Board (FASB) regulate firms' disclosure of financial information which is accessible by their competitors (Denter, Morgan and Sisak, 2012). Those examples mentioned above convey information about the financial support which greatly influences the decision making process. Similarly, scholars compete for grants by submitting research proposals, and presumably they do not know who they are competing against unless the grant-awarding entity, whose purpose is to stimulate research, publicly discloses the list of applicants. It is possible that an ordinary researcher aware of competing for a grant against a leading scholar might give up or exert little effort.

In this paper, we study how to design the optimal information disclosure strategy in all-pay auction contests with one-sided asymmetric information, following the framework of Bayesian persuasion in contests developed in several recent studies (Chen, Kuang and Zheng, 2017*a*, Zhang and Zhou, 2016). Although information disclosure in Tullock contests is well studied, to the best of our knowledge, there is lack of research about the optimal design of information disclosure in all-pay auction contests via the Bayesian persuasion approach. What is more, most existing studies on information disclosure in all-pay auction contests only consider two kinds of strategy, no disclosure (or called full concealment) and full disclosure. We call these two **degenerated** information disclosure strategies later in the paper. With these two strategies, the uninformed player's beliefs either remains the same as his prior (in the no disclosure case) or coincides with the realized state (in the full disclosure case), before a contest starts. This zero-or-one choice seems straightforward. However, this restriction of choice causes loss of generality from the organizer's perspective because the organizer in real life can often choose some disclosure policies in between to partially reveal the information. The Bayesian persuasion approach pioneered by Kamenica and Gentzkow (2011) provides us with new methodology to handle this problem. We say that contest designer **benefit from Bayesian persuasion** if the optimal Bayesian persuasion signal generates strictly higher expected total effort than degenerated information disclosure policy. Zhang and Zhou (2016) first applied this Bayesian persuasion approach to study optimal information disclosure in 2-player one-sided asymmetric-information simultaneous-move Tullock contests, and they found that when the type of informed contestant is binary, it suffices to only compare degenerated strategies. However, in the later study done by Chen, Kuang and Zheng (2017*a*), they adopt the Bayesian persuasion approach to study the optimal information disclosure problem in sequential Tullock contests, and show that even with binary type distributions, Bayesian persuasion can generate higher total effort than degenerated strategies in a sequential move setting.

As mentioned in the previous paragraph, the timing of contests plays an important role and influences the results of optimal Bayesian persuasion signal to a great extent. In this paper, we study the information disclosure problem of all-pay contests, not only in a static setting, but also in a dynamic setting. The simultaneous all-pay auction contests have gained greater attention from researchers compared with the sequential all-pay auction contests which have not been well explored (Segev and Sela, 2014*a*,*b*,*c*). Amann and Leininger (1995, 1996) extend the general auction framework of Milgrom and Weber (1982) into all-pay auction and prove the existence and uniqueness of Bayesian equilibrium for a class of generally asymmetric all-pay auctions with incomplete information. Siegel (2009) builds the link between auctions and contests by studying all-pay contests. All-pay auctions is now treated as one of the three canonical contest models (Dechenaux, Kovenock and Sheremeta, 2015).[1] See Konrad (2011) for a survey on all-pay auction contests.

Our work contributes to the literature on information disclosure in contests. Among many others, some related studies focus on Tullock contests, either under a simultaneous setting or a sequential setting. Serena (2016) studies all the possible type-dependent non-probabilistic disclosure policies (in terms of either concealment for sure or disclosure for sure) in a 2-player simultaneous Tullock contest. Wu and Zheng (2017) consider the type-independent disclosure policies in a similar environment that allows players to share information, and Zhang and Zhou (2016) examine the type-dependent probabilistic disclosure policies (via Bayesian persuasion approach) in 2-player simultaneous contests. Chen, Kuang and Zheng (2017*b*) compare type-dependent non-probabilistic disclosure policies in 2-player sequential Tullock contests, and in a separate study they investigate the optimal Bayesian persuasion signal in 2-player sequential Tullock contests (Chen, Kuang and Zheng, 2017*a*).

The problem on information disclosure in simultaneous all-pay auction contests has attracted more attention recently. For the all-pay auction scenario, previous research shows that bidders do not have incentive to disclose any information if they can only decide within degenerated

[1] The other two frameworks are Tullock contests (Buchanan, Tollison and Tullock, 1980) and rank-order tournament(Lazear and Rosen, 1981).

policies. Szech et al. (2011) find out that partial disclosure dominates degenerated policy. Lu, Ma and Wang (2018) further show that full concealment is the best option for the designer by ranking four different type-dependent non-probabilistic disclosure policies in a 2-player 2-type simultaneous all-pay auction, which shares the same methodology as Serena (2016). Morath and Munster (2013) studies information acquisition prior to an one-sided asymmetric information all-pay auction. Kovenock, Morath and Munster (2015) investigate the incentives to share private information ahead of contests where contestants have independent values or common values, and they show if the decision is made independently, sharing information is dominated.

Our paper also belongs to the literature on equilibrium characterization in all-pay auctions. Hillman and Riley (1989) firstly characterizes the unique mixed strategy equilibrium of 2-player all-pay auction with complete information, and Baye, Kovenock and Vries (1996) further characterize equilibrium for such games with more than two players, and show that the set of equilibria is much larger than has been recognized in the literature. Siegel (2014) studies monotone equilibria in a two-bidder all-pay auction with multiple types. Szech et al. (2011) fully characterize the unique equilibrium of an asymmetric all-pay auction with binary types, assuming that two bidders' types are independently drawn from different two-point distributions and types as well as probabilities may differ among bidders. Equilibrium characterization in our model can be viewed as a special case of Szech et al. (2011) by fixing the value of one player. It is notable that unique mixed strategy equilibrium of 2-player all-pay auction with incomplete information can be constructed by both Siegel and Szech et al.'s work. Most recently, Liu and Chen (2016) finds the unique mixed strategy equilibrium in symmetric two-player all-pay auction contests with correlated information, which may help us extend our result into correlated signals when both players have private values with ex ante symmetric distribution.

Our study is closely related to the studies (Chen, Kuang and Zheng, 2017*a*, Kuang, 2019, Kuang, Zhao and Zheng, 2019, Zhang and Zhou, 2016) which adopt the Bayesian persuasion framework (Kamenica and Gentzkow, 2011) to investigate the information disclosure problem in contests. This approach, compared with the traditional non-probabilistic information disclosure approach, allows the designer to pre-commit to a signal in terms of a distribution, conditional on the realization of the type. Since now the designer can choose to disclosure information partially through a distribution, instead of having to either report the realized type or report nothing, this can potentially achieve a higher payoff for the designer by enlarging his strategy space.

In our all-pay auction contests model, there are two contestants: the contestant A, with commonly known valuation v_A and known as **public** player; contestant B, with privately known valuation on support $\{v_H, v_L\}$ with $v_H > v_L > 0$ and known as **private** player. The contest organizer and contestant A share the same prior belief about contestant B's valuation. The most important feature of this contest model is that the contest designer can **pre-commit** to a signal before the contest starts. This signal can be seen as a conditional distribution on contestant B's valuation. As a result, contestant A can update his belief about contestant B's valuation after observing a realization of signal. Finally, contestant A and contestant B engage in the all-pay auction contest. Contestants are utility-maximizers and the contest designer aims at maximizing his expected total effort. The contest is either a static version (see Section 4) or a dynamic version with public player as the first mover (see Section 5).[2]

Our paper has three theoretical contributions. Firstly, we characterize the optimal Bayesian persuasion strategy for simultaneous one-sided asymmetric-information all-pay auction contests with binary type distributions, and we show that Bayesian persuasion can potentially generate higher total effort than degenerated strategies, in contract to the result under the setup of simultaneous one-sided asymmetric-information Tullock contests (Zhang and Zhou, 2016). Secondly, we characterize the optimal Bayesian persuasion strategy for sequential asymmetric information all-pay auction contests with binary type distributions, and we show that Bayesian persuasion can potentially generate higher total effort than degenerated strategies, consistent with the result under the setup of sequential asymmetric information Tullock contests (Chen, Kuang and Zheng, 2017*a*). Last but not least, we compare optimal Bayesian persuasion strategies in four different settings with two dimensions, all-pay auction contests versus Tullock contests, simultaneous move versus sequential move and we summarize two behaviors that the contest designer may take advantage of, **Threat** and **Fluke**, as well as a unimodal function feature called **Harmony**.

Table 1: Bayesian Persuasion analysis of Four Classes of Contests

	Simultaneous Move	Sequential Move
Tullock	Zhang and Zhou (2016)	Chen, Kuang and Zheng (2017*a*)
All-pay auction	Section 4	Section 5

Table 2: Insights behind Bayesian Persuasion

	Simultaneous Move	Sequential Move
Tullock	-	Threat
All-pay auction	Threat, Harmony	Threat, Fluke

For both simultaneous and sequential setups, when the contestant B's valuation follows a binary distribution (i.e. v_H w.p. p and v_L w.p. $1-p$), it is loss of generality to focus on degenerated strategies. We provide the necessary and sufficient condition for strict dominance of Bayesian persuasion (see Theorem 4.6 and Theorem 5.1). We show that full disclosure is optimal when the winning value v_A of contestant A is less than some threshold (v_L in both setups).[3] On the contrary, no disclosure is optimal policy when v_A is more than some threshold. What is more, this threshold value is also the threshold of v_A for which Bayesian persuasion is effective under both

[2]Note that for a dynamic contest with private player as the first mover, it is essentially equivalent to a dynamic contest with complete information.

[3]Such a threshold is $\sqrt{v_H v_L}$ for simultaneous Tullock contests (Zhang and Zhou, 2016) and $2\sqrt{v_H v_L}$ for sequential Tullock contests (Chen, Kuang and Zheng, 2017*a*).

simultaneous and sequential all-pay auction contests.

Table 3: Threshold I: Apply Degenerated Policy when Lower

	Simultaneous Move	Sequential Move
Tullock	$\sqrt{v_H v_L}$	$2\sqrt{v_H v_L}$
All-pay auction	v_L	v_L

Table 4: Threshold II: Benefit from Bayesian persuasion when Higher

	Simultaneous Move	Sequential Move
Tullock	-	$2v_H$
All-pay auction	v_L	v_L

We know that Bayesian persuasion may be effective for the contest designer under sequential Tullock contests but not simultaneous Tullock contests for binary types (Chen, Kuang and Zheng, 2017*a*, Zhang and Zhou, 2016). When the valuation of contestant A is much greater than the higher value of contestant B ($v_A > 2v_H$), Bayesian persuasion can potentially do better than the degenerated strategies. Note that in the case of $v_A > 2v_H$, under the full disclosure policy, contestant A will always win the game by submitting the realized winning value of contestant B, to stop contestant B from entering the competition. What is more, for strong type B, if we slightly increase the probability of weak type B from zero to some small positive number, contestant A will use the same strategy as if all rivals are still strong type. This key fact creates an opportunity for Bayesian persuasion to generate higher total effort than naive strategies. The contest designer can disguise some of the weak type contestant B as the strong type one such that contestant A will remains his strategy after Bayesian updating. Thus the probability that contestant A is exerting higher effort (when facing a strong type contestant B) increases, and therefore the contest designer can achieve higher expected total effort. Such a logic still holds for both simultaneous and sequential all-pay auction contests except that the threshold becomes v_H instead of $2v_H$.

In both simultaneous and sequential all-pay auction contests, more complicated case can arise. Those features are not captured by the framework of Tullock contests. For simultaneous all-pay auction contests, when v_A is not too much greater than the threshold ($v_L < v_A < v_H$), Bayesian persuasion can do better than the degenerated strategies almost everywhere except for the case when $p \in \{0, p^*, 1\}$ where $p^* = \frac{v_A v_H - v_H v_L}{v_A v_H - v_A v_L}$ under which harmonic mean of winning valuation of contestant B equals v_A. Intuitively, one would expect that a larger proportion of high type player B could lead to higher expected revenue because the contest then would become more severe. However, such an intuition is false when $v_L < v_A < v_H$. The expected total effort function is unimodal with respect to p. Player A may be frustrated when facing too many of high type player B and less motivated when facing too many of low type player B. There exists an optimal value of p^* that provides a harmonic contest environment that can stimulate both players at maximum level. Taking advantage of this property as well as the convexity property, the contest designer can potentially increases his revenue almost everywhere by setting one of the posteriors to p^*.

For sequential all-pay auction contests, when v_A is not too much greater than the threshold ($v_L < v_A < v_H$), Bayesian persuasion can also do better than the degenerated strategies as under the simultaneous setting, but with different insight. Under the full disclosure policy, contestant A will win the contest by submitting v_L when the realized type of B is low and quit the contest when the realized type of B is high. However, for weak type B, if we slightly increase the probability of high type B from zero to some small positive number, contestant A will bid the same strategy as if all rivals are still weak type. The contest designer can disguise some of the strong type contestant B as the weak type one such that contestant A will remain his strategy after Bayesian updating. Thus the probability that contestant A bids positively (when facing a weak type contestant B) increases, so does contestant B. Therefore, the contest designer can achieve higher expected total effort.

In the first extension of our model, we study the incentive of contestant B to share type information. In this setting, contestant B can choose whether or not to reveal type information contingently on signal realization immediately after the Bayesian persuasion stage. If contestant B choose to reveal type information under some Bayesian persuasion signal realization, the contest will hence take place with complete information. Otherwise, the posterior profile associated with Bayesian persuasion signal will remain the same. We find that contestant B has no incentive to reveal type information in simultaneous contests in any case, and has no incentive to reveal type information in sequential contests unless $v_L < v_A < v_H$ and $p < 1 - \frac{v_L}{v_A}$. Further more, when taking into account the incentives of contestant B, the optimal Bayesian persuasion signals in both simultaneous contests and sequential contests are unchanged, which is consistent with the result in Tullock contests (Chen, Kuang and Zheng, 2017*a*).

Our second extension endogenizes timing decisions for the contest organizer. Before Bayesian persuasion stage, we allow contest designer to choose one of the following three mechanisms: simultaneous move contest; sequential contest with the player of private information as the first mover, sequential contest with the player of public information as the first mover. If contest designer chooses simultaneous move contest, then the contest we analyzed in Section 4 occurs. If contest designer chooses sequential contest with the player of private information as the first mover, then the sequential contest of complete information occurs. If contest designer chooses sequential contest with the player of public information as the first mover, then the contest we analyzed in Section 5 occurs. We find that when contestant A has higher winning valuation than both types of contestant B, contestant A is preferred to be the first mover; when contestant A has lower winning valuation than both types of contestant B, contestant B is preferred to be the first mover. When contestant A has winning valuation between two types of contestant B, all timing policy may be optimal depending on the prior on contestant B's types.

The rest of the paper is organized as follows. Section 2 previews our main results through illustrative examples. In Section 3, we describe the model. In Section 4, we solve the optimal signal for the simultaneous all-pay auction contest. In Section 5, we solve the optimal signal for the sequential case. We provide two extensions of main models in Section 6 where we introduce the incentive of contestant B to share type information and Section 7 which endogenizes the timing decision for contest designer. In Section 8, we conclude. Technical proofs are relegated to the appendix.

2 Examples

As mentioned in Table 2, there are two different situations when benefit from Bayesian persuasion in both simultaneous contest and sequential contest.

2.1 Examples in Simultaneous Contests

Example 2.1. *The winning value of contestant A is $v_A = 2$ and the prior belief about v_B is*

$$v_B = \begin{cases} 1 & w.p. \quad 0.5 \\ 3 & w.p. \quad 0.5 \end{cases}$$

The expected total effort under three different information disclosure strategies are shown in the following graph where partial disclosure policy generates posterior $\frac{3}{4}H + \frac{1}{4}L$ with probability $\frac{2}{3}$ and L with probability $\frac{1}{3}$.

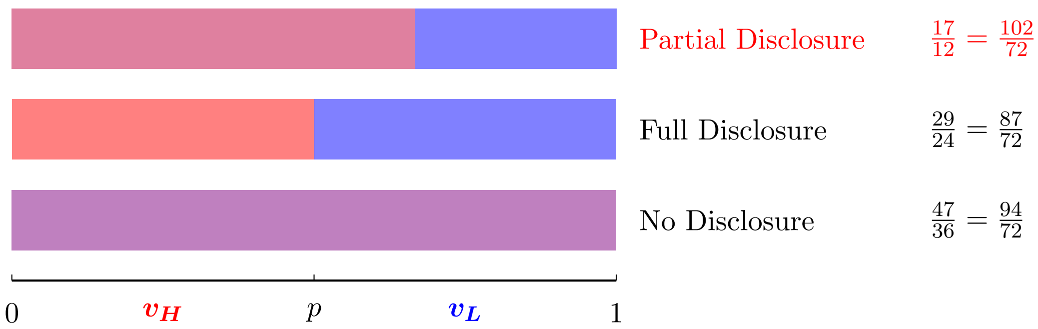


Figure 1: Three Information Disclosure Strategies for Example 2.1

The following signal structure implements prevous partial disclosure policy. Let realization space S consists of two realized signal, $S = \{s_L, s_H\}$. When winning value of contestant B is high, s_H is generated with probability 1. When winning value is low, s_L is generated with probability $\frac{2}{3}$ and s_H is generated with probability $\frac{1}{3}$.

Example 2.2. *The winning value of contestant A is $v_A = 3$ and the prior belief about v_B is*

$$v_B = \begin{cases} 1 & w.p. \quad 0.5 \\ 2 & w.p. \quad 0.5 \end{cases}$$

The expected total effort under three different information disclosure strategies are shown in the following graph where partial disclosure policy generates posterior $\frac{2}{3}H + \frac{1}{3}L$ with probability $\frac{3}{4}$ and L with probability $\frac{1}{4}$.

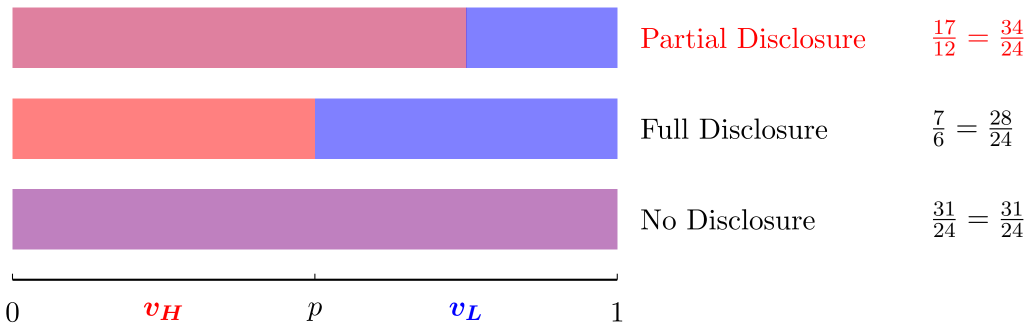


Figure 2: Three Information Disclosure Strategies for Example 2.2

The following signal structure implements prevous partial disclosure policy. Let realization space S consists of two realized signal s_L, s_H as before. When contestant B has winning value v_H, s_H is generated with probability 1. When winning value is v_L, s_L is generated with probability $\frac{1}{2}$ and s_H is generated with probability $\frac{1}{2}$.

In Example 2.1 (where $v_L < v_A < v_H$) and Example 2.2 (where $v_A > v_H$), Bayesian persuasion can generate higher expected effort than the no disclosure or full disclosure policy. We show that although the state of the informed contestant (the following contestant) is as simple as binary, it may be payoff-improving for the designer to apply Bayesian persuasion. Further more, it is worth noting that the contest designer may benefit from Bayesian persuasion as long as $v_A > v_L$.

2.2 Examples in Sequential Contests

Example 2.3. *In a sequential all-pay auction contests, the winning value of contestant A is $v_A = 2$ and the prior belief about v_B is*

$$v_B = \begin{cases} 1 & w.p. \quad 0.25 \\ 3 & w.p. \quad 0.75 \end{cases}$$

The expected total effort under three different information disclosure strategies are shown in the following graph where partial disclosure policy generates posterior $\frac{1}{2}H + \frac{1}{2}L$ with probability $\frac{1}{2}$ and H with probability $\frac{1}{2}$.

The following signal structure implements prevous partial disclosure policy. Let realization

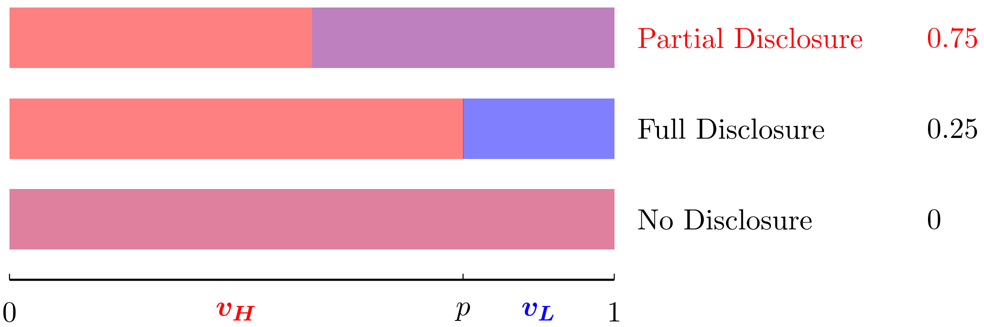


Figure 3: Three Bayesian Persuasion Strategies for Example 2.3

space S consists of two realized signal, $S = \{s_L, s_H\}$. When winning value of contestant B is low, s_L is generated with probability 1. When winning value is high, s_L is generated with probability $\frac{1}{3}$ and s_H is generated with probability $\frac{2}{3}$.

Example 2.4. *In a sequential all-pay auction contests, the winning value of contestant A is $v_A = 4$ and the prior belief about v_B is*

$$v_B = \begin{cases} 1 & w.p. \quad 0.75 \\ 3 & w.p. \quad 0.25 \end{cases}$$

The expected total effort under three different information disclosure strategies are shown in the following graph where partial disclosure policy generates posterior $\frac{1}{2}H + \frac{1}{2}L$ with probability $\frac{1}{2}$ and l with probability $\frac{1}{2}$.

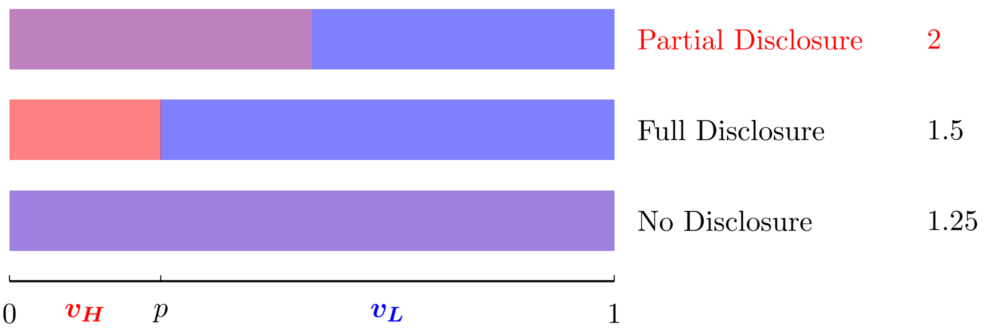


Figure 4: Three Bayesian Persuasion Strategies for Example 2.4

The following signal structure implements prevous partial disclosure policy. Let realization space S consists of two realized signal s_L, s_H as before. When contestant B has winning value v_H, s_H is generated with probability 1. When winning value is v_L, s_L is generated with probability $\frac{2}{3}$ and s_H is generated with probability $\frac{1}{3}$.

In Example 2.3 (where $v_L < v_A < v_H$) and Example 2.4 (where $v_A > v_H$), Bayesian persuasion can generate higher expected effort than the no disclosure or full disclosure policy. Compared with Example 2.1 and Example 2.2, Example 2.3 and Example 2.4 demonstrates that general method of Bayesian persuasion can increase the expected effort to a larger extent than full or no disclosure.

Bayesian persuasion is advantageous if player A's valuation is greater than v_L, where v_L denotes the lower possible valuation of player B, the same as in the simultaneous contest.

In all four examples, we design a stochastic mapping from $\{v_L, v_H\}$ to $\{s_L, s_H\}$ as signal structure to implement corresponding posterior profile. In this paper, we will show that we need at most two realized signal to implement optimal posterior profile for both simultanesou contests and sequential contests. What's more, at least one realized signal can be only generated by only one type but not the other.

3 The Model

3.1 Basic Setup

Information disclosure via Bayesian persuasion has been studied under the framework of both simultaneous and sequential Tullock contests, and we will follow the characterization of incomplete information in those studies (Chen, Kuang and Zheng, 2017*a*, Linster, 1993, Zhang and Zhou, 2016). We consider the following all-pay auction contests with incomplete information. Formally, two risk-neutral participants, A and B, compete for a single prize by exerting irreversible efforts. The success function of contestant $i \in \{A, B\}$ under effort portfolio (x_A, x_B) is given by

$$s_i(x_i, x_{-i}) = \begin{cases} 1 & \text{if} \quad x_i > x_{-i} \\ 0 & \text{if} \quad x_i < x_{-i} \end{cases} \tag{1}$$

Tie breaking rules are different among simultaneous contests and sequential contests, and please refer to subsection 3.3 for details. Player's payoff has a linear form, which is equal to his valuation v_i of winning multiplied by the winning probability s_i, minus the cost of effort.

$$\Pi_i = s_i v_i - x_i \tag{2}$$

Contest designer tries to maximize the expected total effort Π,

$$\Pi = x_A + x_B \tag{3}$$

Contestant A's valuation of winning is commonly known as v_A. Contestant B's value of winning v_B is his private information, but with a common prior shared by the contest designer and contestant A. To be more specific, v_B is a discrete random variable with 2 values $v_L < v_H$. For a binary distribution, it can be uniquely characterized by a single parameter $p = \Pr(v_B = v_H)$, and we use $\Pi(p)$ to denote the expected total effort under the distribution such that $\Pr(v_B = v_H) = p$ and $\Pr(v_B = v_L) = 1 - p$. The common prior is defined by p_0. We may also say that player A is stronger than player B if v_A is greater than v_B.[4]

Bayesian persuasion is studied extensively after Kamenica and Gentzkow (2011). Following the spirit of Bayesian persuasion approach and the previous studies on Tullock contests by Chen, Kuang and Zheng (2017*a*), Zhang and Zhou (2016), we allow the contest designer to pre-commit to a signal before the contest starts in order to maximize expected total effort. A signal π consists of a realization space S and a family of likelihood distributions $\pi = \{\pi(\cdot|v_i)\}_{i=L,H}$ over S. For each possible value of v_B, the signal generates a distribution over the signal space S and the signal π can be represented by an $2 \times |S|$ matrix. Potential instruments for the contest designer are quite rich, including the degenerated no disclosure and full disclosure policies as available choices.

When a signal $s \in S$ is realized, contestant A needs to update his belief about contestant B by applying Bayes' rule. Denote this posterior belief after receiving s as p_s, where $p_s \in [0, 1]$ with boundary scenarios ($p_s \in \{0, 1\}$) also under consideration.

The timing of the game is as follows.

1. The contest designer chooses and pre-commits to a signal π. (Timing of Policy Design)
2. Nature moves and draws a valuation for contestant B, say v_B.
3. The contestant designer carries out his commitment and a signal realization $s \in S$ is generated according to $\pi(s|v_B)$. (Timing of Policy Implementation)
4. The signal realization s is observable by the public and leads to a posterior belief of contestant B, denoted as p_s.
5. The all-pay auction contest takes place, and both contestants choose their efforts.

Note that decisions are made only in stage 1 (by the contest designer) and stage 5 (by the contestants). We call stage 1 the *Bayesian persuasion design stage* and stage 5 the *posterior contest game stage*, following the terminologies in Zhang and Zhou (2016). The posterior game is an all-pay auction with one-sided asymmetric information. In the Bayesian persuasion design stage, the contest designer has incentive to choose the optimal signal π in order to maximize the expected total effort of the contestants.

3.2 Bayesian Persuasion

In stage 1, the contest designer chooses the signal π to maximize the expected total effort in the contest. Given a signal realization s, this leads to a posterior belief p_s and total effort $\Pi(p_s)$. Denote a distribution of posteriors as $\tau \in \Delta(\Delta)$. τ is a random variable that takes value in the simplex Δ. Namely, it assigns a probability measure on the posteriors in the support of τ, $\tau = \{(\Pr(s), p_s)\}_{s\in S}$

[4] Note that it is mathematically equivalent between the current setting and a common-value all-pay auction contest of players with different marginal costs.

where $\Pr(s) > 0$ denotes the probability observing signal s and $\sum_{s\in S}\Pr(s) = 1$. We call τ *Bayes-plausible* if the expected posterior probability equals the prior,

$$\sum_s \Pr(s)p_s = p_0$$

Kamenica and Gentzkow (2011) show that finding optimal signal π is equivalent to searching over Bayes-plausible distribution of posteriors τ, for one that maximizes the expected value of the posterior expected total effort $\mathbb{E}_s(\Pi(p_s)) = \sum_s \Pr(s)\Pi(p_s)$. We can formally define the problem faced by the contest designer as the following

$$\begin{aligned} \max_{\tau\in\Delta(\Delta)} \quad & \mathbb{E}_s(\Pi(p_s)) \\ s.t. \quad & \sum_s \Pr(s)p_s = p_0 \end{aligned}$$

The indirect value function from the above maximization problem is exactly equal to the value of the concave closure of $\Pi(p)$ at the prior, denoted as $\mathbf{cav}\Pi(p)$, as is shown in the following proposition (Kamenica and Gentzkow, 2011).

Proposition 3.1. *The optimal signal always exists and achieves an expected total effort equal to* $\mathbf{cav}\Pi(p_0)$.

The optimal signals can be quite simple in some special cases. When $\Pi(p)$ is concave, then no disclosure is optimal. When $\Pi(p)$ is convex, then full disclosure is optimal. In order to find the optimal signal, we need to construct the concave disclosure of $\Pi(p)$ for $p \in [0, 1]$. In the later analysis, we use $\Pi(p)$ to represent expected total effort as a function of p, where p is the probability of player B being high type.

3.3 Timing

For simultaneous all-pay auction contests, public contestant A and private contestant B choose their effort level simultaneously. If $x_A = x_B$, the tie breaking rule is typically endogenously determined as a part of equilibrium with

$$s_A(x_A, x_B) + s_B(x_A, x_B) = 1 \tag{4}$$

Actually, in simultaneous contests, Bayesian Nash equilibrium is typically mixed-strategy equilibrium. Given the strategy of one contestant, for all possible strategies of the other contestant, the probability of a tie is 0. Therefore, tie breaking rule will not affect the equilibrium.

For sequential all-pay auction contests, public contestant A moves first and private contestant B moves after observing the the effort level of contestant A. If $x_A = x_B < v_B$, prize is assigned to participant B to avoid trivial case because contestant B can exert infinitesimal more $x_A + \varepsilon$

to win the game. If $x_A = x_B = v_B$, prize is assigned to participant A otherwise. Sequential simultaneous contests and sequential all-pay auction is that equilibrium with critical value may not be continuous as v_A changes.

4 Bayesian Persuasion in Simultaneous All-pay Auction Contests

4.1 Equilibrium Characterization

For full characterization of equilibrium, we define a threshold value $p^* = \frac{v_A v_H - v_H v_L}{v_A v_H - v_A v_L}$,. When posterior distribution is p^*, the harmonic mean of v_B equals v_A,

$$\frac{v_A v_H - v_H v_L}{v_A v_H - v_A v_L}\frac{1}{v_H} + \frac{v_H v_L - v_A v_L}{v_A v_H - v_A v_L}\frac{1}{v_L} = \frac{1}{v_A}$$

The equilibrium of discrete-type one-sided asymmetric information all-pay auctions can be considered as a special case that was studied by Siegel (2014). Siegel shows that a unique monotonic equilibrium exists in such an asymmetric two-player all-pay auction with independent valuation and proposes the construction of such an equilibrium. The following Theorem, together with Proposition 4.2, Proposition 4.4 and Proposition 4.5, fully characterizes the equilibrium strategies for contestant A, high-type contestant B and low-type contestant B. See Figure 5 for a graphical illustration.

Theorem 4.1 (Equilibrium Characterization)**.**

1. *Strategy profile shown in Proposition 4.2 is the unique Bayesian Nash equilibrium when $v_A < v_L$ or $v_L < v_A < v_H$ and $p > p^*$;*
2. *Strategy profile shown in Proposition 4.4 is the unique Bayesian Nash equilibrium when $v_L < v_A < v_H$ and $p \leq p^*$ or $v_A > v_H$ and $p \leq \frac{v_H}{v_A}$;*
3. *Strategy profile shown in Proposition 4.5 is the unique Bayesian Nash equilibrium when $v_A > v_H$ and $p \geq \frac{v_H}{v_A}$.*

For all-pay auction contests with two players, at most one of them has positive probability of bidding zero. If contestant A bids zero with positive probability, and further according to the property of monotonic equilibrium, we have the following proposition characterizing the unique equilibrium under this situation.

Proposition 4.2 (Equilibrium Situation I)**.** *The following mixed strategy profile is the unique*

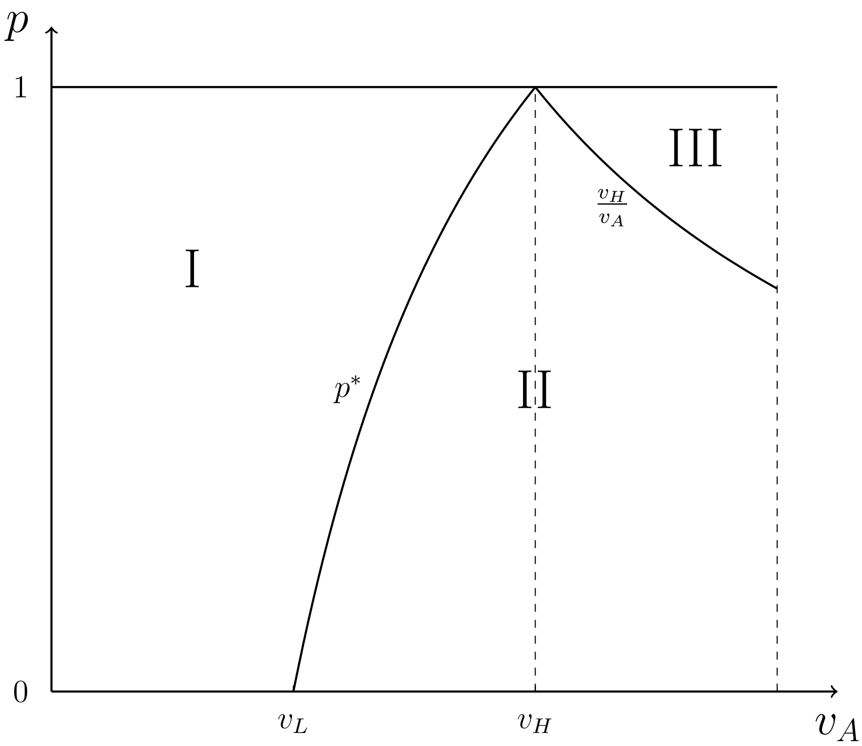
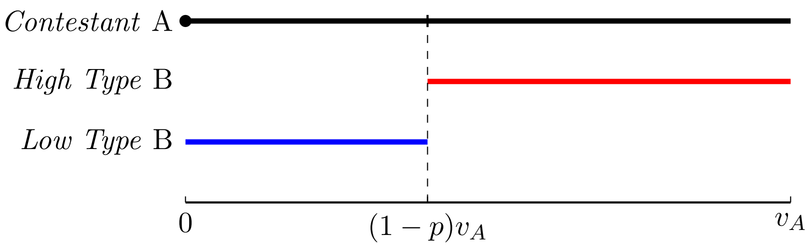


Figure 5: Equilibrium Characterization under Three Situations

equilibrium when $v_A < v_L$ *or* $v_L < v_A < v_H$ *and* $p \geq p^*$,

$$
\begin{aligned}
F_A(x) &= \begin{cases} \frac{x}{v_L} + 1 - \frac{pv_A}{v_H} - \frac{(1-p)v_A}{v_L} & x \in [0, (1-p)v_A] \\ \frac{x}{v_H} + 1 - \frac{v_A}{v_H} & x \in [(1-p)v_A, v_A] \end{cases} \\
F_L(x) &= \frac{x}{(1-p)v_A}, \quad x \in [0, (1-p)v_A] \\
F_H(x) &= \frac{x}{pv_A} + 1 - \frac{1}{p}, \quad x \in [(1-p)v_A, v_A]
\end{aligned}
$$

The following figure gives us an illustration of supports for different players' equilibrium strategies shown in Proposition 4.2.



In this situation, low type player B bids uniformly inside region $[0, v_A(1-p)]$ with probability density $\frac{1}{(1-p)v_A}$ and high type player B bids uniformly inside region $[v_A(1-p), v_A]$ with probability density $\frac{1}{pv_A}$. Contestant A bids uniformly inside region $[0, v_A(1-p)]$ with probability density $\frac{1}{v_L}$

and bids uniformly inside region $[v_A(1-p), v_A]$ with probability density $\frac{1}{v_H}$. The probability densities are derived from the payoff equivalent condition for each pure strategy that belongs to the support of equilibrium mixed strategy. Furthermore, contestant A quits the contest with probability $1 - \frac{pv_A}{v_H} - \frac{(1-p)v_A}{v_L}$, which is indeed greater than zero when $v_A < v_L$ or $v_L < v_A < v_H$ and $p > p^*$. Further more, it is worth noting that the threshold value p^* is solved from

$$1 - \frac{pv_A}{v_H} - \frac{(1-p)v_A}{v_L} = 0 \Longleftrightarrow \frac{1}{v_A} = \frac{p}{v_H} + \frac{1-p}{v_L} \tag{5}$$

Finally, to guarantee that Proposition 4.2 is indeed an equilibrium, we should verify that $(1-p)v_A \leq v_L$. When $v_A < v_L$, this holds trivially. When $v_L < v_A < v_H$ and $p \geq p^*$, the following lemma tells us that $(1-p)v_A \leq (1-p^*)v_A < v_L$

Lemma 4.3. $p^* > 1 - \frac{v_L}{v_A}$

Proof.

$$\frac{v_A v_H - v_H v_L}{v_A v_H - v_A v_L} > 1 - \frac{v_L}{v_A} \Longleftrightarrow \frac{v_H(v_A - v_L)}{v_A(v_H - v_L)} > \frac{v_A - v_L}{v_A} \Longleftrightarrow \frac{v_H}{v_H - v_L} > 1$$

□

The expected total effort function under situation I is defined as

$$\begin{aligned} \Pi_{\mathrm{I}}(p) &= \mathbb{E}(x_A) + p\mathbb{E}(x_{BH}) + (1-p)\mathbb{E}(x_{BL}) \\ &= \frac{pv_A^2}{v_H} - \frac{p^2 v_A^2}{2v_H} + \frac{(1-p)^2 v_A^2}{2v_L} + \frac{v_A}{2} \end{aligned} \tag{6}$$

The expected revenue is strictly decreasing for $p \in [0,1]$ since the first order derivative is negative.

$$\Pi_{\mathrm{I}}'(p) = (1-p)v_A^2\Big(\frac{1}{v_H} - \frac{1}{v_L}\Big) < 0$$

By the positiveness of second order derivative $\frac{\partial^2 \Pi_{\mathrm{I}}}{\partial p^2}$, we can easily verify the convexity of $\Pi_{\mathrm{I}}(p)$,

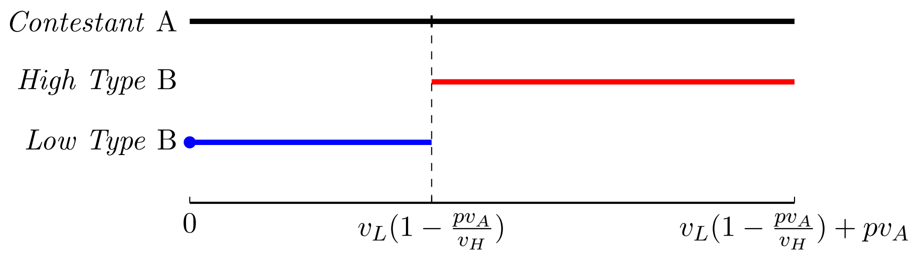
$$\Pi_{\mathrm{I}}''(p) = -\frac{v_A^2}{v_H} + \frac{v_A^2}{v_L} > 0$$

If contestant B bids zero with positive probability, and further according to the property of monotonic equilibrium, equilibrium may have two different forms according to the threshold type. If the threshold type is v_L, then only low type B bids zero with positive probability and high type B always bids positively, which is shown in Proposition 4.4. If the threshold type is v_H, then low type B bids zero with probability 1 and high type B bids zero with positive probability, which is shown in Proposition 4.5.

Proposition 4.4 (Equilibrium Situation II)**.** *The following mixed strategy profile is the unique* *equilibrium when* $v_L < v_A < v_H$ *and* $p \leq p^*$ *or* $v_A > v_H$ *and* $p \leq \frac{v_H}{v_A}$. *Let* $V = v_L - \frac{pv_Av_L}{v_H} + pv_A$,

$$\begin{aligned}
F_A(x) &= \begin{cases} \frac{x}{v_L} & x \in [0, V - pv_A] \\ \frac{x}{v_H} + 1 - \frac{V}{v_H} & x \in [V - pv_A, V] \end{cases} \\
F_L(x) &= \frac{x}{(1-p)v_A} + \frac{1}{1-p} - \frac{V}{(1-p)v_A}, \quad x \in [0, V - pv_A] \\
F_H(x) &= \frac{x}{pv_A} + 1 - \frac{V}{pv_A}, x \in [V - pv_A, V]
\end{aligned}$$

The following figure gives us an illustration of supports for different players' equilibrium strategies shown in Proposition 4.4.



In this situation, low type player B quits the contest with positive probability and bids uniformly inside region $[0, V - pv_A]$ with probability density $\frac{1}{(1-p)v_A}$. High type player B bids uniformly inside region $[V - pv_A, V]$ with probability density $\frac{1}{pv_A}$. Contestant A bids uniformly inside region $[0, V - pv_A]$ with probability density $\frac{1}{v_L}$ and bids uniformly inside region $[V - pv_A, V]$ with probability density $\frac{1}{v_H}$.

To derive such an equilibrium, we first know that high type player B adopts a uniform bidding strategy inside some interval $[V - pv_A, V]$ with density $\frac{1}{pv_A}$ such that contestant A is indifferent in bidding any value inside $[V - pv_A, V]$. Meanwhile, to make high type player B indifferent in bidding any value inside $[V - pv_A, V]$, contestant A should set the probability density to $\frac{1}{v_H}$ inside $[V - pv_A, V]$. Then the probability of bidding inside $[0, V - pv_A]$ should be one minus the probability of bidding inside $[V - pv_A, V]$ and hence should be $1 - \frac{pv_A}{v_H}$. In order to make low type player B indifferent in bidding any value inside $[0, V - pv_A]$, contestant A should set the probability density to $\frac{1}{v_L}$ inside $[0, V - pv_A]$. Then we can conclude that

$$\frac{V - pv_A}{v_L} = 1 - \frac{pv_A}{v_H} \Longrightarrow V = (1 - \frac{pv_A}{v_H})v_L + pv_A \tag{7}$$

The expected total effort function under situation II is defined as

$$\begin{aligned}
\Pi_{\text{II}}(p) &= \mathbb{E}(x_A) + p\mathbb{E}(x_H) + (1-p)\mathbb{E}(x_L) \\
&= \frac{pv_AV}{2v_H} + \frac{V}{2} - \frac{pv_A}{2} + \frac{V^2}{2v_A}
\end{aligned} \tag{8}$$

The second order derivative for $\Pi_{\text{II}}(p)$ is positive,

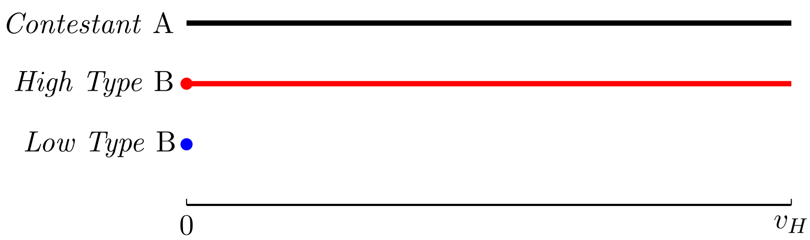
$$\Pi''_{\text{II}}(p) = \frac{v_A^2}{v_H}(1 - \frac{v_L}{v_H}) + v_A(1 - \frac{v_L}{v_H})^2 > 0$$

which implies the convexity of $\Pi_{\text{II}}(p)$.

Proposition 4.5 (Equilibrium Situation III)**.** *The following mixed strategy profile is the unique equilibrium when $v_A > v_H$ and $p \geq \frac{v_H}{v_A}$*

$$\begin{aligned}
F_A(x) &= \frac{x}{v_H}, \quad x \in [0, v_H] \\
F_L(x) &= 1, \quad x \geq 0 \\
F_H(x) &= \frac{x}{pv_A} + 1 - \frac{v_H}{pv_A}, \quad x \in [0, v_H]
\end{aligned}$$

The following figure gives us an illustration of supports for different players' equilibrium strategies shown in Proposition 4.5.



In this situation, low type player B quits the contest for sure. High type player B bids uniformly inside region $[0, v_H]$ with probability density $\frac{1}{pv_A}$ and quits the contest with probability $1 - \frac{v_H}{pv_A}$. Contestant A bids uniformly inside region $[0, v_H]$ with probability density $\frac{1}{v_H}$.

The expected total effort function under situation III is defined as

$$\Pi_{\text{III}}(p) = \frac{v_H}{2} + \frac{v_H^2}{2v_A} = \frac{v_A v_H + v_H^2}{2v_A} \tag{9}$$

4.2 Optimal Bayesian Persuasion Signal

We first present the optimal Bayesian persuasion signal through the following Theorem, and then provide the analysis process according to the value of v_A.

Theorem 4.6. *The optimal information disclosure strategy is shown in the following table.*

range of v_A	p	*optimal Bayesian persuasion*
$v_A < v_L$	$[0, 1]$	*Full disclosure*
$v_A \in (v_L, v_H)$	$p \in (0, p^*)$	*Partial Disclosure with belief* $p_m = p^*$ *and* $p_l = 0$
	$p \in (p^*, 1)$	*Partial Disclosure with belief* $p_m = p^*$ *and* $p_h = 1$
	$p \in \{0, p^*, 1\}$	*No disclosure*
$v_A > v_H$	$p \in (0, \frac{v_H}{v_A})$	*Partial Disclosure with belief* $p_h = \frac{v_H}{v_A}$ *and* $p_l = 0$
	$p \in \{0\} \cup [\frac{v_H}{v_A}, 1]$	*No disclosure*

where $p^* = \frac{v_A v_H - v_H v_L}{v_A v_H - v_A v_L}$. *The contest designer can benefit from Bayesian persuasion if* $v_A > v_L$.

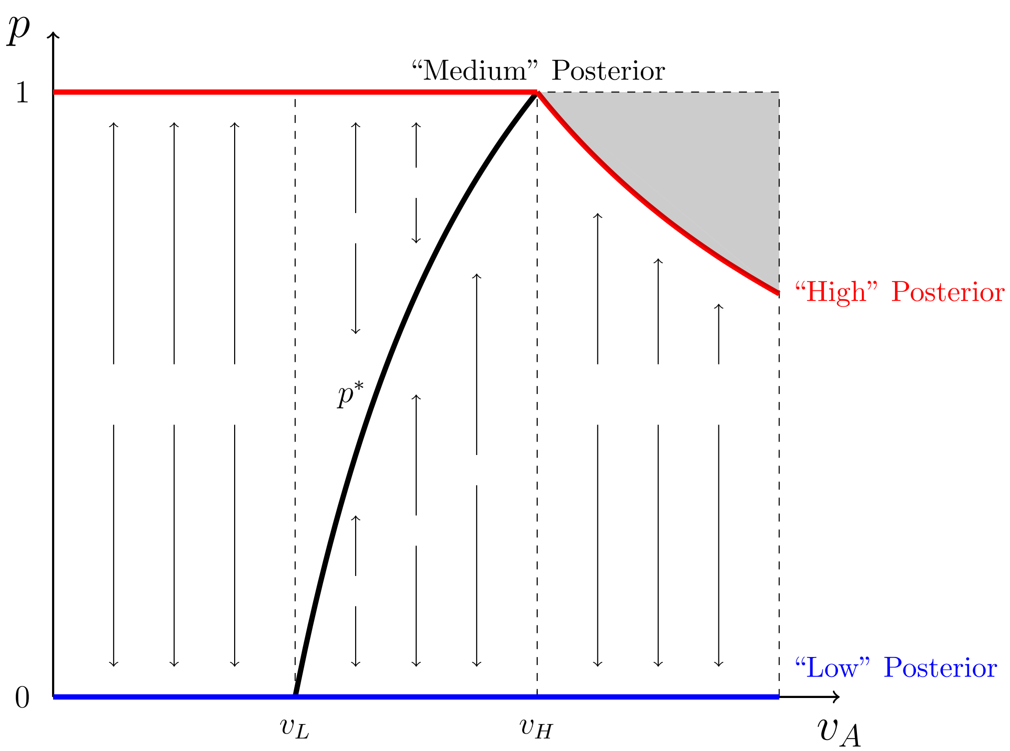


Figure 6: Optimal Bayesian Persuasion in Simultaneous Contests

Let $S = \{s_L, s_H\}$ where s_L denotes "Low" Signal and s_H denotes "High" Signal.

- When $v_A \in (v_L, v_H)$ and $p \in (0, p^*)$, $\Pr(s_H|v_L) = \frac{p(1-p^*)}{p^*(1-p)}$ and $\Pr(s_H|v_H) = 1$ implements the optimal disclosure policy.
- When $v_A \in (v_L, v_H)$ and $p \in (p^*, 1)$, $\Pr(s_H|v_L) = 0$ and $\Pr(s_H|v_H) = \frac{p^*(1-p)}{p(1-p^*)}$ implements the optimal disclosure policy.
- When $v_A > v_H$ and $p \in (0, \frac{v_H}{v_A})$, $\Pr(s_H|v_L) = \frac{p(v_A - v_H)}{(1-p)v_H}$ and $\Pr(s_H|v_H) = 1$ implements the optimal disclosure policy.

Figure 6 graphically illustrates Theorem 4.6. We use the horizontal axis to represent different values of v_A and the vertical axis to represent the probability p. The contest designer applies

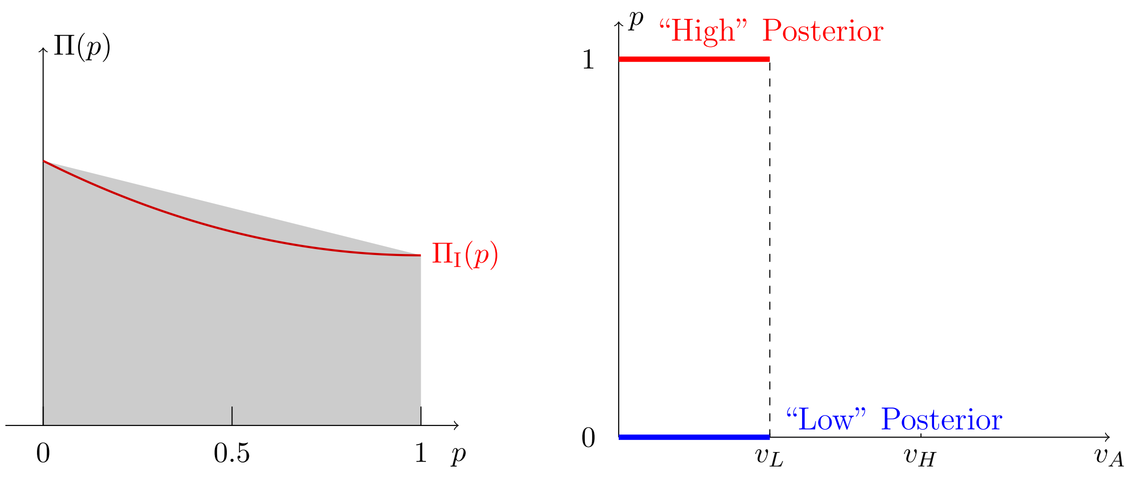


Figure 7: Simultaneous Contests, $v_A < v_L$

full concealment strategy when 2-tuple (v_A, q) locates in gray area. The contest designer delivers two signals in the white area, denoted as s_H (“High” Signal) and s_L (“Low” Signal). There exist three possible posteriors, the “Low” posterior, the “Medium” posterior and the “High” posterior. “Low” posterior is always associated with “Low” Signal. “High” posterior is always associated with “High” Signal. However, “Medium” posterior is sometimes associated with “Low” Signal and sometimes associated with “High” Signal. In “Low” posterior, contestant B must has winning value v_L. On the contrary, in “High” posterior, contestant B has winning value v_H deterministically only if v_A does not exceed v_H, otherwise, player A cannot accurately infer the type of opponent. The above two posteriors are available for all possible values of v_A. The third possibility is the “Medium” posterior, which is meaningless when $v_A \notin (v_L, v_H)$. When receiving corresponding signal (“High” Signal or “Low” Signal), contestant B has winning value v_H with probability p^* and winning value v_L with probability $1 - p^*$. What's more, when receiving the “Medium” posterior, contestant B is evenly matched with contestant A because the harmonic mean of winning valuation of different types for contestant B equals the winning valuation of contestant A.

4.2.1 $v_A < v_L$

When $v_A < v_L$, the equilibrium is characterized by Proposition 4.2. Then by the convexity of $\Pi_{\mathrm{I}}(p)$, full disclosure is optimal because the concave closure of a convex function is the line segment between two endpoints. Hence, we have the following proposition,

Proposition 4.7. *In simultaneous all-pay auction when $v_A < v_L$, the full disclosure policy is optimal.*

Left panel of Figure 7 shows graphically that the concave closure of $\Pi_{\mathrm{I}}(p)$ when $v_A < v_L$.

4.2.2 $v_L < v_A < v_H$

When $v_L < v_A < v_H$, from the equilibrium characterization, the expected total effort function is

$$\Pi(p) = \begin{cases} \Pi_{\text{II}}(p) & p \leq p^* \\ \Pi_{\text{I}}(p) & p \geq p^* \end{cases}$$

The continuity of $\Pi(p)$ at cutoff value p^* is given by the following lemma,

Lemma 4.8.

$$\lim_{p \xrightarrow{+} p^*} \Pi_{\text{I}}(p) = \lim_{p \xrightarrow{-} p^*} \Pi_{\text{II}}(p) \tag{10}$$

Until now, we have recognized that $\Pi(p)$ is a continuous two-stage function, both stages are convex, but the shape of complete function is still unknown. The following lemma proves the local optimality of p^*.

Lemma 4.9.

$$\lim_{p \xrightarrow{+} p^*} \Pi'_{\text{I}}(p) < 0 < \lim_{p \xrightarrow{-} p^*} \Pi'_{\text{II}}(p)$$

Combining the property of local maximum and piecewise convexity, contest designer may always have the incentive to include p^* as one of the potential posteriors compared with no disclosure. But full disclosure is still not ruled out. The last key step of the analysis focuses on the violation of the full disclosure condition, that is,

$$\Pi(p^*) > p^*\Pi(1) + (1 - p^*)\Pi(0) \tag{11}$$

By proving the global optimality of $\Pi(p^*)$, we can argue that $\Pi(p^*)$ is greater than both of $\Pi(1)$ and $\Pi(0)$. Hence, it violates the full disclosure condition.

Lemma 4.10. $\Pi(p^*) > \Pi(1)$ *and* $\Pi(p^*) > \Pi(0)$.

Finally, the following proposition concludes the analysis when $v_L < v_A < v_H$.

Proposition 4.11. *In simultaneous all-pay auction when $v_L < v_A < v_H$, the designer benefits from Bayesian persuasion when $p \in (0, p^*) \cup (p^*, 1)$.*

If applying Bayesian persuasion, belief $p = p^* = \frac{v_A v_H - v_H v_L}{v_A v_H - v_A v_L}$ is always generated because the global optimality of $\Pi(p^*)$. One interesting fact is that when $p = p^*$, the harmonic mean of v_B is identical to v_A,

$$\frac{p^*}{v_H} + \frac{1 - p^*}{v_L} = \frac{1}{v_A}$$

Intuitively, we may expect that the larger proportion of high type player B may lead to higher expected revenue. Nonetheless, such an intuition is wrong. Player A may be frustrated when facing too many of high type player B and less motivated when facing too many of low type player

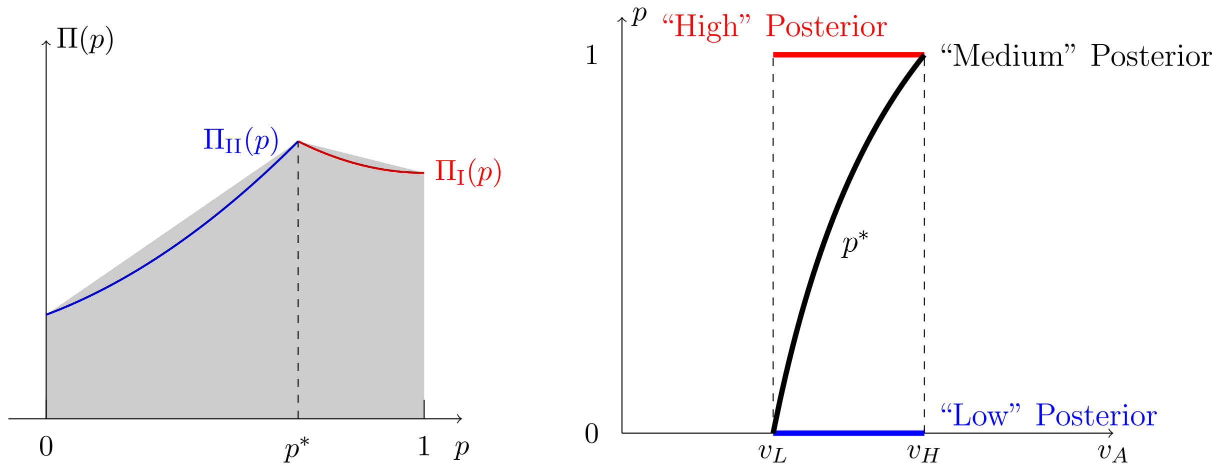


Figure 8: Simultaneous Contests, $v_L < v_A < v_H$

B. The optimal value of p^* provides a harmonic contest environment that can stimulate both players at maximum level. The expected effort of both players, $\mathbb{E}(x_A)$ for player A and

$$\mathbb{E}(x_B) = p\mathbb{E}(x_H) + (1-p)\mathbb{E}(x_L)$$

for player B reaches its optimal at p^*, indicating that p^* motivates both players at the maximum level. The proof can be derived similarly by calculating first order derivatives from both sides.

Left panel of Figure 8 shows graphically that the concave closure of $\Pi(p)$ when $v_L < v_A < v_H$.

4.2.3 $v_A > v_H$

When $v_A > v_H$, from the equilibrium characterization, the expected total effort function is

$$\Pi(p) = \begin{cases} \Pi_{\text{II}}(p) & p \leq \frac{v_H}{v_A} \\ \Pi_{\text{III}}(p) & p \geq \frac{v_H}{v_A} \end{cases}$$

The continuity of $\Pi(p)$ at cutoff value $\frac{v_H}{v_A}$ is given by the following lemma,

Lemma 4.12.

$$\lim_{p \xrightarrow{-} \frac{v_H}{v_A}} \Pi_{\text{II}}(p) = \frac{v_A v_H + v_H^2}{2v_A} \tag{12}$$

Proof. When $p \xrightarrow{-} \frac{v_H}{v_A}$, $V = pv_A$. Then,

$$\lim_{p \xrightarrow{-} \frac{v_H}{v_A}} \Pi_{\text{II}}(p) = \frac{p^2 v_A^2}{2v_H} + \frac{p^2 v_A}{2} = \frac{v_A v_H + v_H^2}{2v_A}$$

□

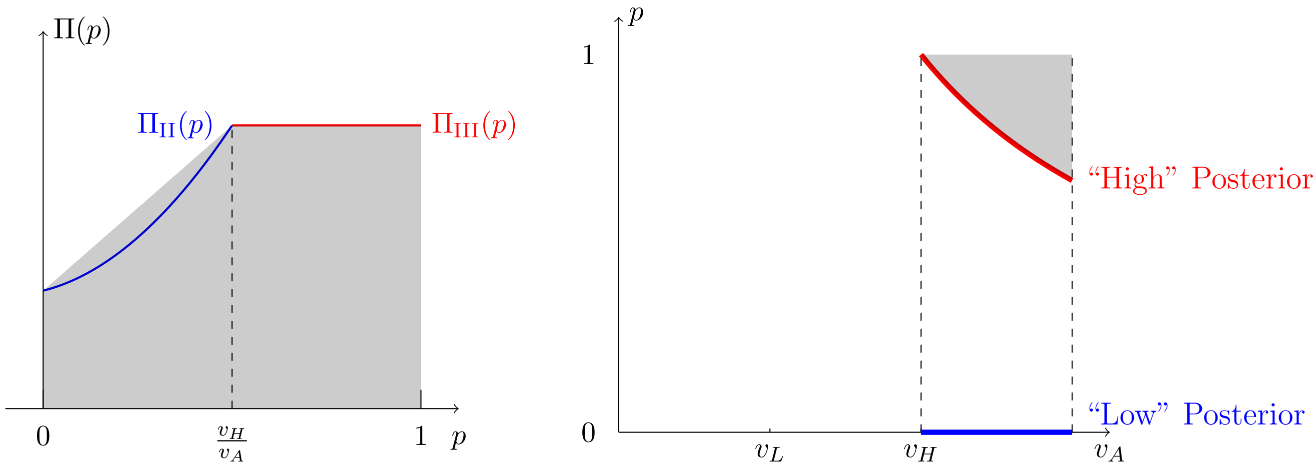


Figure 9: Simultaneous Contests, $v_A > v_H$

At last, we need to rule out the full disclosure policy, which is trivial according to the following lemma.

Lemma 4.13.

$$\Pi(0) = \frac{v_A v_L + v_L^2}{2v_A} < \frac{v_A v_H + v_H^2}{2v_A} \tag{13}$$

Finally, the following proposition concludes the analysis when $v_A > v_H$.

Proposition 4.14. *In simultaneous all-pay auction when $v_A > v_H$, the designer benefits from Bayesian persuasion when $p \in (0, \frac{v_H}{v_A})$.*

If applying Bayesian persuasion, belief $p_h = \frac{v_H}{v_A}$ and $p_l = 0$ are always generated. Otherwise, the designer should apply no disclosure policy.

Left panel of Figure 9 shows graphically that the concave closure of $\Pi(p)$ when $v_A > v_H$.

5 Bayesian Persuasion in Sequential All-pay Auction Contests

5.1 Equilibrium Characterization

For sequential all-pay auction contests, we first characterize the strategies adopted by contestant B. Recall that when $x_A = x_B < v_B$, the prize is assigned to participant B to avoid trivial case since contestant B can exert infinitesimal more effort $x_A + \varepsilon$ to win the game; when $x_A = x_B = v_B$, the prize is assigned to participant A because the previous infinitesimal demonstration no longer works. Therefore, we can conclude that participant B exert effort level at

$$x_B^* = \begin{cases} x_A & x_A < v_B \\ 0 & x_A \geq v_B \end{cases}$$

Then the winning probability for contestant A when bidding x_A can be expressed as

$$p\mathbb{I}(x_A \geq v_H) + (1-p)\mathbb{I}(x_A \geq v_L)$$

Therefore, three possible strategies are bidding $0, v_L$ and v_H. If contestant A is indifferent between two strategies, let him adopt the contest-designer-preferred strategy. Figure 10 shows the strategy of contestant A graphically. For those three indifference curves, contestant A always choose the higher one because it is preferred by the contest designer.

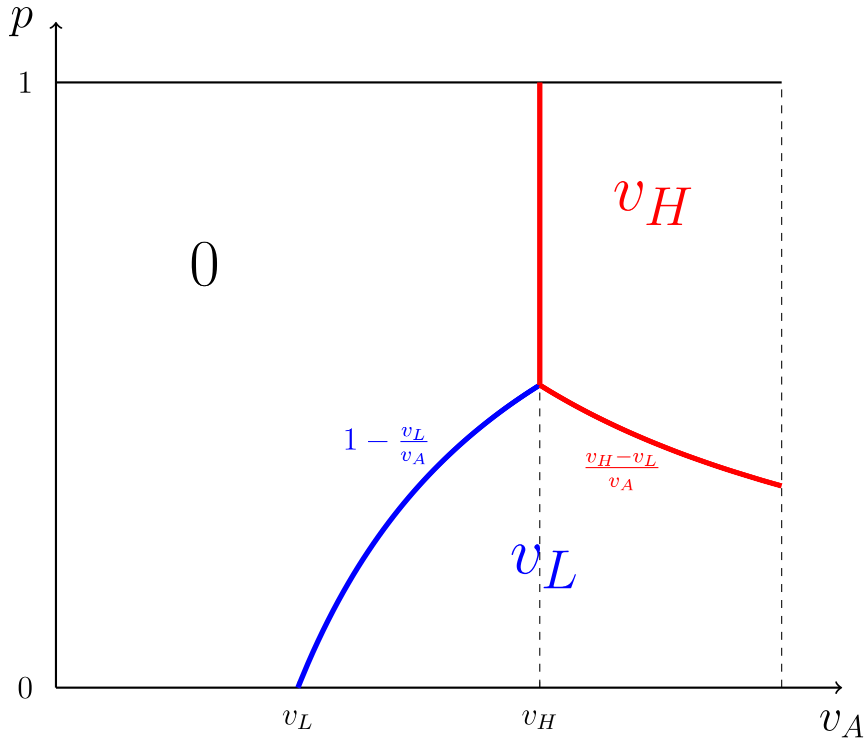


Figure 10: Strategy of Contestant A

If contestant A bids zero, then contestant B will bid infinitesimal ε to win the contest. Therefore, both contestants exert zero effort. We call this equilibrium an *inactive* one.

5.2 Optimal Bayesian Persuasion Signal

We first present the optimal Bayesian persuasion signal through the following Theorem, and then provide the analysis process according to the value of v_A.

Theorem 5.1. *The optimal Bayesian persuasion is shown in the following table.*

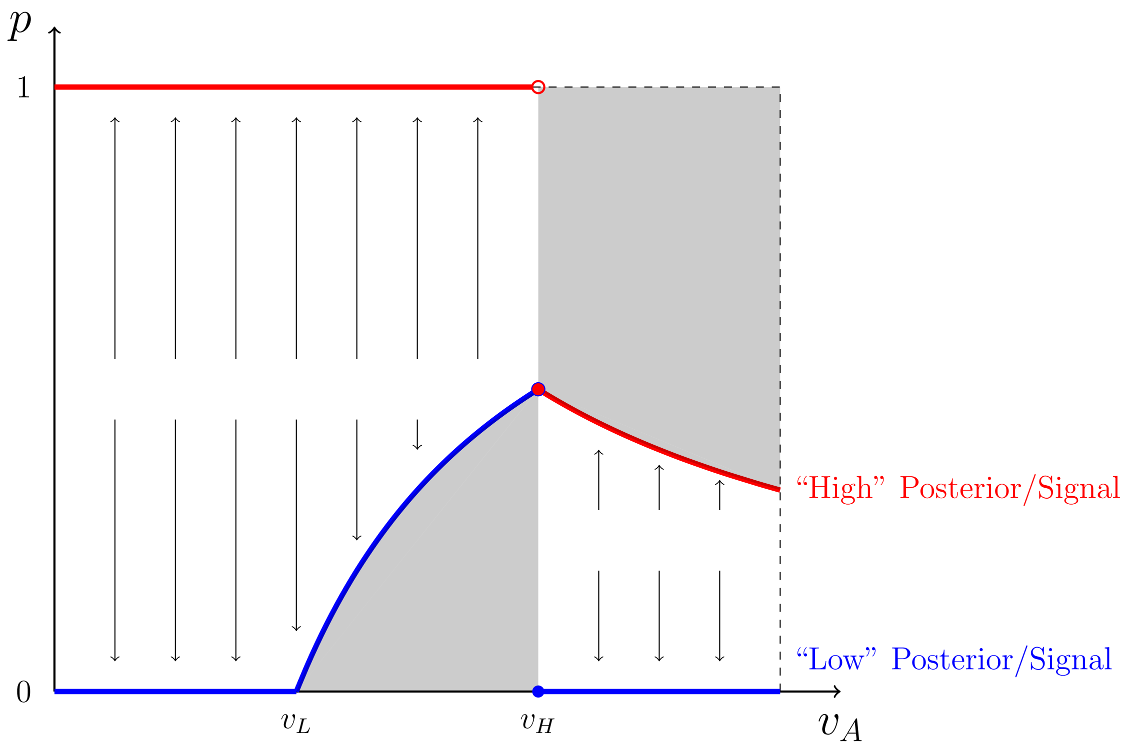


Figure 11: Optimal Bayesian Persuasion in Sequential Contests

range of v_A	p	*Optimal Bayesian Persuasion*
$v_A \leq v_L$	*[0,1]*	*Full disclosure*
$v_A \in (v_L, v_H)$	$p \in (1 - \frac{v_L}{v_A}, 1)$	*Partial Disclosure with belief* $p_h = 1$ *and* $p_l = 1 - \frac{v_L}{v_A}$
	$p \in (0, 1 - \frac{v_L}{v_A})$	*No disclosure*
$v_A > v_H$	$p \in (0, \frac{v_H - v_L}{v_A})$	*Partial Disclosure with belief* $p_h = \frac{v_H - v_L}{v_A}$ *and* $p_l = 0$
	$p \in \{0\} \cup [\frac{v_H - v_L}{v_A}, 1]$	*No disclosure*

The contest designer can benefit from Bayesian persuasion if $v_A > v_L$.

Let $S = \{s_L, s_H\}$ where s_L denotes “Low” Signal and s_H denotes “High” Signal. Unlike the simultaneous contests, the relationship between signal and posterior is straightforward.“Low” Signal is always associated with “Low” Posterior and “High” Signal is always associated with “High” Posterior.

- When $v_A \in (v_L, v_H)$ and $p \in (1 - \frac{v_L}{v_A}, 1)$, $\Pr(s_H | v_L) = 0$ and $\Pr(s_H | v_H) = \frac{v_L - (1-p)v_A}{p v_L}$ implements the optimal disclosure policy.
- When $v_A > v_H$ and $p \in (0, \frac{v_H - v_L}{v_A})$, $\Pr(s_H | v_L) = \frac{p(v_A + v_L - v_H)}{(1-p)(v_H - v_L)}$ and $\Pr(s_H | v_H) = 1$ implements the optimal disclosure policy.

Figure 11 graphically illustrates Theorem 5.1. We use the horizontal axis to represent different values of v_A and the vertical axis to represent the probability p. The contest designer applies

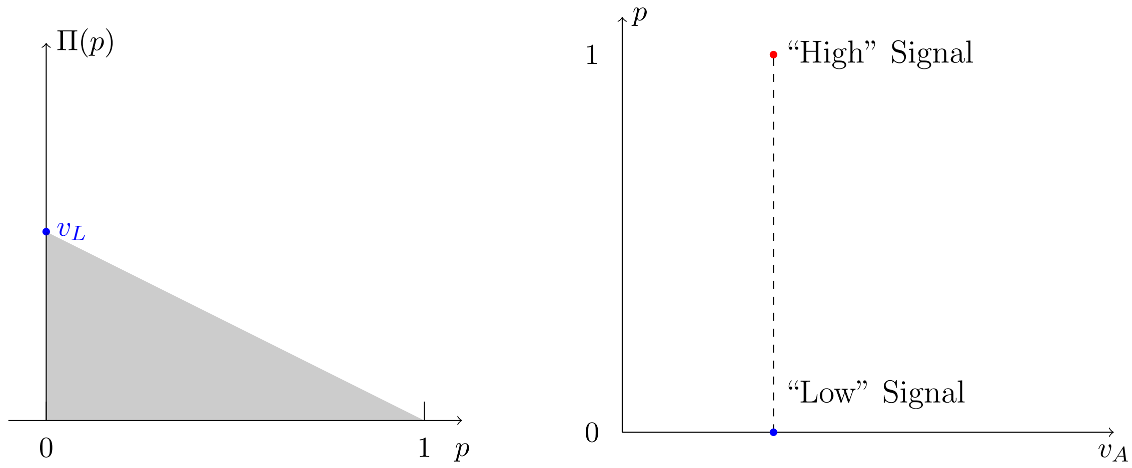


Figure 12: Sequential Contests, $v_A = v_L$

full concealment strategy when 2-tuple (v_A, q) locates in gray area. The contest designer delivers two signals in the white area. There exists two possible signals, the "Low" signal and the "High" signal. Compared with optimal Bayesian persuasion signal in simultaneous contests where "Low" signal is always accurate, contestant B may not have winning value v_L when $v_L < v_A < v_H$ and receiving "Low" signal. Meanwhile, when receiving "High" signal, contestant B has winning value v_H deterministically only if v_A does not exceed v_H, otherwise, player A cannot accurately infer the type of opponent. The above two signals are available for all possible values of v_A.

5.2.1 $v_A < v_L$

When $v_A < v_L$, the equilibrium is always inactive, $\Pi(p) = 0$. Therefore, we can conclude the following that all Bayesian persuasion signals are outcome equivalent.

Proposition 5.2. *In sequential all-pay auction when $v_A < v_L$, the contest designer cannot get positive expected total effort by Bayesian persuasion.*

Both no disclosure and full disclosure are optimal here. To make it consistent with boundary value, we use full disclosure to represent the optimal policy.

5.3 $v_A = v_L$

In this circumstance, contestant A bids v_A if and only if $p = 0$. In such an equilibrium, player A bids $v_A = v_L$ and player B bids 0. The equilibrium expected total effort is v_A for the contest designer. Otherwise, the equilibrium is inactive.

Proposition 5.3. *In sequential all-pay auction when $v_A = v_L$, the full disclosure policy is optimal.*

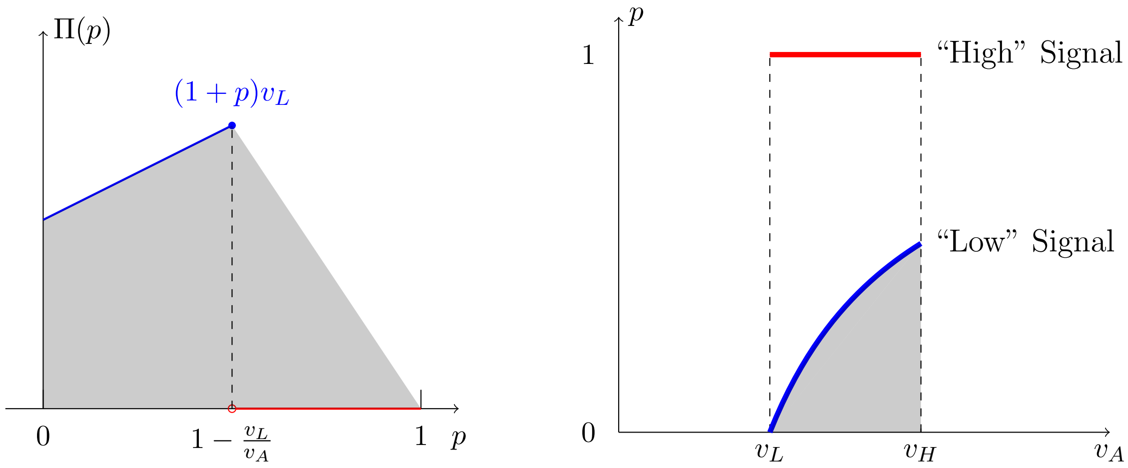


Figure 13: Sequential Contests, $v_L < v_A < v_H$

5.4 $v_L < v_A < v_H$

The optimal strategy for player A is

$$x_A = \begin{cases} v_L & p \in [0, 1 - \frac{v_L}{v_A}] \\ 0 & p \in (1 - \frac{v_L}{v_A}, 1] \end{cases}$$

The threshold is given by the following equation.

$$(1-p)v_A - v_L = 0$$

High type player B will bid exactly what A bids and low type player B will bid 0 all the time. Equilibrium expected total effort is described as

$$\Pi(p) = \begin{cases} (1+p)v_L & p \in [0, 1 - \frac{v_L}{v_A}] \\ 0 & p \in (1 - \frac{v_L}{v_A}, 1] \end{cases} \tag{14}$$

Proposition 5.4. *In sequential all-pay auction when $v_L < v_A < v_H$, the contest designer benefits from Bayesian persuasion if $p \in (1 - \frac{v_L}{v_A}, 1]$.*

Contest designer can apply Bayesian persuasion to avoid inactive equilibrium by setting posteriors to $p_l = 1 - \frac{v_L}{v_A}$ ("Low Signal") and $p_h = 1$ ("High Signal").

5.5 $v_A = v_H$

The optimal strategy for player A is

$$x_A = \begin{cases} v_L & p \in [0, 1 - \frac{v_L}{v_A}) \\ v_H & p \in [1 - \frac{v_L}{v_A}, 1] \end{cases}$$

High type player B will bid v_L when A bids v_L and quits the contest when A bids v_H. Low type player B quits the contest all the time. Equilibrium expected total effort is described as

$$\Pi(p) = \begin{cases} (1+p)v_L & p \in [0, 1 - \frac{v_L}{v_A}) \\ v_H & p \in [1 - \frac{v_L}{v_A}, 1] \end{cases} \tag{15}$$

Proposition 5.5. *In sequential all-pay auction when $v_A = v_H$, the contest designer benefits from Bayesian persuasion if $p \in [0, 1 - \frac{v_L}{v_A}]$.*

The avoidance of inactiveness is similar to the previous analysis.

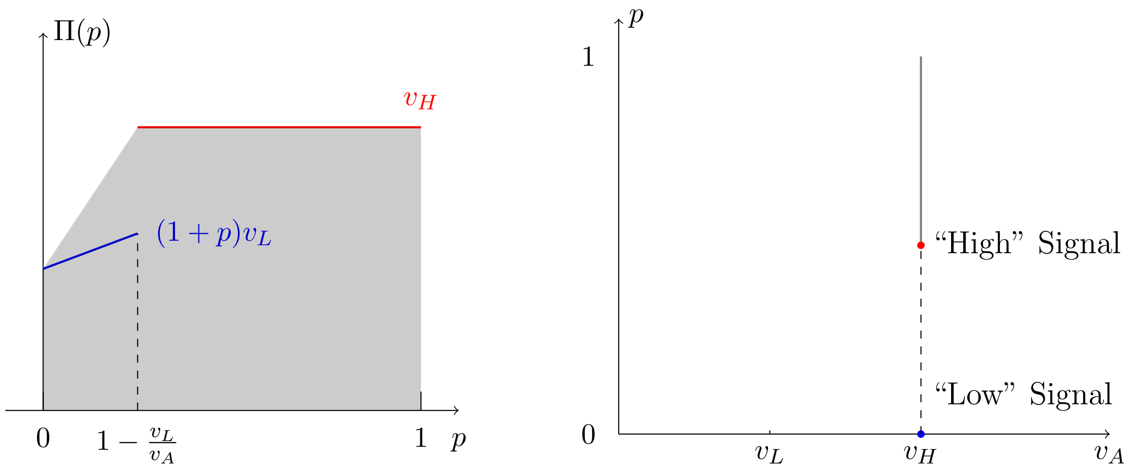


Figure 14: Sequential Contests, $v_A = v_H$

5.6 $v_A > v_H$

The optimal strategy for player A is

$$x_A = \begin{cases} v_L & p \in [0, \frac{v_H - v_L}{v_A}) \\ v_H & p \in [\frac{v_H - v_L}{v_A}, 1] \end{cases}$$

The threshold is given by the following equation.

$$(1-p)v_A - v_L = v_A - v_H$$

Equilibrium expected total effort is described as

$$\Pi(p) = \begin{cases} (1+p)v_L & p \in [0, \frac{v_H - v_L}{v_A}) \\ v_H & p \in [\frac{v_H - v_L}{v_A}, 1] \end{cases} \tag{16}$$

Hence, we conclude that

Proposition 5.6. *In sequential all-pay auction when $v_A > v_H$, the contest designer benefits from Bayesian persuasion if $p \in [0, \frac{v_H - v_L}{v_A})$.*

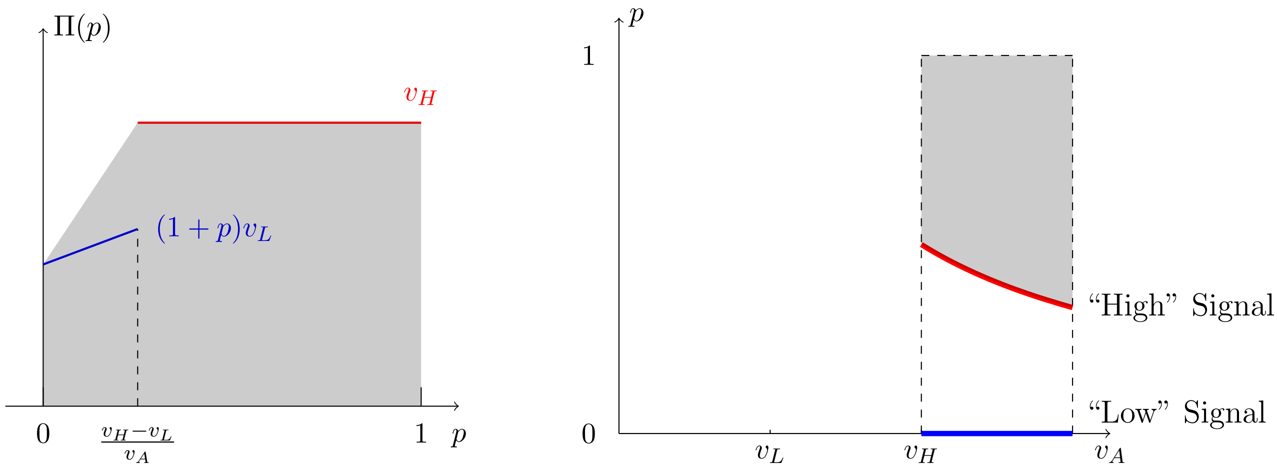


Figure 15: Sequential Contests, $v_A > v_H$

6 Extension I: Information Revealing Incentive of the Informed Player

In this section, we slightly modify the setup in the benchmark model by incorporating the incentives of contestant B to reveal own type information. One more stage is included in this game, called *information revealing stage*, in which contestant B chooses whether to reveal his type or not conditional on signal realization. This stage is added immediately after the Bayesian persuasion design stage. Using backward induction, we first study the incentive to share information for a given posterior contest game. In this section, for realistic considerations, we only analyze two possible strategies for contestant B, fully revealing and fully concealing, without considering the stochastic signals by contestant B via the Bayesian persuasion approach.

6.1 Simultaneous Contests

Recall that there are three potential equilibrium situations, as shown in Figure 5. In this part, we first derive the ex ante expected utility for contestant B under those three situations and then study the incentive to share type information.

6.1.1 Three Equilibrium Situations

In situation I, by Proposition 4.2, both types of player B have positive utilities. The ex ante expected utility for player B is

$$\begin{aligned}\Pi_{B,\mathrm{I}}(p) &= \Pr(v_B = v_H)\underbrace{(v_H - v_A)}_{\text{High Type}} + \Pr(v_B = v_L)\underbrace{v_L\left(1 - \frac{pv_A}{v_H} - \frac{(1-p)v_A}{v_L}\right)}_{\text{Low Type}} \\ &= p(v_H - v_A) + (1-p)v_L(1 - \frac{pv_A}{v_H} - \frac{(1-p)v_A}{v_L}) \\ &= -\frac{(v_H - v_L)v_A}{v_H}p^2 + \frac{(v_H - v_L)(v_H + v_A)}{v_H}p + (v_L - v_A)\end{aligned}$$

The first order derivative and the second order derivative are

$$\begin{aligned}\Pi'_{B,\mathrm{I}}(p) &= -\frac{2(v_H - v_L)v_A}{v_H}p + \frac{(v_H - v_L)(v_H + v_A)}{v_H} \\ \Pi''_{B,\mathrm{I}}(p) &= -\frac{2(v_H - v_L)v_A}{v_H}\end{aligned}$$

In situation I, we have $v_A \le v_H$ for sure. Hence, $\Pi'_{B,\mathrm{I}}(1) = \frac{(v_H - v_L)(v_H - v_A)}{v_H}$. Therefore, in situation I, expected utility for player B is an increasing concave function.

In situation II, by Proposition 4.4, only high type player B have positive utilities. The ex ante expected utility for player B is

$$\begin{aligned}\Pi_{B,\mathrm{II}}(p) &= \Pr(v_B = v_H)\underbrace{(v_H - v_L + \frac{pv_A v_L}{v_H} - pv_A)}_{\text{High Type}} \\ &= \frac{p(v_H - v_L)(v_H - pv_A)}{v_H} \\ &= -\frac{(v_H - v_L)v_A}{v_H}p^2 + (v_H - v_L)p\end{aligned}$$

The first order derivative and the second order derivative are

$$\begin{aligned}\Pi'_{B,\mathrm{II}}(p) &= -\frac{2(v_H - v_L)v_A}{v_H}p + (v_H - v_L) \\ \Pi''_{B,\mathrm{II}}(p) &= -\frac{2(v_H - v_L)v_A}{v_H}\end{aligned}$$

When $v_A < v_H$, p^* minimizes $\Pi'_{B,\mathrm{II}}(p)$ with value $\Pi'_{B,\mathrm{II}}(p^*) = v_H + v_L - 2v_A$, the sign of which is undetermined. When $v_A > v_H$, $\frac{v_H}{v_A}$ minimizes $\Pi'_{B,\mathrm{II}}(p)$ with value $\Pi'_{B,\mathrm{II}}(\frac{v_H}{v_A}) = v_H - v_L < 0$. Therefore, expected utility for player B is a concave function in situation II while it is not monotonic increasing as in situation I.

In situation III, by Proposition 4.5, both types of player B has zero utility,

$$\Pi_{B,\mathrm{III}}(p) = 0$$

6.1.2 Incentive Analysis

When $v_A < v_L$, contestant B has no incentive to reveal his type for any posterior distribution p because situation I happens in entire domain and the expected utility function is concave, $\Pi''_{B,\mathrm{I}}(p) < 0$.

When $v_L < v_A < v_H$, $\Pi_B(p)$ consists of two segments,

$$\Pi_B(p) = \begin{cases} \Pi_{B,\mathrm{II}}(p) & p \leq p^* \\ \Pi_{B,\mathrm{I}}(p) & p \geq p^* \end{cases} \tag{17}$$

where $p^* = \frac{v_A v_H - v_H v_L}{v_A v_H - v_A v_L}$, as shown in Figure 5. Both parts of $\Pi_B(p)$ are concave function, and we need to compare the geometric relationship between $\Pi_B(p^*)$ and line segment connecting $(0, \Pi_B(0))$ and $(0, \Pi_B(1))$. We find out that $(0, \Pi_B(0)), (p^*, \Pi_B(0)), (p^*, \Pi_B(1))$ are collinear (see Figure 16) because

$$\begin{aligned} \Pi_B(0) &= 0 \\ \Pi_B(p^*) &= \frac{v_H(v_H - v_A)(v_A - v_L)}{v_A(v_H - v_L)} = (v_H - v_A)p^* \\ \Pi_B(1) &= v_H - v_A \end{aligned}$$

which implies that contestant B has no incentive to reveal his type for any posterior distribution p since $\Pi_B(p) \geq \Pi_B(0) + p(\Pi_B(1) - \Pi_B(0))$ for any posterior p.

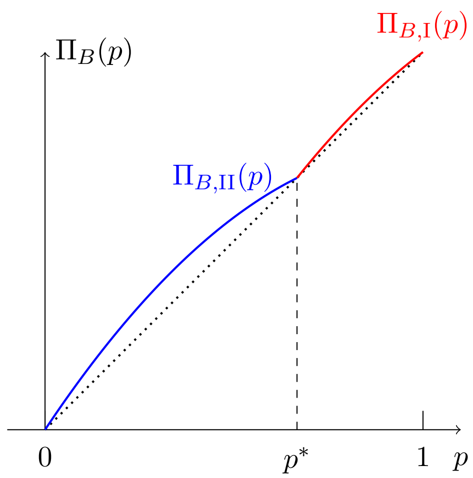


Figure 16: Incentive of Contestant B, simultaneous contests, $v_L < v_A < v_H$

When $v_A > v_H$, $\Pi_B(p)$ consists of two segments,

$$\Pi_B(p) = \begin{cases} \Pi_{B,\text{II}}(p) & p \leq \frac{v_H}{v_A} \\ \Pi_{B,\text{III}}(p) & p \geq \frac{v_H}{v_A} \end{cases} \tag{18}$$

since $\Pi_B(0) = \Pi_B(1) = 0$, contestant B has no incentive to reveal his type for any posterior distribution p.

To summarize the results of different values of v_A, we have the following theorem.

Theorem 6.1. *Contestant B has no incentive to reveal type information in simultaneous contests.*

Since contestant B do not want to share his or her type information in all posteriors, contest organizer will not change his optimal signal when introducing the incentives of B, as shown in the following corollary.

Corollary 6.2. *For simultaneous contests, the optimal Bayesian persuasion signal will not change when incorporating incentive to reveal type information for contestant B.*

6.2 Sequential Contests

In sequential contests, there are three potential bidding strategy for contestant A, $0, v_L$ and v_H. We classift those equilibria into three situations based on the optimal strategy of A, namely the situation when bidding 0 is optimal, the situation when bidding v_L is optimal and the situation when bidding v_H is optimal, as shown in Figure 10. In this part, we first derive the ex ante expected utility for contestant B under those three situations and then study the incentive to share type information.

6.2.1 Three Equilibrium Situations

When optimal strategy for contestant A is 0, the ex ante expected utility for player B is

$$\Pi_{B,1}(p) = pv_H + (1-p)v_L = v_L + p(v_H - v_L)$$

When optimal strategy for contestant A is v_L, the ex ante expected utility for player B is

$$\Pi_{B,2}(p) = p(v_H - v_L)$$

When optimal strategy for contestant A is v_H, the ex ante expected utility for player B is

$$\Pi_{B,3}(p) = 0$$

6.3 Incentive Analysis

When $v_A < v_L$, optimal strategy for contestant A is always 0, contestant B has no incentive to reveal his type for any posterior distribution p since $\Pi(p) = \Pi_{B,1}(p)$ is a linear function.

When $v_L \le v_A < v_H$, $\Pi_B(p)$ consists of two segments,

$$\Pi_B(p) = \begin{cases} \Pi_{B,2}(p) & p < 1 - \frac{v_L}{v_A} \\ \Pi_{B,1}(p) & p \ge 1 - \frac{v_L}{v_A} \end{cases} \tag{19}$$

since slopes of both parts are the same and the utility function $\Pi_B(p)$ has a step on $p = 1 - \frac{v_L}{v_A}$, contestant B has incentive to reveal his type if and only if $p < 1 - \frac{v_L}{v_A}$. See Figure 17 for a graphical illustration.

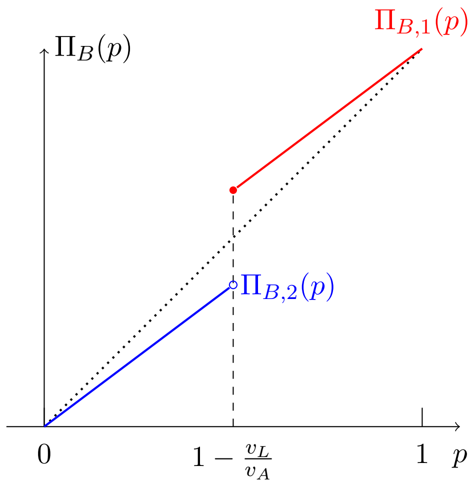


Figure 17: Incentive of Contestant B, sequential contests, $v_L < v_A < v_H$

When $v_A \ge v_H$, $\Pi_B(p)$ consists of two segments,

$$\Pi_B(p) = \begin{cases} \Pi_{B,2}(p) & p \le \frac{v_H}{v_A} \\ \Pi_{B,3}(p) & p \ge \frac{v_H}{v_A} \end{cases} \tag{20}$$

since $\Pi_B(0) = \Pi_B(1) = 0$, contestant B has no incentive to reveal his type for any posterior distribution p.

To summarize the results of different values of v_A, we have the following theorem.

Theorem 6.3. *Contestant B has incentive to reveal type information in sequential contests if $v_L < v_A < v_H$ and $p < 1 - \frac{v_L}{v_A}$.*

Since contestant B has incentive to share his or her type information only if $v_L < v_A < v_H$ and $p < 1 - \frac{v_L}{v_A}$, we need to modify the expected total effort function under such circumstance, as shown in left panel of Figure 18. Since the concave closure of expected total effort function will

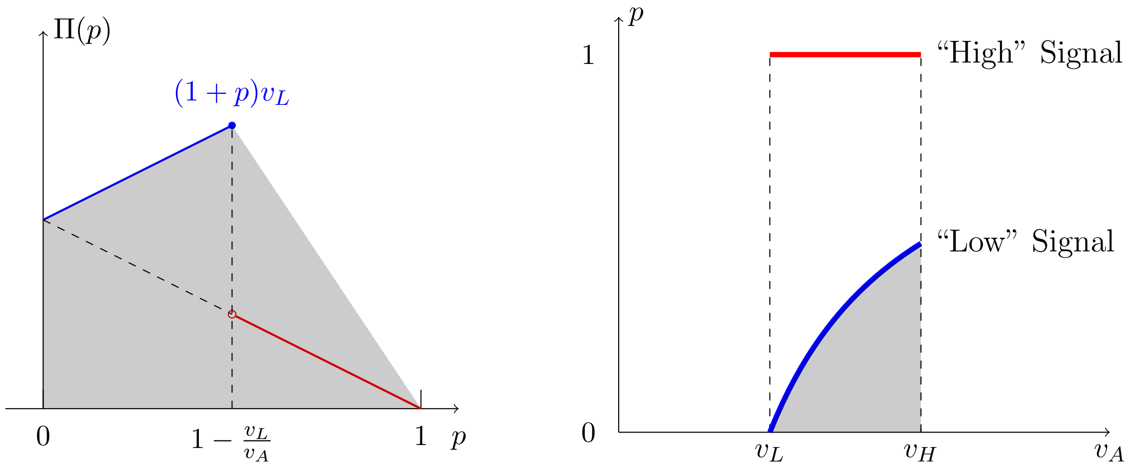


Figure 18: Bayesian Persuasion in Sequential Contests with Incentive of B, $v_L < v_A < v_H$

not change, contest organizer will not change his optimal signal when introducing the incentives of B, as shown in the following corollary.

Corollary 6.4. *For sequential contests, the optimal Bayesian persuasion signal will not change when incorporating incentive to reveal type information for contestant B.*

7 Extension II: Endogenized Order of Move

In this section, we assume that contest designer can endogenize the order of move for the contest. For all-pay auction contests we analyzed in this paper, three potential schemes arise:

1. Simultaneous move contest that has been analyzed in Section 4, denoted as **Sim**.
2. Sequential move contest with the player of public information as the first mover that has been analyzed in Section 5, denoted as **Pub**.
3. Sequential move contest with the player of private information as the first mover, denoted as **Pri**.

We slightly abuse the notation by letting Π_{Sim} denote the expected total effort under optimal Bayesian persuasion signal when the contest is simultaneous move, Π_{Pub} the expected total effort under optimal Bayesian persuasion signal when the player with public information moves first and Π_{Pri} the expected total effort under optimal Bayesian persuasion signal when the player with private information moves first.

According to the result in Section 5, we can conclude that when public player moves first and the contest designer applying the optimal Bayesian persuasion signal,

Case (1) $v_A < v_L$, $\Pi_{\text{Pub}}(p) = 0$.

Case (2) $v_L \leq v_A < v_H$, $\Pi_{\text{Pub}}(p) = \begin{cases} (1+p)v_L & p \leq 1 - \frac{v_L}{v_A} \\ (2v_A - v_L)(1-p) & p > 1 - \frac{v_L}{v_A} \end{cases}$

Case (3) $v_A \geq v_H$, $\Pi_{\text{Pub}}(p) = \begin{cases} pv_A + v_L & p < \frac{v_H - v_L}{v_A} \\ v_H & p \geq \frac{v_H - v_L}{v_A} \end{cases}$

7.1 Private Player as the First Mover

With probability p, the contest happens between high type player B and contestant A. With probability $1-p$, the contest happens between low type player B and contestant A. Since the information is complete, there is no need for Bayesian persuasion. The expected total effort function can be expressed as

$$\Pi_{\text{Pri}}(p) = pv_A \mathbb{I}(v_A \leq v_H) + (1-p)v_A \mathbb{I}(v_A \leq v_L)$$

with different values of v_A, $\Pi_{\text{Pri}}(p)$ has different expressions,

Case (1) $v_A \leq v_L$, $\Pi_{\text{Pri}}(p) = v_A$.

Case (2) $v_L < v_A \leq v_H$, $\Pi_{\text{Pri}}(p) = pv_A$.

Case (3) $v_A > v_H$, $\Pi_{\text{Pri}}(p) = 0$.

7.2 Comparison

7.2.1 $v_A \leq v_L$

Proposition 7.1. *When $v_A \leq v_L$,* **Pri** *is optimal.*

Proof. We first claim that when $v_A \leq v_L$, **Pub** is dominated. When $v_A < v_L$, $\Pi_{\text{Pub}}(p) = 0 < v_A = \Pi_{\text{Pri}}(p)$. When $v_A = v_L$, $\Pi_{\text{Pub}}(p) = v_A(1-p) \leq v_A = \Pi_{\text{Pri}}(p)$. Then we need to compare the expected total effort function between remaining two schemes. Since $\Pi_{\text{Sim}}(p)$ is a linear function[5] and

$$\begin{aligned} \Pi_{\text{Sim}}(0) = \frac{v_A v_L + v_A^2}{2v_L} &\leq v_A = \Pi_{\text{Pri}}(0) \\ \Pi_{\text{Sim}}(1) = \frac{v_A v_H + v_A^2}{2v_H} &< v_A = \Pi_{\text{Pri}}(1) \end{aligned}$$

we can conlude that **Pri** is optimal. □

In this situation, private player has higher winning value than public player for sure. And the optimal scheme is letting the player with higher winning value moves first. Note that when

[5] Because full disclosure is optimal for contest designer.

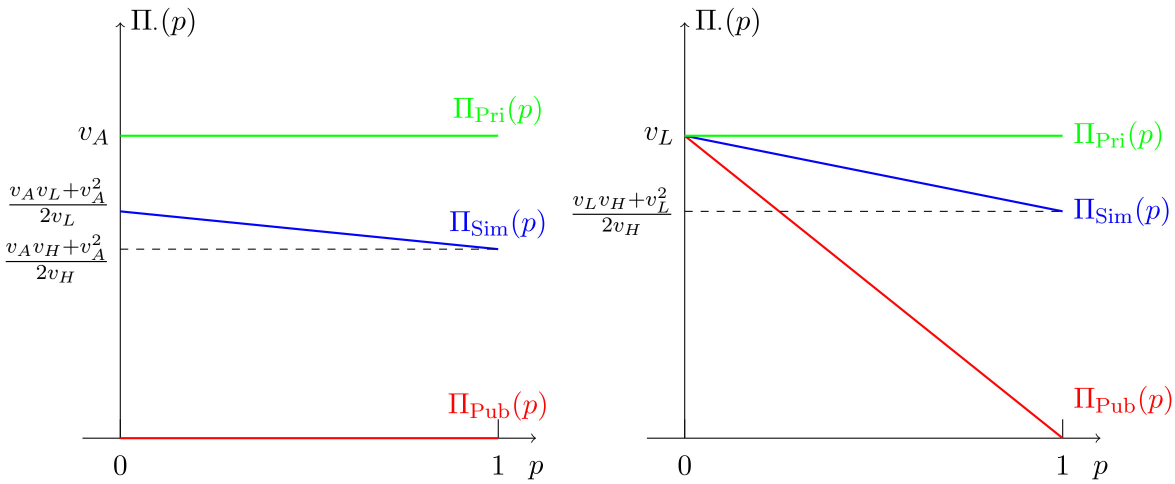


Figure 19: Endogenized Timing, $v_A < v_L$ (left) and $v_A = v_L$ (right)

$v_A = v_L$ and $p = 0$, all three schemes generate the same expected total effort function because it is actually a complete information all-pay auction contest with same winning values.

7.2.2 $v_A \geq v_H$

Proposition 7.2. *When $v_A \geq v_H$,* **Pub** *is optimal.*

Proof. We first claim that when $v_A \geq v_H$, **Pri** is dominated. When $v_A > v_H$, $\Pi_{\text{Pub}}(p) > 0 = \Pi_{\text{Pri}}(p)$. When $v_A = v_H$, $\Pi_{\text{Pub}}(p) \geq pv_A = \Pi_{\text{Pri}}(p)$. Then we need to compare the expected total effort function between remaining two schemes. Since $\Pi_{\text{Sim}}(p)$ is a piecewise linear function, and the following two comparisons imply that $\Pi_{\text{Sim}}(p)$ is lower than $\Pi_{\text{Pub}}(p)$,

$$\Pi_{\text{Sim}}(0) = \frac{v_A v_L + v_L^2}{2v_A} \quad < \quad v_L = \Pi_{\text{Pub}}(0)$$
$$\Pi_{\text{Sim}}(\frac{v_H}{v_A}) = \frac{v_A v_H + v_H^2}{2v_A} \quad < \quad v_H = \Pi_{\text{Pub}}(\frac{v_H}{v_A})$$

we can conlude that **Pub** is optimal. □

In this situation, public player has higher winning value than private player for sure. Similar to the previous case, optimal scheme is still letting the player with higher winning value moves first. Note that when $v_A = v_H$ and $p = 1$, all three schemes generate the same expected total effort function because it is actually a complete information all-pay auction contest with same winning values.

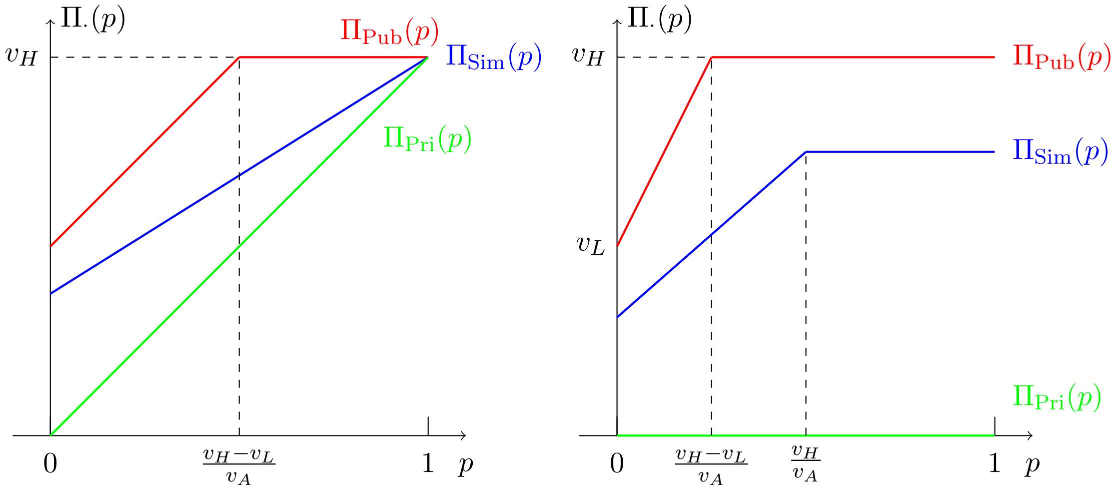


Figure 20: Endogenized Timing, $v_A = v_H$ (left) and $v_A > v_H$ (right)

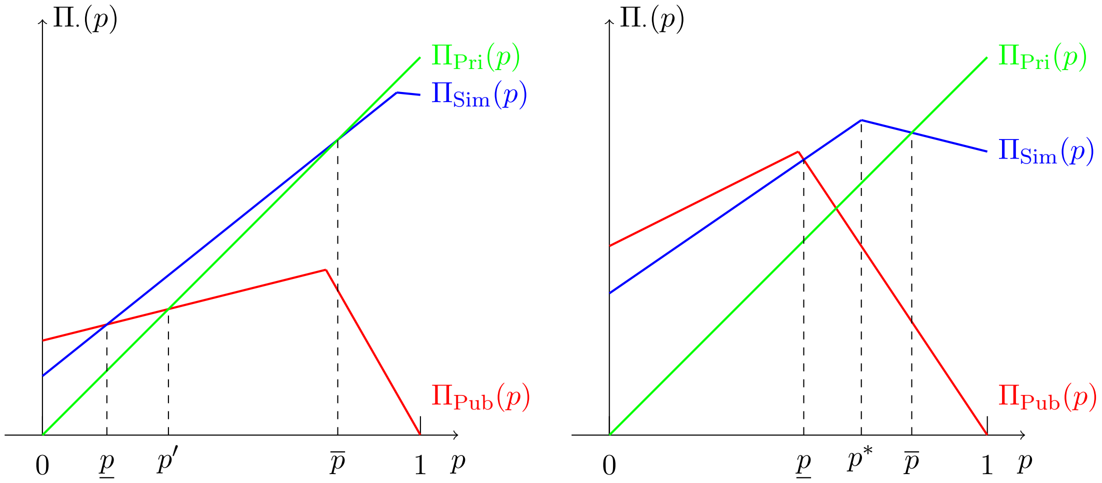


Figure 21: Endogenized Timing, $v_L < v_A < v_H$, (left $v_A < \frac{3+\sqrt{5}}{2} v_L$, right $v_A > \frac{3+\sqrt{5}}{2} v_L$)

7.2.3 $v_L < v_A < v_H$

$v_L < v_A < v_H$ is the most complicated case.The following proposition summarizes the optimal scheme under different prior, please refer to appendix for proof details.

Proposition 7.3. *When $v_L < v_A < v_H$, there exist two threshold values $0 < \underline{p} < \overline{p} < 1$ such that (1)* **Pub** *is optimal for $p \in [0, \underline{p}]$, (2)* **Sim** *is optimal for $p \in [\underline{p}, \overline{p}]$ and (3)* **Pri** *is optimal for $p \in [\overline{p}, 1]$.*

In this situation, public player is relatively stronger when p is lower and relatively weaker when p is higher. Similar to the previous two cases, optimal scheme lets the player with relatively higher winning value moves first. Hence, **Pub** is optimal when p is lower and **Pri** when p is higher. It is worth mentioning that when p is moderate, **Sim** is the optimal scheme.

8 Conclusion and Discussion

8.1 Discussion

Based on the research in this paper as well as previous studies on Bayesian persuaion in Tullock contests, we can conclude that in those four (2×2) classes of contest, there exist five different scenarios where Bayesian persuasion dominates degenerated policies.

1. Sequential Tullock contest, very strong player A, low probability of low type player B.
2. Simultaneous all-pay auction contest, medium player A, almost everywhere.
3. Simultaneous all-pay auction contest, strong player A, low probability of low type player B.
4. Sequential all-pay auction contest, medium player A, low probability of high type player B.
5. Sequential all-pay auction contest, strong player A, low probability of low type player B.

We can characterize them into three different classes, each of them exhibits an important economic and psychological insight.

Threat. When player A is strong ($v_A > v_H$ for all-pay and $v_A > 2v_H$ for Tullock), and the probability of low type player B is lower than one threshold ($1 - \frac{2v_H}{v_A}$ for sequential Tullock, $1 - \frac{v_H}{v_A}$ for simultaneous all-pay and $1 - \frac{v_H - v_L}{v_A}$ for sequential all-pay), player A will beat player B by using the same strategy as if all rivals are high type. We call this behavior **threat** because player B will give up for sure in the contest. Therefore, the contest designer can disguise some of the weak type contestant B as the strong type one such that contestant A will still exert the same level of effort after Bayesian updating. Thus the probability that contestant A is exerting higher effort (when facing a strong type contestant B) increases, and therefore the contest designer can achieve higher expected total effort.

Fluke. When player A is medium ($v_L < v_A < v_H$ for sequential all-pay), and the probability of high type player B is lower than one threshold ($1 - \frac{v_L}{v_A}$), player A will play against player B using the same strategy as if all rivals are low type. We call this behavior **fluke** because player A ignores the small probability of high type player B and focuses on beating low type player B. When $p > 1 - \frac{v_L}{v_A}$, compared with full disclosure policy, contest designer can disguise some of the strong type contestant B as the weak type one such that contestant A will still exert the same level of effort after Bayesian updating. Therefore, the contest designer can achieve higher expected payoff because (1) the probability that contestant A is exerting positive effort (when facing a weak type contestant B) increases; (2) the probability that contestant B is exerting the same effort (given player A entering the competition) increases.

Harmony. When player A is medium ($v_L < v_A < v_H$ for simultaneous all-pay), not only expected total effort $\Pi(p)$, but also $\mathbb{E}(x_A)$ and $\mathbb{E}(x_B)$, reach its maximum when harmonic mean of v_B equals v_A. The optimal value of p^* provides a harmonic contest environment that can stimulate both players at maximum level. The insights behind this phenomenon is noteworthy. One contestant will be less motivated not only when facing too many of strong rivals but also when facing too many of weak rivals. Consider the following real world example. A sales manager wants to maximize his employees' total efforts in a sales contest, for the sake of stimulating employees, manager should not involve a wide competence difference among a sales group. To achieve that, the sales manager can divide employees into different ability-based groups.

8.2 Conclusion

In this paper, we fully characterize the optimal Bayesian persuasion signal in an all-pay auction contest of 2 players with binary type distributions, and show that Bayesian persuasion can potentially generate higher total effort than degenerated strategies, in both simultaneous move and sequential move setups. In our model, one contestant's valuation is commonly known and the other contestant's valuation is private information. Our findings differ from the result for simultaneous Tullock contests in Zhang and Zhou (2016) and echo with the result for sequential Tullock contests in Chen, Kuang and Zheng (2017*a*). We restrict our attention to the binary case in this paper, and please refer to Kuang (2019) for analysis on more general type distributions.

In Kamenica and Gentzkow (2011), their first key question is when the sender could benefit from persuasion. The same question can be asked in our framework. Theorem 4.6 and Theorem 5.1 actually illustrate that contest designer can benefit from persuasion for both simultaneous and sequential all-pay auction contests, and this happens when $v_A > v_L$, while in sequential Tullock contests it is required that v_A far exceeds v_L, namely, $v_A > 2v_H$. The contest designer can take advantage of *threat* and *fluke* behavior of player A and use the property of *harmony* to enhance his objective.

We allow for the player with private information to voluntarily reveal own type in our first extension of the model and we show that the designer's optimal information disclosure policy is robust regardless of the player's information revealing incentive. We also consider about the possibility that the design can decide on the order of move for the contest, and we fully characterize the optimal disclosure policy as our second extension of the model.

8.3 Future Study

In simultaneous contests, we may be interested in correlated signals when both players have private values. Concept of correlated signal is first studied by Bergemann and Morris (2016). They focus on normal form game with state-dependent utility functions for both players. However, this seems very challenging, except for the symmetric prior and symmetric signal case, because we are not able to solve for Bayesian Nash equilibrium for the general case of incomplete information contests, not only in Tullock framework but also in all-pay auction framework. For sequential contests, it is meaningless to consider correlated signals because contestant B only takes into account the effort of contestant A, rather than the type of contestant A. One possible direction for future work can be that the designer can choose which contestant to be persuaded, assuming both players have asymmetric distributions of types.

A Proofs

Proof of Lemma 4.8.

$$
\begin{aligned}
\lim_{p \xrightarrow{+} p^*} \Pi_{\mathrm{I}}(p) &= \frac{pv_A^2}{v_H} - \frac{p^2 v_A^2}{2v_H} + \frac{(1-p)^2 v_A^2}{2v_L} + \frac{v_A}{2} \\
&= \frac{v_A(v_A - v_L)}{v_H - v_L} - \frac{v_H(v_A - v_L)^2}{2(v_H - v_L)^2} + \frac{v_L(v_H - v_A)^2}{2(v_H - v_L)^2} + \frac{v_A}{2} \\
&= \frac{2v_A(v_A - v_L)(v_H - v_L) - v_H(v_A - v_L)^2 + v_L(v_H - v_A)^2}{2(v_H - v_L)^2} + \frac{v_A}{2} \\
&= \frac{v_A^2 v_H - v_A^2 v_L - 2v_A v_H v_L + 2v_A v_L^2 - v_H v_L^2 + v_H^2 v_L}{2(v_H - v_L)^2} + \frac{v_A}{2} \\
&= \frac{v_A^2 + v_H v_L - 2v_A v_L}{2(v_H - v_L)} + \frac{v_A}{2}
\end{aligned} \tag{21}
$$

For $p \xrightarrow{-} p^*$, we have $\tilde{x} = v_L - \frac{pv_A v_L}{v_H} = \frac{v_L(v_H - v_A)}{v_H - v_L}$

$$
\begin{aligned}
\lim_{p \xrightarrow{-} p^*} \Pi_{\mathrm{II}}(p) &= \frac{pv_A \tilde{x}}{2v_H} + \frac{p^2 v_A^2}{2v_H} + \frac{\tilde{x}}{2} + p\tilde{x} + \frac{p^2 v_A}{2} + \frac{\tilde{x}^2}{2v_A} \\
&= \frac{v_A^2 - v_A v_L}{2(v_H - v_L)} + \frac{\tilde{x}}{2} + p\tilde{x} + \frac{p^2 v_A}{2} + \frac{\tilde{x}^2}{2v_A} \\
&= \frac{v_A^2 + v_H v_L - 2v_A v_L}{2(v_H - v_L)} + p\tilde{x} + \frac{p^2 v_A}{2} + \frac{\tilde{x}^2}{2v_A} \\
&= \frac{v_A^2 + v_H v_L - 2v_A v_L}{2(v_H - v_L)} + \frac{v_A}{2}
\end{aligned} \tag{22}
$$

□

Proof of Lemma 4.9.

$$
\begin{aligned}
\lim_{p \xrightarrow{+} p^*} \Pi'_{\mathrm{I}}(p) &= (1-p)v_A^2 \left(\frac{1}{v_H} - \frac{1}{v_L} \right) \\
&= -\frac{v_A v_L (v_H - v_A)}{v_H - v_L} \frac{v_H - v_L}{v_H v_L} \\
&= -\frac{v_A(v_H - v_A)}{v_H} < 0
\end{aligned} \tag{23}
$$

For $p \overset{-}{\longrightarrow} p^*$, we have $\tilde{x} = v_L - \frac{pv_Av_L}{v_H} = \frac{v_L(v_H-v_A)}{v_H-v_L}$ and $\tilde{x}'(p) = -\frac{v_Av_L}{v_H}$.

$$\begin{aligned}
\lim_{p \overset{-}{\longrightarrow} p^*} \Pi'_{\text{II}}(p) &= \frac{v_A\tilde{x}}{2v_H} - \frac{pv_A^2v_L}{2v_H^2} + \frac{pv_A^2}{v_H} + \frac{2v_Av_H - 3v_Av_L}{2v_H} \\
&= \frac{v_Av_L(v_H-v_A)}{2v_H(v_H-v_L)} - \frac{v_Av_L(v_A-v_L)}{2v_H(v_H-v_L)} + \frac{v_A(v_A-v_L)}{v_H-v_L} + \frac{2v_Av_H - 3v_Av_L}{2v_H} \\
&= \frac{v_Av_L(v_H-v_A) - v_Av_L(v_A-v_L) + 2v_Hv_A(v_A-v_L)}{2v_H(v_H-v_L)} + \frac{2v_Av_H - 3v_Av_L}{2v_H} \\
&= \frac{v_Av_Hv_L - 4v_A^2v_L + v_Av_L^2 + 2v_Hv_A^2 + 2v_Av_H^2 - 3v_Av_Hv_L - 2v_Av_Hv_L + 3v_Av_L^2}{2v_H(v_H-v_L)} \\
&= \frac{-2v_Av_Hv_L - 2v_A^2v_L + 2v_Av_L^2 + v_Hv_A^2 + v_Av_H^2}{v_H(v_H-v_L)} > 0
\end{aligned} \tag{24}$$

□

Proof of Lemma 4.10. (1) $\Pi(p^*) > \Pi(1)$.

$$\begin{aligned}
& \frac{v_A^2 + v_Hv_L - 2v_Av_L}{2(v_H-v_L)} + \frac{v_A}{2} > \frac{v_A^2 + v_Hv_A}{2v_H} \\
\iff \quad & \frac{v_A^2 + v_Hv_L - 2v_Av_L}{2(v_H-v_L)} > \frac{v_A^2}{2v_H} \\
\iff \quad & v_A^2v_H + v_H^2v_L - 2v_Av_Hv_L > v_A^2v_H - v_A^2v_L \\
\iff \quad & v_A^2v_L + v_H^2v_L > 2v_Av_Hv_L
\end{aligned}$$

(2) $\Pi(p^*) > \Pi(0)$.

$$\begin{aligned}
& \frac{v_A^2 + v_Hv_L - 2v_Av_L}{2(v_H-v_L)} + \frac{v_A}{2} > \frac{v_L^2 + v_Av_L}{2v_A} \\
\iff \quad & \frac{v_A^2 + v_Av_H + v_Hv_L - 3v_Av_L}{2(v_H-v_L)} > \frac{v_L^2}{2v_A} + \frac{v_L}{2} \\
\iff \quad & \frac{v_A^2 + v_Av_H - 3v_Av_L + v_L^2}{2(v_H-v_L)} > \frac{v_L^2}{2v_A}
\end{aligned}$$

then we can prove the last inequality holds,

$$\frac{v_A^2 + v_Av_H - 3v_Av_L + v_L^2}{2(v_H-v_L)} > \frac{v_Av_H - v_Av_L}{2(v_H-v_L)} = \frac{v_A}{2} > \frac{v_L^2}{2v_A}$$

□

B Extention II, Analysis of $v_L < v_A < v_H$ Case

Since both Π_{Pri} and Π_{Pub} is unrelated to v_H, we first compare these two timing schemes. Π_{Pri} is monotonically increasing in p, while Π_{Pub} is increasing in p when $p < 1 - \frac{v_L}{v_A}$ and decreasing in

p when $p > 1 - \frac{v_L}{v_A}$.

$$\begin{array}{rcl} 0 = \Pi_{\text{Pri}}(0) & < & \Pi_{\text{Pub}}(0) = v_L \\ v_A = \Pi_{\text{Pri}}(1) & > & \Pi_{\text{Pub}}(1) = 0 \\ \Pi_{\text{Pri}}(1 - \dfrac{v_L}{v_A}) & = & v_A - v_L \\ \Pi_{\text{Pub}}(1 - \dfrac{v_L}{v_A}) & = & 2v_L - \dfrac{v_L^2}{v_A} \end{array}$$

Those boundary conditions implies that there exists unique p' such that **Pri** and **Pub** are indifferent when $p = p'$, **Pub** is better when $p < p'$ and **Pri** is better when $p > p'$. The following lemma compare the intersection point p' with the inflection point $\frac{v_A - v_L}{v_A}$ of Π_{Pub}.

Lemma B.1. *Let p' denote the intersection point of $\Pi_{\text{Pub}}(p)$ and $\Pi_{\text{Pri}}(p)$,*

- *When $v_A > \frac{3+\sqrt{5}}{2} v_L$, $\Pi_{\text{Pri}}(1 - \frac{v_L}{v_A}) > \Pi_{\text{Pub}}(1 - \frac{v_L}{v_A})$, p' is lower than $\frac{v_A - v_L}{v_A}$.*
- *When $v_A = \frac{3+\sqrt{5}}{2} v_L$, $\Pi_{\text{Pri}}(1 - \frac{v_L}{v_A}) = \Pi_{\text{Pub}}(1 - \frac{v_L}{v_A})$, p' is equal to $\frac{v_A - v_L}{v_A}$.*
- *When $v_A < \frac{3+\sqrt{5}}{2} v_L$, $\Pi_{\text{Pri}}(1 - \frac{v_L}{v_A}) < \Pi_{\text{Pub}}(1 - \frac{v_L}{v_A})$, p' is higher than $\frac{v_A - v_L}{v_A}$.*

Proof. Let $x = \frac{v_A}{v_L}$, rearrange $v_A - v_L = 2v_L - \frac{v_L^2}{v_A}$ to $x^2 - 3x + 1 = 0$, the solution of which is $\frac{3+\sqrt{5}}{2}$. $\square$

The following lemma compute the intersection point p' of $\Pi_{\text{Pub}}(p)$ and $\Pi_{\text{Pri}}(p)$.

Lemma B.2. *Let p' denote the intersection point of $\Pi_{\text{Pub}}(p)$ and $\Pi_{\text{Pri}}(p)$,*

- *When $v_A > \frac{3+\sqrt{5}}{2} v_L$, $p' = \frac{v_L}{v_A - v_L}$ and $\Pi_{\text{Pri}}(p') = \Pi_{\text{Pub}}(p') = \frac{v_A v_L}{v_A - v_L}$.*
- *When $v_A = \frac{3+\sqrt{5}}{2} v_L$, $p' = 1 - \frac{v_L}{v_A}$ and $\Pi_{\text{Pri}}(p') = \Pi_{\text{Pub}}(p') = \frac{2+\sqrt{5}}{2} v_L$.*
- *When $v_A < \frac{3+\sqrt{5}}{2} v_L$, $p' = \frac{2v_A - v_L}{3v_A - v_L}$ and $\Pi_{\text{Pri}}(p') = \Pi_{\text{Pub}}(p') = \frac{v_A(2v_A - v_L)}{3v_A - v_L}$.*

The following lemma proves that when $v_A \geq \frac{3+\sqrt{5}}{2}$, simultaneous contest may be the optimal scheme under some prior.

Lemma B.3. *When $v_A \geq \frac{3+\sqrt{5}}{2} v_L$, $\Pi_{\text{Sim}}(\frac{v_L}{v_A - v_L}) > \frac{v_A v_L}{v_A - v_L}$.*

Proof. For $v_A \geq \frac{3+\sqrt{5}}{2} v_L$,

$$\begin{aligned}
\Pi_{\text{Sim}}(\frac{v_L}{v_A - v_L}) &= \Pi_{\text{Sim}}(0) + (\Pi_{\text{Sim}}(p^*) - \Pi_{\text{Sim}}(0))\frac{v_L}{v_A - v_L}\frac{v_A(v_H - v_L)}{v_H(v_A - v_L)} \\
&= \frac{v_A v_L + v_L^2}{2v_A} + (\frac{v_A^2 + v_A v_H + v_H v_L - 3v_A v_L}{2(v_H - v_L)} - \frac{v_A v_L + v_L^2}{2v_A})\frac{v_L}{v_A - v_L}\frac{v_A(v_H - v_L)}{v_H(v_A - v_L)} \\
&= \frac{v_A v_L + v_L^2}{2v_A} + \frac{v_L(v_A^3 + v_A^2 v_H + v_A v_H v_L - 3v_A^2 v_L - v_A v_H v_L - v_H v_L^2 + v_A v_L^2 + v_L^3)}{2v_H(v_A - v_L)^2} \\
&= \frac{v_A v_L + v_L^2}{2v_A} + \frac{v_L(v_A + v_L)}{2(v_A - v_L)} + \frac{v_L(v_A^3 - 3v_A^2 v_L + v_A v_L^2 + v_L^3)}{2v_H(v_A - v_L)^2} \\
&= \frac{v_A v_L + v_L^2}{2v_A} + \frac{v_L(v_A + v_L)}{2(v_A - v_L)} + \frac{v_L(v_A^2 - 2v_A v_L - v_L^2)}{2v_H(v_A - v_L)} \\
&= \frac{v_A v_L + v_L^2}{2v_A} + \frac{v_L(v_A + v_L)}{2(v_A - v_L)} + \frac{v_L(v_A^2 - 2v_A v_L - v_L^2)}{2v_H(v_A - v_L)} \\
&> \frac{v_L(v_A + v_L)(2v_A - v_L)}{2v_A(v_A - v_L)} \\
&= \frac{v_A v_L}{v_A - v_L}\frac{2v_A^2 + v_A v_L - v_L^2}{2v_A^2} \\
&> \frac{v_A v_L}{v_A - v_L}
\end{aligned}$$

□

The following lemma proves that when $v_A < \frac{3+\sqrt{5}}{2} < 3$, simultaneous contest may be the optimal scheme under some prior.

Lemma B.4. *If $v_A < 3v_L$, $\Pi_{\text{Sim}}(p^*) > \Pi_{\text{Pub}}(p^*)$ and $\Pi_{\text{Sim}}(p^*) > \Pi_{\text{Pri}}(p^*)$.*

Proof.

$$\begin{aligned}
\Pi_{\text{Sim}}(p^*) &= \frac{v_A^2 + v_A v_H + v_H v_L - 3v_A v_L}{2(v_H - v_L)} \\
\Pi_{\text{Pub}}(p^*) &= \frac{v_L(v_H - v_A)(2v_A - v_L)}{v_A(v_H - v_L)} \\
\Pi_{\text{Pri}}(p^*) &= \frac{v_H(v_A - v_L)}{v_H - v_L}
\end{aligned}$$

Part (1)

$$\begin{aligned}
\Pi_{\text{Sim}}(p^*) - \Pi_{\text{Pub}}(p^*) &= \frac{v_A^3 + v_A^2 v_H + v_A v_H v_L - 3v_A^2 v_L - 2v_L(v_H - v_A)(2v_A - v_L)}{2v_A(v_H - v_L)} \\
&= \frac{v_A^3 + v_A^2 v_H + v_A v_H v_L - 3v_A^2 v_L - (4v_A v_H v_L - 4v_A^2 v_L - v_H v_L^2 + v_A v_L^2)}{2v_A(v_H - v_L)} \\
&= \frac{v_A^3 + v_A^2 v_H - 3v_A v_H v_L + v_A^2 v_L + v_H v_L^2 - v_A v_L^2}{2v_A(v_H - v_L)} \\
&= \frac{v_H(v_A^2 - 3v_A v_L + v_L^2)}{2v_A(v_H - v_L)} + \frac{v_A^3 + v_A^2 v_L - v_A v_L^2}{2v_A(v_H - v_L)} \\
&= \frac{v_A^2 - 3v_A v_L + v_L^2}{2v_A} + \frac{v_A^3 + 2v_A^2 v_L - 4v_A v_L^2 + v_L^3}{2v_A(v_H - v_L)} \\
&> \frac{v_A^2 - 3v_A v_L + v_L^2}{2v_A} + \frac{v_A^2 + 2v_A v_L - v_L^2}{2v_A} \\
&= \frac{2v_A^2 - v_A v_L}{2v_A} > 0
\end{aligned}$$

Part (2)

$$\begin{aligned}
\Pi_{\text{Sim}}(p^*) - \Pi_{\text{Pri}}(p^*) &= \frac{v_A^2 + v_A v_H + v_H v_L - 3v_A v_L - 2v_H(v_A - v_L)}{2(v_H - v_L)} \\
&= \frac{(v_H - v_A)(3v_L - v_A)}{2(v_H - v_L)} < 0
\end{aligned}$$

□

Last but not the least, we prove Proposition 7.3.

For **Sim**, $\Pi_{\textbf{Sim}}(0) < \Pi_{\textbf{Pub}}(0)$ and $\Pi_{\textbf{Sim}}(1) < \Pi_{\textbf{Pri}}(1)$.

If the intersection point p' of $\Pi_{\textbf{Pub}}(p)$ and $\Pi_{\textbf{Pri}}(p)$ is lower than the inflection point of $\Pi_{\textbf{Pub}}(p)$, $1 - \frac{v_L}{v_A}$, we need only to prove that $\Pi_{\textbf{Sim}}(p')$ is larger than $\Pi_{\textbf{Pub}}(p') = \Pi_{\textbf{Pri}}(p')$, which is correct according to Lemma B.3. Hence, (1) $\Pi_{\textbf{Sim}}(p)$ has unique intersection point with $\Pi_{\textbf{Pub}}(p)$, denoted as $\underline{p} < p^*$, (2) $\Pi_{\textbf{Sim}}(p)$ has unique intersection point with $\Pi_{\textbf{Pri}}(p)$, denoted as $\overline{p} > p^*$ (see the left panel of Figure 21).

If the intersection point p' of $\Pi_{\textbf{Pub}}(p)$ and $\Pi_{\textbf{Pri}}(p)$ is higher than the inflection point of $\Pi_{\textbf{Pub}}(p)$, $1 - \frac{v_L}{v_A}$, then we have $v_A < \frac{3+\sqrt{5}}{2} v_L < 3v_L$. By Lemma B.4, we have $\Pi_{\textbf{Sim}}(p^*)$ is larger than both $\Pi_{\textbf{Pub}}(p^*)$ and $\Pi_{\textbf{Pri}}(p^*)$. Since $p^* > 1 - \frac{v_L}{v_A}$, we can conclude that (1) $\Pi_{\textbf{Sim}}(p)$ has unique intersection point with $\Pi_{\textbf{Pub}}(p)$, denoted as $\underline{p} < p^*$, (2) $\Pi_{\textbf{Sim}}(p)$ has unique intersection point with $\Pi_{\textbf{Pri}}(p)$, denoted as $\overline{p} > p^*$ (see right panel of Figure 21).

References

Amann, Erwin, and Wolfgang Leininger. 1995. "Expected revenue of all-pay and first-price sealed-bid auctions with affiliated signals." *Journal of Economics*, 61(3): 273–279.

Amann, Erwin, and Wolfgang Leininger. 1996. "Asymmetric All-Pay Auctions with Incomplete Information: The Two-Player Case." *Games & Economic Behavior*, 14(1): 1–18.

Baye, Michael R., Dan Kovenock, and Casper G. De Vries. 1996. "The all-pay auction with complete information." *Economic Theory*, 8(2): 291–305.

Bergemann, Dirk, and Stephen Morris. 2016. "Information Design, Bayesian Persuasion and Bayes Correlated Equilibrium." *American Economic Review*, 106(5): 586–591.

Buchanan, James M, Robert D Tollison, and Gordon Tullock. 1980. *Toward a theory of the rent-seeking society.* Texas A & M Univ Pr.

Chen, Jian, Zhonghong Kuang, and Jie Zheng. 2017*a*. "Bayesian Persuasion in Sequential Tullock Contests." *Working Paper.*

Chen, Jian, Zhonghong Kuang, and Jie Zheng. 2017*b*. "Optimal Joint Design of Timing and Information Disclosure in Tullock Contest." *Working Paper.*

Dechenaux, Emmanuel, Dan Kovenock, and Roman M. Sheremeta. 2015. "A survey of experimental research on contests, all-pay auctions and tournaments." *Experimental Economics*, 18(4): 609–669.

Denter, Philipp, John Morgan, and Dana Sisak. 2012. "'Where Ignorance is Bliss, 'Tis Folly to Be Wise': Transparency in Contests." *Working Paper.*

Hillman, Arye L., and John G. Riley. 1989. "Politicallly Contestable Rents and Transfers." *Economics & Politics*, 1(1): 17–39.

Kamenica, Emir, and Matthew Gentzkow. 2011. "Bayesian Persuasion." *American Economic Review*, 101(6): 2590–2615.

Konrad, Kai A. 2011. *Strategy and Dynamics in Contests.* Oxford University Press.

Kovenock, Dan, Florian Morath, and Johannes Munster. 2015. "Information Sharing in Contests." *Journal of Economics & Management Strategy*, 24(3): 570–596.

Kuang, Zhonghong. 2019. "Bayesian Persuasion in One-sided Asymmetric All-Pay Auction Contests." *Working Paper.*

Kuang, Zhonghong, Hangcheng Zhao, and Jie Zheng. 2019. "Information Design in All-pay Auction Contests." *Working Paper.*

Lazear, Edward P., and Sherwin Rosen. 1981. "Rank-Order Tournaments as Optimum Labor Contracts." *Journal of Political Economy*, 89(5): 841–864.

Linster, Bruce G. 1993. "Stackelberg rent-seeking." *Public Choice*, 77(2): 307–321.

Liu, Zhiyang, and Bo Chen. 2016. "A symmetric two-player all-pay contest with correlated information." *Economics Letters*, 145: 6–10.

Lu, Jingfeng, Hongkun Ma, and Zhe Wang. 2018. "Ranking Disclosure Policies in All-Pay Auctions with Incomplete Information." *Economic Inquiry*, 56(3): 1464–1485.

Milgrom, Paul R., and Robert J. Weber. 1982. "A Theory of Auctions and Competitive Bidding." *Econometrica*, 50(5): 1089–1122.

Morath, Florian, and Johannes Munster. 2013. "Information acquisition in conflicts." *Economic Theory*, 54(1): 99–129.

Segev, Ella, and Aner Sela. 2014*a*. "Multi-stage sequential all-pay auctions." *European Economic Review*, 70: 371–382.

Segev, Ella, and Aner Sela. 2014*b*. "Sequential all-pay auctions with head starts." *Social Choice & Welfare*, 43(4): 893–923.

Segev, Ella, and Aner Sela. 2014*c*. "Sequential All-Pay Auctions with Noisy Outputs." *Journal of Mathematical Economics*, 50(1): 251–261.

Serena, Marco. 2016. "Harnessing Beliefs to Stimulate Efforts." *Social Science Electronic Publishing*.

Siegel, Ron. 2009. "All-Pay Contests." *Econometrica*, 77(1): 71–92.

Siegel, Ron. 2014. "Asymmetric all-pay auctions with interdependent valuations." *Journal of Economic Theory*, 153(C): 684–702.

Szech, Nora, Olivier Bos, Konrad Kai, Benny Moldovanu, Johannes Mnster, Patrick Nora Schmitz, and Szech. 2011. "Asymmetric All-Pay Auctions with Two Types." *University of Bonn*.

Wu, Zenan, and Jie Zheng. 2017. "Information sharing in private value lottery contest ." *Economics Letters*, 157: 36–40.

Zhang, Jun, and Junjie Zhou. 2016. "Information Disclosure in Contests: A Bayesian Persuasion Approach." *Economic Journal*, 126: 2197–2217.