Information Asymmetry, Job Switching Mobility, and Screening of Abilities: A Tale of Two Sectors

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Initial Version January 29, 2014; Updated March 31, 2015

Abstract

How does information asymmetry between firms regarding the quality (ability) of workers, determine the distribution of workers’ qualities in those firms? We build a game theoretic model of information asymmetry between 2 representative firms competing in the labor market for labor inputs. In the benchmark model where one firm is perfectly informed about the quality of workers, while the other firm is fully uninformed, we show the existence and uniqueness of the equilibrium in which the informed firm obtains the high quality workers, while the uninformed firm obtains the low quality workers. We then consider an extension of the model where the uninformed firm is partially informed, in that high quality workers are distinguishable, while low quality workers are not. In equilibrium, the partially informed firm obtains both the highest and lowest quality workers, while the fully informed firm obtains the middle quality workers. We also consider a version of the baseline model with worker mobility friction, finding that in equilibrium, the results are similar to the model with the partially informed firm. For welfare concern, a higher technology level of the partially informed sector, an wider screening range, or a lower job switching friction level, will all lead to a higher level of the social surplus. While firms are able to work on the improvement of the first two factors, reducing labor market rigidity may have to

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heavily reply on the government policy. Our results have particular applications to the interactions between the state-owned and private firms in China’s labor market, as well as the interactions between foreign and domestic firms competing in the labor market in general.

JEL Classification Codes: C72, D24, D31, D82, J31, J62

Keywords: Screening, Asymmetric Information, Frictions, Job Mobility, Wage, Ability Distribution

1 Introduction

How do workers and firms interact when some firms in the market have an informational advantage over other firms regarding the quality of production inputs? For example, when local firms and global firms compete, global firms may suffer from lack of precise information about the potential pool of workers, while local firms can observe worker abilities first hand. Another example is the competition between State-owned Enterprises and private enterprises in China. Although State-owned firms may be larger and more powerful, they are generally less accurate at assessing the potential worker population.

In this paper, we build a game theoretic model of information asymmetry between two representative firms that are competing for labor inputs – that is, firms differ in terms of their ability to assess the quality (ability) of workers. In our benchmark model, Firm 1 (labelled as non-SOE) is perfectly informed about the quality of inputs, while Firm 2 (labelled as SOE) is uniformed, knowing only about the distribution of workers’ abilities. In the unique equilibrium of the game, there exists a cutoff point of the ability level such that the more informed firm obtains all the workers whose abilities are higher than the cutoff level, while less informed firm obtains the rest of the workers with relatively lower abilities.

We then consider a variant of the model with an information structure in which the less informed firm is partially informed about workers’ abilities, by being able to perfectly infer worker ability beyond a threshold level (ie. for sufficiently low abilities, the firm cannot distinguish), while the other firm is fully informed. In equilibrium, there exist two cutoff points of the ability levels such that Firm 2 obtains the high quality and low quality workers, while Firm 1 obtains those of middle-quality. We also consider an extension of the baseline model in which workers have the option to switch between firms with a mobility friction cost. In equilibrium, the model reduces to a static one, with the same outcome as in the model with a partially informed firm. We also show that there is a one-to-one mapping between and given an equilibrium allocation in the labor market, when is relatively large.

We also provide a concrete example with a simple triangular distribution of workers’ ability to illustrate the differences in the equilibrium outcomes among three model setups that have different settings for firms’ partial information and workers’ switching
Furthermore, we show that our results still hold qualitatively when introducing a finite screening cost or endogenizing the screening range.

Our paper relates to the literature on determinants of wage in the labor market, where firms are heterogeneous in their ability to obtain the relevant information about potential workers. Garen (1985)[2] considers a model of job screening, in which large firms have higher information acquisition costs than small firms, which can explain the positive correlations between wage, education and firm size in US labor markets. Other relevant papers explore the distribution of ability and earnings in the labor markets. In particular, Costrell and Loury (2004)[1] examine the impact of inequality in worker abilities on the earnings distribution. Grossman (2004)[4] considers the impact of international trade on labor force polarization when workers have private information about their own abilities. Another related strand of literature focuses on dual labor markets in developing countries. Zenou (2008)[8] studies a two-sector economy consisting of formal and informal sectors, in which the formal sector faces search frictions. The effect of tax policies on firms’ profits is examined.

Since our paper is inspired by the interaction of state-owned enterprises (SOE) and private firms in China’s labor market, we refer to the un-informed firm as the SOE and the informed firm as the non-SOE firm, throughout the paper. These are merely labels, and the key difference between the two types of firms in our framework is the information asymmetry.

The rest of the paper is organized as follows: Section 2 introduces the general model setup and provides motivating empirical support; Section 3 establishes the benchmark model; Section 4 extends the benchmark model by adding to SOE partial information on worker quality; Section 5 incorporates worker mobility into the benchmark model. For each version of the model, the equilibrium is derived, and comparative statics are examined. Section 6 provides a concrete example with triangular distribution for workers’ ability and solves for closed form solutions. In Section 7 we discuss a few issues including adding screening cost and endogenizing screening range. Section 8 concludes. And appendix gives out some technical proofs we consider important and not obvious.

2 The Environment

We consider an economy with an infinite number of workers with heterogeneous ability levels and two types of firms that vary in their information about the workers’ ability levels, for each of which there exists a representative firm.

2.1 Ability Distribution of Workers

Normalizing the mass of workers to a measure of one, we assume that a typical worker’ ability level $r$ follows a distribution with a differentiable cumulative distribution function $F(\cdot)$ and a continuous probability density function $f(\cdot)$. This means that the
number of workers\(^1\) whose abilities are less than \(r\) is measured by \(F(r)\). We introduce some reasonable assumptions regarding the ability distribution below.

**Assumption 1:** \(r \in [\varepsilon, +\infty)\) where \(\varepsilon > 0\). This assumption guarantees a positive bound from below for the distribution of ability in the population of workers, assuming all workers are able to work and hence can contribute to the production of goods.

**Assumption 2:** \(f\) is continuous on \([\varepsilon, +\infty)\), and \(f(\varepsilon) = 0, f'(\varepsilon) > 0\). This implies that that both the least able and most able workers are of a very small portion.

**Assumption 3:** \(\overline{r} = \int_{\varepsilon}^{+\infty} rdF(r) < +\infty\). That is, the average level of ability is limited.

**Assumption 4:** \(G(r^*) \equiv \frac{\int_{r^*}^{+\infty} r dF(r)}{r^* F(r^*)}\) is decreasing in \(r^*\).\(^2\) By letting \(H(r^*) = \int_{\varepsilon}^{r^*} r dF(r)\), an equivalent way of stating this assumption is that the elasticity of \(H(r^*)\) with respect to \(r^*\) is decreasing in \(r^*\).

Concerning the validity of the assumptions above, we use the SAT grades from 2006 to 2013 as a proxy of ability, and the statistical results are shown in Figures 2.1 and 2.2. It is easy to see that the data is quite consistent with Assumptions 1-4. We also tested our assumptions by using the Chinese high school students’ grades in the National College Entrance Examination (samples from Shandong Province in the year of 2013) for robustness check across countries, and the result is very positive.

\(^1\)To be precise, “the number of workers” actually means “the measure of workers” since we assume there is a continuum of workers.

\(^2\)Under the “Decreasing Strong Reverse Hazard Rate” assumption, i.e., \(\frac{rf(r)}{F(r)}\) decreasing in \(r\), it is easy to show that Assumption 2.4 holds.
2.2 Information Asymmetry on Workers’ Abilities

We assume that the workers’ ability distribution is common knowledge and every worker knows his own ability level. However, the two types of firms, type 1 and type 2, are assumed to have asymmetric information regarding workers’ ability levels. In reality, a firm’s information regarding a typical worker’s ability does not only depend on the type of the firm, but also depends on ability level of the worker. For example, regarding a typical worker’s ability level a type-1 firm’s information may be in general more precise than that of a type-2 firm, but it is also commonly agreed that a typical firm may be more informed of the ability level of worker A than that of worker B. In our model, for simplicity, we make the following assumptions about firms’ information on workers’ abilities.

Assumption 5: All the type-1 firms have the complete information regarding all the workers’ ability levels.

Assumption 6: All the type-2 firms have the incomplete information regarding all the workers’ ability levels.

Regarding Assumption 6, to keep our analysis tractable, without loss of generality, we are particularly interested in two kinds of incomplete information: no information and partial information, in this paper. Sections 3 and 5 will study the no information case and Section 4 will focus on the partial information case.

To come up with real world examples that fit the above setup with information asymmetry across different firms, one can either think of type-1 firms as local firms and type-2 firms as global firms, or the former as the non-state-owned enterprises (non-SOE) and the latter as the state-owned enterprises (SOE). From now on, to make the
type information self-evident, we will label the representative type-1 firm as non-SOE and the representative type-2 firm as SOE.

2.3 Production and Firm Profits

For both SOE and non-SOE, assume that their production functions are in the form of $AN^\alpha$ where $A \in (0, +\infty)$ measures the technology, $N$ is a factor of effective labor, defined as the following

$$N = \int_{-\infty}^{+\infty} rdF(r)$$ (2.1)

and $\alpha \in (0, 1)$ measures the contribution of effective labor to total output.

Denote the set including all workers in SOE by $\Sigma_s$ and that including all workers in non-SOE by $\Sigma_n$. Normalizing the price of the final good to 1, we can write non-SOE's profit as

$$\Pi_n = A_n \left[ \int_{\Sigma_n} rdF(r) \right]^\alpha - \int_{\Sigma_n} \omega_n (r) dF(r)$$ (2.2)

where $\omega_n (r)$ is the wage of non-SOE's worker with ability $r$.

For SOE, since it has incomplete information on workers' abilities, the wage level it sets for a typical worker cannot sorely depend on that worker's ability, thus its profit is given by the following expression:

$$\Pi_s = A_s \left[ \int_{\Sigma_s} rdF(r) \right]^\alpha - \int_{\Sigma_s} \omega_s (I(r)) dF(r)$$ (2.3)

where $\omega_s (I(r))$ is the wage of SOE's worker with ability $r$, and $\omega_s (I(r))$ depends on $I(r)$, its incomplete information on worker's ability.

2.4 Workers' Payoffs

We assume that a worker's per period payoff is simply the difference between his salary $\omega (r)$ and his job switching cost $f$ (if applicable). To be more specific,

$$u (r, t, y) = \begin{cases} 
\omega_n (r) - f & t = n, y = 1 \\
\omega_s (I(r)) - f & t = s, y = 1 \\
\omega_n (r) & t = n, y = 0 \\
\omega_s (I(r)) & t = s, y = 0 
\end{cases}$$

where $t$ is the type of the firm the worker works in and $y$ is the indicator of job switching status. In Sections 3 and 4 we consider the case where there is no job switching cost, and in Section 5 we allow for the existence of mobility frictions.
3 The Benchmark Model

In the benchmark case, we assume that (1) there is no job switching cost: \( f = 0 \), and (2) SOE’s incomplete information is described by the following assumption.

**Assumption 6**: SOE has no information regarding all the workers’ ability levels, i.e., \( I(r) = \emptyset \).

Given \( f = 0 \), obviously a worker will choose to work in a firm with a higher wage. If two firms provide the same wage, then the worker will randomly pick a firm to work in.

The non-SOE’s profit is simply described by (2.2). However, for SOE, it cannot distinguish workers’ abilities at all, so a universal wage \( \omega_s(r) \) will be set for all the workers SOE hires. Thus its profit is given by the following expression:

\[
\Pi_s = A_s \left[ \int_{\Sigma_s} r dF(r) \right]^{\alpha} - \omega_s(r) \int_{\Sigma_s} dF(r) \tag{3.1}
\]

### 3.1 Equilibrium Analysis

#### 3.1.1 The wage rule

To maximize non-SOE’s profit, the wage of worker with ability \( r \) is decided as

\[
\omega_n(r) = \alpha A_n \left[ \int_{\Sigma_n} r dF(r) \right]^{\alpha - 1} r \tag{3.2}
\]

It is derived from the fact that the wage equals the marginal value of production. On one hand, the firm’s profit will shrink with higher wage, so no firm has incentive to increase wage offers. On the other hand, there are lots of firms in SOE and non-SOE, so price competition will lead to the exact equality between wage and marginal value of production.

However, SOE can only provide a level of the same wage across all the workers it hires. Similar to the expression above, we can write SOE’s wage rule as

\[
\omega_s(r) = \alpha A_s \left[ \int_{\Sigma_s} r dF(r) \right]^{\alpha - 1} \tilde{r} \tag{3.3}
\]

where \( \tilde{r} \) is the average ability of workers satisfying

\[
\int_{\Sigma_s} \tilde{r} dF(r) = \int_{\Sigma_s} r dF(r) \tag{3.4}
\]

Substitute (3.4) back into (3.3) and we get

\[
\omega_s(r) = \frac{\alpha A_s \left[ \int_{\Sigma_s} r dF(r) \right]^\alpha}{\int_{\Sigma_s} dF(r)} \tag{3.5}
\]
3.1.2 The existence of cutoff

**Lemma 1** In equilibrium, the lowest ability level in $\Sigma_n$ is weakly higher than the highest ability level in $\Sigma_s$.

**Corollary 1** In equilibrium, there must exist a $r^*$ so that workers with ability lower than $r^*$ choose to work in SOE, while those with ability higher than $r^*$ choose to work in non-SOE.

For a high ability worker, non-SOE is a better choice since it will offer a salary exactly matching his ability, which exceeds the pooling wage provided by SOE. While for a low ability worker, pooling wage appears more appealing, due to the fact that the relatively high ability workers in SOE pull up the average ability level and make it possible to free ride. What described by Corollary 1 allows us to better understand the structure of equilibrium. A table is shown below to summarize the result:

<table>
<thead>
<tr>
<th>Ability</th>
<th>Work in</th>
<th>Number</th>
<th>Wage</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r \leq r^*$</td>
<td>SOE</td>
<td>$F (r^*)$</td>
<td>$\frac{\alpha A_s \left[ \int_{r^<em>}^{+\infty} rdF(r) \right]^\alpha}{F(r^</em>)}$</td>
</tr>
<tr>
<td>$r &gt; r^*$</td>
<td>Non-SOE</td>
<td>$1 - F (r^<em>) \alpha A_n \left[ \int_{r^</em>}^{+\infty} rdF (r) \right]^{\alpha - 1} r$</td>
<td></td>
</tr>
</tbody>
</table>

In Table 3.1, $r^*$ is decided by

$$
\frac{\alpha A_s \left[ \int_{r^*}^{+\infty} rdF (r) \right]^\alpha}{F(r^*)} = \alpha A_n \left[ \int_{r^*}^{+\infty} rdF (r) \right]^{\alpha - 1} r^* \quad (3.6)
$$

The wage distribution as a function of workers’ ability is shown in Figure 3.1.

![Figure 3.1: Wage Distribution as a Function of Workers’ Ability](image)
3.1.3 The existence and uniqueness of equilibrium

However, we should make sure that (3.6) has at least one solution, which prevents the analysis from ending up with the trivial situation where all workers want to work in SOE or all want to work in non-SOE. Also, whether the equilibrium remains unique is of our interest. Fortunately, we can prove both existence and uniqueness of equilibrium under some regular conditions.

**Proposition 1** Given the differentiability of $F(r)$ and $f(0) > 0$, (3.6) has one unique positive solution for the cutoff value $r^*$. 

3.2 Comparative Statics

3.2.1 Increase in $A_s$

First, more workers will crowd into SOE, since now technology advance in SOE can enhance wage level, leading to an increase in $r^*$. Then, a higher cutoff value of ability will enlarge the wage multiplier in non-SOE, since fewer workers in non-SOE result in higher marginal production value. Finally, the total effect of more advanced technology and shift of the cutoff ability on SOE workers’ wage appears positive, though the pure effect of the cutoff ability shift remains ambiguous.
Figure 3.2: Comparative Statics with Respect to an Increase in $A_s$

We can see clearly from the graph above that an increase in SOE’s technology will make everyone better off.

3.2.2 Increase in $A_n$

First, advance in non-SOE’s technology will drive workers to move from SOE to non-SOE, decreasing the cutoff ability. Then, three kinds of patterns will possibly happen, depending on the value of $\alpha$, the contribution of effective labor to total output.

To be more specific, a two-panel graph is drawn as below:
When $\alpha$ is sufficiently large, i.e. $\alpha \to 1$, the wage level in SOE rises. As a matter of fact, shrinking in SOE’s size will lead to two different effects: lower expected average ability by SOE and higher marginal production value, which drive SOE’s wage move towards opposite directions. However, a large allows that the marginal production value effect will dominate. At the meantime, non-SOE’s wage multiplier also ascends, since the advance in technology dominates the decrease in marginal production value. Now everyone in the economy gets better off.

When $\alpha$ is sufficiently small, i.e. $\alpha \to 0$, the wage level in SOE falls, since now lower expected average ability dominates. Pretty much to the same, non-SOE’s wage multiplier shrinks since the advantage brought by the advanced technology cannot fully offset the disadvantage of a lower marginal production value. Under this condition, all workers are worse off.

When $\alpha$ is at some moderate value between 0 and 1, there is still one other state that may occur: the wage level in SOE decreases but the multiplier in non-SOE still increases. Now the low ability workers get worse off while those with high ability are happier than before.

To conclude, the technology advance in SOE always benefits the whole population while it is hard to judge whether non-SOE’s innovation does good to workers. In fact, it does good only if the labor factor accounts for a share large enough in production.
3.2.3 Increase in \( \alpha \)

We can change the form of (??) into:

\[
k (r^*; \alpha) = \frac{\int_{r^*}^{+\infty} r dF (r)}{r^* F (r^*)} \left( \frac{\int_{r^*}^{+\infty} r dF (r)}{\int_{r^*}^{+\infty} r dF (r)} \right)^{\alpha}
\]

So we can find some fixed point \( r_f^* \) satisfying \( \int_{r^*}^{r_f^*} r dF (r) = \int_{r_f^*}^{+\infty} r dF (r) \) that \( k (r_f^*; \alpha) \) doesn’t change value when \( \alpha \) goes through \((0, 1)\).

Furthermore, \( \frac{\partial k(r^*; \alpha)}{\partial \alpha} < 0 \) for \( r^* < r_f^* \) and \( \frac{\partial k(r^*; \alpha)}{\partial \alpha} > 0 \) for \( r^* > r_f^* \).

In this case, when the technology in non-SOE is relatively advanced compared to that in SOE, an increase in will drive more workers to shift from SOE to non-SOE; When non-SOE’s technology level is relatively lower, a rise in will push workers in the opposite direction.

4 A Model With Partial Information

In the model with partial information, everything remains unchanged as in Section 3, except for the SOE’s information pattern. We assume that there exists a cutoff level \( L \) such that SOE can precisely know the ability of workers whose ability is above or equal to \( L \) while knows nothing about the ability of those whose ability is below \( L \).

To be specific, we assume that (1) there is no job switching cost: \( f = 0 \), and (2) SOE’s incomplete information is described by the following assumption.

**Assumption 6”**: SOE has no information regarding a worker’s ability level \( r \) if \( r < L \), i.e., \( I (r) = \emptyset \), and has full information regarding a worker’s ability level \( r \) if \( r \geq L \), i.e., \( I (r) = r \).
The non-SOE’s profit is simply described by (2.2). However, for SOE, it cannot
distinguish workers’ abilities when \( r < L \) and can precisely tell the workers’ abilities
when \( r \geq L \). So a universal wage will be set for all the workers with \( r < L \), and an
ability-specific wage rule will be set for those with \( r \geq L \).

4.1 Equilibrium Analysis

4.1.1 The wage rule

Similarly to the basic model, non-SOE’s wage rule is:

\[
\omega_n (r) = \alpha A_n \left[ \int_{\Sigma_n} r dF (r) \right]^{\alpha - 1} r
\]

(4.1)

However, SOE’s wage rule is now a segmented function. For workers whose ability
is beyond \( L \), it will pay a wage proportional to their ability. For workers whose ability
is lower than \( L \), SOE cannot judge what their abilities are, so it will pay a wage
corresponding to the expected ability of those who are in SOE with abilities below \( L \):

\[
\omega_s (r) = \begin{cases} 
\alpha A_s \left[ \int_{\Sigma_s} r dF (r) \right]^{\alpha - 1} r & \forall r > L \\
\alpha A_s \left[ \int_{\Sigma_s} r dF (r) \right]^{\alpha - 1} \tilde{r} & \forall r \leq L 
\end{cases}
\]

(4.2)

where

\[
\int_{\Sigma_s \cap [0,L]} \tilde{r} dF (r) = \int_{\Sigma_s \cap [0,L]} r dF (r)
\]

(4.3)

Obviously, \( \tilde{r} < L \), which brings a discontinuity of SOE’s wage at the ability \( L \). We
can see that for those with abilities above \( L \), their wages are proportional to abilities,
while the coefficient is different between SOE and non-SOE.

4.1.2 Possible Equilibrium

**Proposition 2** If the equilibrium exists with a mixture of SOE and non-SOE for work-
ers’ abilities lower than \( L \), it must be the case that the coefficient in non-SOE is less
than that in SOE, i.e. both low-ability and high-ability workers choose SOE while those
in the middle range go to non-SOE.

The pattern and the intuition for workers with abilities lower than \( L \) exactly re-
sembles those in Section 3. But the preference of SOE over non-SOE by highest ability
workers indicates the existence of wage premium. In fact, if there is no wage premium,
SOE is not capable of attracting any worker, which in equilibrium will be denied by
limited labor force and Inada condition. Now the wage path is shown as below:

![Figure 4.1: Wage Distribution as a Function of Workers' Ability](image)

**Proposition 3** With a $C^1$ continuous probability density function and $f(\varepsilon) = 0$, for an $L$ large enough, the above equilibrium exists, it is uniquely determined by

$$A_n \left[ \int_{r^*}^{L} r dF(r) \right]^{\alpha-1} r^* = A_s \left[ \int_{\varepsilon}^{r^*} r dF(r) + \int_{L}^{+\infty} r dF(r) \right]^{\alpha-1} \frac{\int_{\varepsilon}^{r^*} r dF(r)}{F(r^*)} \quad (4.4)$$

**4.1.3 The profit and social welfare**

Now the profits of non-SOE and SOE are output minus wage:

$$\pi_n = (1 - \alpha) A_n \left[ \int_{\varepsilon}^{L} r dF(r) \right]^\alpha$$

$$\pi_s = (1 - \alpha) A_s \left[ \int_{\varepsilon}^{r^*} r dF(r) + \int_{L}^{+\infty} r dF(r) \right]^\alpha$$

And social welfare is represented by total output:

$$W = A_n \left[ \int_{\varepsilon}^{L} r dF(r) \right]^\alpha + A_s \left[ \int_{\varepsilon}^{r^*} r dF(r) + \int_{L}^{+\infty} r dF(r) \right]^\alpha$$

**4.2 Comparative statics**

**4.2.1 Shock on $L$**

**Lemma 2** In equilibrium, when SOE expands the screening range, i.e., lowers $L$, the cutoff $r^*$ will move in the same direction with $0 < \frac{\partial r^*}{\partial L} \frac{L f(L)}{r^* f(r^*)}$.
The outcome of Lemma 2 is quite intuitive: When SOE improves screening technology to expand the screening range, free riding behavior becomes less popular.

**Proposition 4** In equilibrium, SOE’s profit will increase if lowering \( L \), while non-SOE’s profit will shrink. Furthermore, the total profit will increase, and the social welfare, i.e., the total output, will improve.

From firms' perspective, enlarging screening range will prevent SOE from free riding, thus SOE will gain more. However, it is not good news to non-SOE, because more fierce competition is going to cause a profit squeeze. From the social planner’s standpoint, more complete information should reduce efficiency loss and hence improve social welfare.

Fortunately, the goals of social planner and SOE are consistent in our setting, which means that no government intervention is needed to induce SOE to carry out more efficient screening policy.

### 4.2.2 Increase in \( A_s \)

**Lemma 3** If SOE improves the technology, the cutoff \( r^* \) will also increase, leading to the result where more low-ability workers enter SOE.

The result from Lemma 3 is also very straightforward, for technology enhancement gives a raise in SOE’s wage, attracting more and more workers who have originally been working in non-SOE.

**Proposition 5** If SOE improves the technology, the profit of SOE will rise while that of non-SOE will drop. Furthermore, the social welfare will increase.

More workers in SOE will definitely bring more revenue, but to draw a conclusion on how profit will change, we still need the analysis of SOE’s wage. There are two impacts concerning wage: decrease in per unit wage and increase in labor force, with the latter dominating the former. As a result, the revenue gain fully offsets the loss from higher salary, leading to an increase in profit. As for non-SOE, the analysis is to the contrary. Also, not surprisingly, one-sided technology promotion in SOE will do good to social welfare.

### 4.2.3 Increase in \( A_n \)

**Lemma 4** If non-SOE improves the technology, the cutoff \( r^* \) will decrease, leading to the result where more low-ability workers enter non-SOE.

**Proposition 6** If non-SOE improves the technology, the profit of non-SOE will rise while that of SOE will drop.
The main result from SOE’s technology improvement can apply here, by merely replacing the word "SOE" by "non-SOE", except for the conclusion on the social welfare change.

Therefore, it is always beneficial to upgrade the SOE’s technology, but whether or not we should encourage active innovation in non-SOE sector remains an ambiguous question requiring more detailed and careful consideration.

4.3 Discussion about the size of $L$

The result of $L$ being large enough is interesting and consistent with reality, but when $L$ is relatively small, i.e. $1 \geq \frac{A_s}{A_n} \left[ \frac{\int_{L}^{\infty} r dF(r)}{\int_{L_{-}}^{\infty} r dF(r)} \right]^{1-\alpha}$, note that there is still equilibrium.

Let $L^*$ be such that $\frac{A_s}{A_n} \left[ \frac{\int_{L^*}^{\infty} r dF(r)}{\int_{L_{-}}^{\infty} r dF(r)} \right]^{1-\alpha} = 1$, and we have the following results.

**Lemma 5** If $L \leq L^*$, in equilibrium there are no mixture of SOE and non-SOE workers whose ability are below $L$. All of them choose to work in non-SOE.

**Proposition 7** If $L \leq L^*$, SOE and non-SOE have the same proportional coefficient, with some of the fully screened workers in SOE and the rest in non-SOE.

However, in this case we are only able to know the measure of the total ability in both firms, without being able to justify a specific fully screened worker’s choice.

We summarize the results in the following table and provide graphs that show the results in a more straightforward way.

<table>
<thead>
<tr>
<th>Table 4.1: Equilibrium Allocations of Labor in SOE and Non-SOE when $L \leq L^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Range</strong></td>
</tr>
<tr>
<td>SOE</td>
</tr>
<tr>
<td>Non-SOE</td>
</tr>
</tbody>
</table>
To guarantee that all workers with ability lower than $L$ are in non-SOE, we should have
\[ \frac{A_s^{1-\alpha}}{A_s^{1-\alpha} + A_n^{1-\alpha}} \int_{\epsilon}^{+\infty} r d F(r) \geq \int_{\epsilon}^{L} r d F(r), \] i.e. $1 \geq \frac{A_s}{A_n} \left[ \frac{\int_{\epsilon}^{L} r d F(r)}{\int_{\epsilon}^{+\infty} r d F(r)} \right]^{1-\alpha}$, which is consistent with the condition.

Moreover, we can see that now the change in $L$ does not have influence on the total ability in both firms, thus has no influence on both profits and social welfare, since all the workers are offered with the wage exactly corresponding to abilities.

The expressions for profits and the total output are the following:

\[
\pi_s = \frac{(1 - \alpha) A_s^{1-\alpha}}{A_s^{1-\alpha} + A_n^{1-\alpha}} \left( \int_{\epsilon}^{+\infty} r d F(r) \right)^\alpha
\]

\[
\pi_n = \frac{(1 - \alpha) A_n^{1-\alpha}}{A_s^{1-\alpha} + A_n^{1-\alpha}} \left( \int_{\epsilon}^{+\infty} r d F(r) \right)^\alpha
\]

\[
W = \left( A_s^{1-\alpha} + A_n^{1-\alpha} \right)^{1-\alpha} \left( \int_{\epsilon}^{+\infty} r d F(r) \right)^\alpha
\]

Therefore, an increase of technology $A_t$ will increase the profit in the corresponding sector $t$ and lower that in the other sector. However, the technology improvement can definitely improve the social welfare.
5 A Model with Mobility Friction

In the model with worker mobility friction, we assume that the SOE can’t screen any specific worker’s ability, which is similar to Section 3, instead, there is a new feature introduced in this section: when any worker enters the labor market, he first chooses to work in SOE or non-SOE. If one chooses to enter SOE first, he can only get an average wage since SOE are not able to acquire the information about his ability. However, if one chooses to enter non-SOE first, he has two alternatives thereafter: to keep working in non-SOE or to switch into SOE. Due to the working experience in non-SOE, he reveals his true ability level according to the wage given by non-SOE. Thus he can earn a wage exactly matching his ability if switching to work in SOE. But the job switching imposes a permanent mobility friction cost \( f \). The choice tree is shown below:

\[
\begin{align*}
\text{SOE} & \{ \text{non-SOE}, \ldots \} \\
\text{non-SOE} & \{ \text{SOE}, \ldots \}
\end{align*}
\]

To simplify the analysis, we assume that the economy will come to an equilibrium immediately, with rational expectations by both firms and works, and we are interested in such a stable equilibrium. Also, it is assumed that the transition from non-SOE to SOE takes place in a very short time so that the temporary wage in non-SOE almost counts for nothing.

5.1 Equilibrium Analysis

5.1.1 The wage rule

In the economy, there are three different types of workers divided by their choices: those directly working in SOE, whose set is denoted by \( \Sigma_s \); those directly working in non-SOE, denoted by \( \Sigma_n \); those switching from non-SOE to SOE, denoted by \( \Sigma_{ns} \).

As in the benchmark model and the model with partial information, here we also solve for the wage rules of the three types:

\[
\begin{align*}
\omega_s(r) &= \alpha A_s \left( \int_{\Sigma_s+\Sigma_{ns}} r dF (r) \right)^{\alpha-1} \tilde{r}, r \in \Sigma_s \\
\omega_n(r) &= \alpha A_n \left( \int_{\Sigma_n} r dF (r) \right)^{\alpha-1} r, r \in \Sigma_n \\
\omega_{ns}(r) &= \alpha A_s \left( \int_{\Sigma_s+\Sigma_{ns}} r dF (r) \right)^{\alpha-1} r - f, r \in \Sigma_{ns}
\end{align*}
\]

where

\[
\int_{\Sigma_s} \tilde{r} dF (r) = \int_{\Sigma_s} r dF (r)
\]
defines the average ability.
5.1.2 Existence and uniqueness of equilibrium

Since firms’ profits increase with the new entrance of workers, there are no rejections when job switching occurs. The first step is to determine the pattern of equilibrium.

**Lemma 6** For any randomly picked workers with ability $r_s$ from $\Sigma_s$, $r_n$ from $\Sigma_n$, $r_{ns}$ from $\Sigma_{ns}$, the relationships are $r_s \geq r_n$, $r_s \geq r_{ns}$, and $r_n \leq r_{ns}$.

We can easily conclude that $\Sigma_n$ is non-empty, otherwise the marginal entrance into non-SOE will get an infinitely high wage level, providing a strong incentive to deviate. Similar reasoning can show that $\Sigma_s$ and $\Sigma_{ns}$ are also non-empty. In fact, if $\Sigma_{ns}$ is empty, according to Lemma 26, the equilibrium will be the same as in the benchmark model. However, now the worker with sufficient high ability will find out that switching to SOE will gain a benefit exceeding the friction cost $f$.

In addition, if $\Sigma_s$ is empty, according to Lemma 26, the equilibrium will be described as below, shown in Figure 5.1.

![Figure 5.1: Ability Distribution in SOE and Non-SOE with Empty $\Sigma_s$](image)

However, now for those workers in non-SOE, they are regretting about not choosing directly to enter SOE, for then an average wage exactly matches their ability and the coefficient in SOE is greater than that in non-SOE.
Next, consider the situation where all three types’ sets are non-empty. Now the equilibrium will be like the following:

There are two cutoffs, so we can pin down two cutoff conditions:

\[
\begin{align*}
\alpha A_s & \left( \int_{\varepsilon}^{r_1^*} r dF(r) + \int_{r_2^*}^{+\infty} r dF(r) \right)^{\alpha-1} \frac{\int_{\varepsilon}^{r_1^*} r dF(r)}{F(r_1^*)} \quad (5.1) \\
& = \alpha A_n \left( \int_{r_1^*}^{r_2^*} r dF(r) \right)^{\alpha-1} r_1^* \\
& = \alpha A_s \left( \int_{\varepsilon}^{r_1^*} r dF(r) + \int_{r_2^*}^{+\infty} r dF(r) \right)^{\alpha-1} r_2^* \\
& = \alpha A_n \left( \int_{r_1^*}^{r_2^*} r dF(r) \right)^{\alpha-1} r_2^* + f \\
\end{align*}
\]

**Proposition 8** Given the friction cost \( f \) large enough, the economy has a unique equilibrium determined by (5.1)/(5.2).

**5.1.3 Equivalence between partial information setting and the mobility friction setting**

In fact, due to the uniqueness of the equilibrium, there is a one-to-one relationship between the cutoff value of the screening range \( L \) in the model with partial information.
and the value of the friction cost \( f \) in the model with mobility friction, characterized by the following condition:

\[
\alpha \left[ A_s \left( \int_{r_L}^{r^*} r dF(r) + \int_{L}^{+\infty} r dF(r) \right)^{\alpha-1} - A_n \left( \int_{r^*}^{L} r dF(r) \right)^{\alpha-1} \right] L = f
\]

where \( L \) is equivalent to the cutoff \( r_2^* \) in the model with mobility friction.

### 5.2 Comparative statics

#### 5.2.1 Shock on \( f \)

**Proposition 9** In equilibrium, SOE’s profit will rise if lowering \( f \), while non-SOE’s profit will drop. Furthermore, the total profit will increase, and the social welfare, i.e., the total output, will improve.

The friction cost \( f \) here acts like a wedge between the salaries of SOE and non-SOE sectors. A smaller \( f \) enables SOE to pay a slimmer wage premium, thus is welcomed by SOE but does harm to non-SOE. However, for the entire society, a less rigid labor market will definitely improve the efficiency of the labor resource allocation.

#### 5.2.2 Increase in \( A_s \)

**Proposition 10** In equilibrium, an increase in \( A_s \) will lead to decreases in \( r_2^* \) and non-SOE’s profit, but increases in SOE’s profit and total output.

Very much like the case in Section 4, SOE’s technology improvement will benefit SOE and improve social welfare, at the cost of hurting non-SOE. But the propagation seems a little bit different in that now more high ability workers swarm into SOE to raise profit, while in Section 4 technology improvement only brings more low ability workers to SOE.

#### 5.2.3 Increase in \( A_n \)

**Proposition 11** In equilibrium, an increase in \( A_n \) will lead to decreases in \( r_1^* \) and SOE’s profit, and an increase in non-SOE’s profit.

The main results here are the same as those in Section 4 with no labor market friction and pre-determined screening pattern. Still, one-sided technology advance in non-SOE sector is not always a good idea.

It is worth mentioning that with the existence of labor mobility cost, contract design by SOE can fully replicate the economy pattern where SOE screens workers all by itself. In this case, non-SOE unintentionally helps screen workers for SOE.
6 A Simple Example with Triangular Distribution

In this section, we provide a concrete setup for the story of two sectors with a triangular distribution of workers’ ability. To be specific, we assume \( A_n = A_s, \alpha = \frac{1}{2}, \) and the probability density function \( f(x) \) is defined below

\[
f(x) = \begin{cases} 
\frac{x}{a^2}, & 0 \leq x \leq a \\
\frac{2a-x}{a^2}, & a \leq x \leq 2a \\
0, & \text{otherwise}
\end{cases}
\]

which implies

\[
F(x) = \begin{cases} 
0, & x < 0 \\
\frac{x^2}{2a^2}, & 0 \leq x \leq a \\
\frac{4ax-2a^2-x^2}{2a^2}, & a \leq x \leq 2a \\
1, & x > 2a
\end{cases}
\]

Figure 6.1: Triangular Distribution

It is easy to calculate that all the assumptions regarding the ability distributions hold under this triangular distribution. In particular, the critical variable \( G(r) \) defined in Assumption 4 is weakly decreasing in \( r^* \), which is crucial to guarantee the uniqueness condition for the equilibrium.

\[
G(r^*) = \int_{r^*}^{r} r dF(r) = \begin{cases} 
\frac{2}{3}, & 0 \leq r \leq a \\
\frac{2}{3} \frac{2a^2-r^3-a^3}{a^2r^3-2a^2r}, & a \leq r \leq 2a
\end{cases}
\]

6.1 Equilibrium for the Benchmark Model

Following the analysis in Section 3, if the cutoff value satisfies \( r^* \leq a \), then the cutoff condition can be described as \( \frac{3}{2} = \left( \frac{a}{\frac{a^3}{3a^2}} \right)^{\frac{3}{2}} \). Solving this equation for \( r^* \), we have
\( r^* = \sqrt[3]{\frac{12}{13}}a \), which is indeed less than \( a \). Since the solution is unique, we can conclude that it is the only solution.

### 6.2 Equilibrium for the Model with Partial Information

If SOE has full information about a worker’s ability for \( r \in [L, +\infty) \) and has no information about a worker’s ability for \( r \in [\varepsilon, L) \), the cutoff value should be less than that in the benchmark case, so \( r^* < a \) is also satisfied in this case. For the existence of a pure-strategy equilibrium, \( L \) should be large enough such that

\[
1 \leq \frac{A_s}{A_n} \left[ \int_{L}^{\infty} r dF(r) \right]^{-1} \left( \int_{\varepsilon}^{L} r dF(r) \right)^{-1} \left( \int_{r^*}^{L} r dF(r) \right) \frac{L}{2} = f
\]

which implies \( L > a \). Given that, the cutoff condition becomes

\[
R_{\frac{3}{2}} = \frac{L^3}{a^3} \left( \frac{L^3}{a} + \frac{L^2}{a^2} - \frac{L}{a} + \frac{r^3}{a^3} \right) \frac{1}{2}
\]

with the solution

\[
r^* = \sqrt[3]{3aL^2 - L^3 - \frac{40}{13}a^3}
\]

### 6.3 Equilibrium for the Model with Mobility Friction

According to the equivalence relationship between the cutoff value of the screening range \( L \) in the model with partial information and the switching cost \( f \) in model with mobility friction, it is true that \( r^* = r_1^* < a < r_2^* = L < 2a \) and

\[
\left[ A_s \left( \int_{\varepsilon}^{r^*} r dF(r) + \int_{L}^{+\infty} r dF(r) \right)^{-\frac{1}{2}} - A_n \left( \int_{r^*}^{L} r dF(r) \right)^{-\frac{1}{2}} \right] \frac{L}{2} = f
\]

The equation above can be transformed into the following one:

\[
\left( \int_{\varepsilon}^{r^*} r dF(r) + \int_{L}^{+\infty} r dF(r) \right)^{-\frac{1}{2}} \left( 1 - \frac{\int_{\varepsilon}^{r^*} r dF(r)}{r^* F(r^*)} \right) \frac{L}{2} = \frac{f}{A_s}
\]

Utilizing the distribution information, we can have the following two equations:

\[
\left( \frac{r^3 + L^3}{3a^2} + \frac{4a}{3} - \frac{L^2}{a} \right)^{-\frac{1}{2}} L = \frac{6f}{A_s}
\]

\[
\sqrt{3aL^2 - L^3 - \frac{40}{13}a^3} = r^*
\]

Solving for the two equations above, we can have

\[
r_1^* = \sqrt[3]{3aL^2 - L^3 - \frac{40}{13}a^3}
\]

\[
r_2^* = \frac{6f}{A_s} \sqrt[3]{\frac{4}{13}a}
\]
7 Discussion

7.1 Finite Screening Cost

So far in the paper, we assume that once SOE has no information regarding a worker’s ability, it has no way to identify this worker’s ability. In other words, the screening cost is infinity in the current setting. In this subsection, we allow for a moderate level of the screening cost and would like to check if our previous results still hold qualitatively. Since the benchmark model (Section 3) can be viewed as a special case of the model with partial information (Section 4) when \( L \) approaches infinity, we only consider how the assumption of screening cost plays a role in the latter.

7.1.1 Constant screening cost

Assume that for non-SOE the cost for screening every single worker is \( \delta \), where \( 0 < \delta < +\infty \), and for SOE the cost for screening a worker with ability \( r \) is \( \delta \) if \( r \geq L \), and it is infinite if \( r < L \). So the profits for SOE and non-SOE are

\[
\Pi_n = A_n \left( \int_{\Sigma_n} r dF(r) \right)^\alpha - \delta \int_{\Sigma_n} F(r) - \int_{\Sigma_n} \omega_n(r) dF(r)
\]

\[
\Pi_s = A_s \left( \int_{\Sigma_s} r dF(r) \right)^\alpha - \delta \int_{\Sigma_s \cap (L, +\infty)} F(r) - \int_{\Sigma_s} \omega_s(r) dF(r)
\]

Thus the wage rules are

\[
\omega_n(r) = \alpha A_n \left( \int_{\Sigma_n} r dF(r) \right)^{\alpha-1} r - \delta
\]

\[
\omega_s(r) = \begin{cases} 
\alpha A_s \left( \int_{\Sigma_s} r dF(r) \right)^{\alpha-1} r - \delta, r > L \\
\alpha A_s \left( \int_{\Sigma_s} r dF(r) \right)^{\alpha-1} \frac{\int_{\Sigma_s \cap (L, +\infty)} F(r)}{\int_{\Sigma_s \cap (L, +\infty)} dF(r)}, r \leq L
\end{cases}
\]

and the corollary about the cutoff point still holds, with the cutoff condition shown below as

\[
\alpha A_n \left( \int_{r^*}^L r dF(r) \right)^{\alpha-1} r^* - \delta = \alpha A_s \left( \int_{\epsilon}^{r^*} r dF(r) + \int_{L}^{+\infty} r dF(r) \right)^{\alpha-1} \frac{\int_{\epsilon}^{r^*} r dF(r)}{F(r^*)}
\]

Rearranging the terms, we get

\[
1 = \frac{\delta}{\alpha A_n \left( \int_{r^*}^L r dF(r) \right)^{\alpha-1} r^*} + \frac{A_s}{A_n} \left( \frac{\int_{\epsilon}^{r^*} r dF(r) + \int_{L}^{+\infty} r dF(r)}{\int_{r^*}^L r dF(r)} \right)^{\alpha-1} \frac{\int_{\epsilon}^{r^*} r dF(r)}{r^* F(r^*)}
\]

The right hand decreases in \( r^* \), so the uniqueness of equilibrium is guaranteed. And the existence of equilibrium is guaranteed automatically when \( \epsilon \) is sufficiently small.

The profit levels are the same as before, which means that the firm transfer all the screening cost to workers.
7.1.2 Linear screening cost

Suppose instead that the cost of screening every single worker is linear to the total amount of workers screened, i.e. \( \delta (\int_{\Sigma} A dF(r)) \), where \( A \) denotes the set of workers who have been screened.

In this case, it can also be shown that the existence and uniqueness of equilibrium is guaranteed, and the firms actually takes an advantage through screening.

7.1.3 Diminishing screening cost

It is most likely in reality that the cost of screening every single worker is diminishing as the total amount of workers screened increases, i.e. \( \delta (\int_{\Sigma} A dF(r))^{\beta-1} \), where \( A \) denotes the set of workers who have been screened and \( \beta \in (0, 1) \). In this case the condition for the uniqueness of equilibrium becomes stronger while the firms and the workers share the cost of efficiency due to the existence of screening cost.

7.2 Endogenizing the Screening Range

In all the models we have studied, the screening range for SOE is exogenously given. However, it is more plausible in the real world that firms may have incentive to choose which workers to identify their ability and which not to and that this may be feasible through costly information acquisition. In such scenarios, we need a model where the screening range can be endogenously decided by the firms.

In this subsection, we assume that non-SOE has no screening cost and for the SOE, the cost for screening every single worker is \( \delta \), where \( 0 < \delta < +\infty \). So the profits for SOE and non-SOE are

\[
\Pi_n = A_n \left( \int_{\Sigma_n} r dF(r) \right)^{\alpha} - \int_{\Sigma_n} \omega_n(r) dF(r)
\]

\[
\Pi_s = A_s \left( \int_{\Sigma_s} r dF(r) \right)^{\alpha} - \delta \int_{\Sigma_s \cap (L, +\infty)} dF(r) - \int_{\Sigma_s} \omega_s(r) dF(r)
\]

and the wage rules are

\[
\omega_n(r) = \alpha A_n \left( \int_{\Sigma_n} r dF(r) \right)^{\alpha-1} r
\]

\[
\omega_s(r) = \alpha A_s \left( \int_{\Sigma_s} r dF(r) \right)^{\alpha-1} r - \delta, r > L
\]

\[
\omega_s(r) = \alpha A_s \left( \frac{\int_{\Sigma_s} r dF(r)}{\int_{\Sigma_s \cap (\epsilon, L)} dF(r)} \right)^{\alpha-1}, r \leq L
\]

When \( \delta \) goes to infinity, it is the exact case of the benchmark model (Section 3). When \( \delta \) becomes larger, there will be a regime switching. We consider two possible cases here.
7.2.1 $\delta$ is relatively small

As can be easily seen from the figure above, the equilibrium pattern when $\delta$ is relatively small resembles the results in the model with partial information (Section 4) except that there is a downward shift of $\delta$ for SOE’s wage path with ability greater than $L$. Obviously, the cutoff value is the same as in Section 4 and is determined by

$$A_n \left( \int_{r^*}^{L} r dF (r) \right)^{\alpha-1} r^* = A_s \left( \int_{\varepsilon}^{r^*} r dF (r) + \int_{L}^{+\infty} r dF (r) \right)^{\alpha-1} \frac{\int_{\varepsilon}^{r^*} r dF (r)}{F (r^*)}$$

The profits are

$$\Pi_n = (1 - \alpha) A_n \left( \int_{r^*}^{L} r dF (r) \right)^{\alpha}$$

$$\Pi_s = (1 - \alpha) A_s \left( \int_{\varepsilon}^{r^*} r dF (r) + \int_{L}^{+\infty} r dF (r) \right)^{\alpha}$$

and the social welfare is

$$W = A_n \left( \int_{r^*}^{L} r dF (r) \right)^{\alpha} + A_s \left( \int_{\varepsilon}^{r^*} r dF (r) + \int_{L}^{+\infty} r dF (r) \right)^{\alpha} - \delta (1 - F (L))$$

As can be easily seen, when $\delta$ is relatively small, increasing screening cost brings no effect on the cutoff point, and thus have no effect on the profits for both firms, but indeed decreases the social welfare.
7.2.2 \( \delta \) is relatively large

When \( \delta \) is relatively large, there are two cutoff points, one smaller than \( L \) while the other one larger than \( L \). The following two equations simultaneously determine the pattern of the economy:

\[
\alpha A_n \left( \int_\varepsilon^{r_1^*} rdF(r) + \int_{r_2^*}^{+\infty} rdF(r) \right)^{\alpha - 1} \frac{\int_\varepsilon^{r_1^*} rdF(r)}{F(r_1^*)} - \alpha A_n \left( \int_{r_2^*}^{+\infty} rdF(r) \right)^{\alpha - 1} r_1^* = 0
\]

\[
\alpha A_s \left( \int_\varepsilon^{r_1^*} rdF(r) + \int_{r_2^*}^{+\infty} rdF(r) \right)^{\alpha - 1} r_2^* - \alpha A_s \left( \int_{r_2^*}^{+\infty} rdF(r) \right)^{\alpha - 1} r_2^* - \delta = 0
\]

We find out that the results are the almost the same as what we have in the model with mobility frictions (Section 5), except that the screening cost \( \delta \) here plays the role of the friction cost \( f \) in Section 5. The profits are

\[
\Pi_n = (1 - \alpha) A_n \left( \int_{r_1^*}^{r_2^*} rdF(r) \right)^\alpha
\]

\[
\Pi_s = (1 - \alpha) A_s \left( \int_\varepsilon^{r_1^*} rdF(r) + \int_{r_2^*}^{+\infty} rdF(r) \right)^\alpha
\]

And the social welfare is

\[
W = A_n \left( \int_{r_1^*}^{r_2^*} rdF(r) \right)^\alpha + A_s \left( \int_\varepsilon^{r_2^*} rdF(r) + \int_{L}^{+\infty} rdF(r) \right)^\alpha - \delta \left( 1 - F(L) \right)
\]
Similarly to what we have in the model with mobility frictions, here we can conclude that increasing $\delta$ will increase $r_1^*, r_2^*, \Pi_n$ but decrease $\Pi_s$.

Although in this case it is hard to derive the analytic comparative statics for social welfare with respect to changes in size of screening cost, numerically we can show that the following results are robust: (1) In the first regime (with small $\delta$), social welfare drops with an increase in $\delta$; (2) In the second regime (with large $\delta$), social welfare rises with an increase in $\delta$. The pattern can be shown as follows.

![Figure 7.3: Change of Social Welfare in $\delta$](image)

The first result is intuitive in a sense that when the screening cost is small, greater screening cost does not affect the pattern of the economy pattern but will induce more waste on wages, and hence results in a decrease in social welfare. However, part of the second result is quite surprising that when the screening cost is large greater screening cost leads to an increase in social welfare. To understand this, note that increasing screening cost has two impacts that move towards the opposite directions: i) cutting the number of screened workers, which will help increase social welfare; ii) raising the screening cost for screened workers, which will reduce social welfare. When the screening cost already reaches a relatively high level, the first impact dominates the second due to the diminishing cost effect.

However, it is worth mentioning that the second regime may not occur in the real world. Note that in reality when $\delta$ is large, this implies the cost of information acquisition between SOE and Non-SOE is substantial, hence SOE will have strong incentive to reduce this information asymmetry through various channels, which are not incorporated in our model. When $\delta$ is small, such an incentive is weak and hence our results should make good sense in the first regime.
8 Conclusion

In this paper, we adopt a game theoretical approach to study the workers’ job choices and firms hiring decisions in an economy with asymmetric information about the workers’ abilities. We show the uniqueness and existence of equilibrium, solve for the wage rule and labor allocation, as well as conduct the comparative statics analysis. An equivalence result is established between the role of screening range cutoff value $L$ and the role of job switching cost $f$, which helps us to better understand how workers’ job switching mobility and firms’ information asymmetry in screening workers’ ability interact with each other and how each of these two factors affects the equilibrium outcome. We also provide a concrete example with a simple triangular distribution of workers’ ability to illustrate the differences in the equilibrium outcomes among three setups that have different settings for firms’ partial information and workers’ switching cost. Furthermore, we show that our results still hold qualitatively when introducing a finite screening cost or endogenizing the screening range.

Basically, our paper suggests that a wider screening range or a lower labor mobility friction cost will attract more workers to the sector with incomplete information, thus increase that sector’s profit and social welfare at the cost of squeezing the other sector’s gain. Besides, one sector’s technology enhancement shifts profit toward itself.

Therefore, the government should focus on reducing the labor market rigidity, for example, by offering re-employment training program, establishing and improving platforms containing amplified employment information and so on. Note that there is no need to intervene firms’ screening mechanism, because firms with incomplete information will spontaneously seek a way to enlarge the screening range, consistent with social planner’s objective. Also, productivity improvement in the incomplete information sector always helps while the result is ambiguous for productivity improvement in the complete information sector.

Our results have a wide range of applications to the interactions between firms with different levels of information precision about the quality of the production inputs. Such scenarios includes the competition between the state-owned and private firms in China’s labor market, as well as the interactions between foreign/global and domestic/local firms competing in the factor markets in general.

One direction for future study is to incorporate information acquisition into the model, which will improve the model’s power to explain why such an information asymmetry exist among different firms. Another area which might be promising is to build a dynamic model with mobility frictions and screening cost, where both commitment and learning will play important roles.
Appendix: Technical Proofs:

Proof. of Lemma 1: Suppose not, then there exists a \( r \) in \( \Sigma_s \) and a \( r' \) in \( \Sigma_n \) satisfying \( r > r' \). Then according to the workers’ decision rule we have \( \omega_n (r) \leq \omega_s (r) \) and \( \omega_n (r') \geq \omega_s (r') \). However, according to the firms’ wage rules we have: \( \omega_s (r) = \omega_s (r') \) and \( \omega_n (r') = \alpha A_n \left[ \int_{\Sigma_n} r dF(r) \right]^{\alpha-1} r' < \alpha A_n \left[ \int_{\Sigma_n} r dF(r) \right]^{\alpha-1} r = \omega_n (r) \), contradicting \( \omega_n (r) \leq \omega_s (r) \).

Proof. of Proposition 3: To prove existence, denote

\[
k (r^*; \alpha) = \frac{\left[ \int_{r}^{+\infty} r dF(r) \right]^\alpha \left[ \int_{r^*}^{+\infty} r dF(r) \right]^{1-\alpha}}{r^* F(r^*)}
\]

Since \( \int_{r}^{+\infty} r dF(r) < +\infty \), we know \( \lim_{r^* \to +\infty} k (r^*; \alpha) = 0 \). We also have

\[
\lim_{r^* \to 0} k (r^*; \alpha) = \lim_{r^* \to 0} \frac{r^* f(r^*) \left\{ \alpha \left[ \int_{r}^{+\infty} r dF(r) \right]^{1-\alpha} - (1-\alpha) \left[ \int_{r}^{+\infty} r dF(r) \right]^\alpha \right\}}{F(r^*) + r^* f(r^*)} = +\infty
\]

Due to the continuity of \( k (r^*; \alpha) \), there must be a solution for \( k (r^*; \alpha) = \frac{A_n}{A_s} \) for \( r^* > 0 \). To prove uniqueness, rewrite the equation (3.6) and we have

\[
\frac{A_n}{A_s} \left( \int_{r}^{+\infty} r dF(r) \right)^{1-\alpha} = \frac{r^* F(r^*)}{\int_{r}^{+\infty} r dF(r)}
\]

The left side decreases in \( r^* \) while the right side increases in \( r^* \), thus there is at most one equilibrium solution. Therefore, we know that there is a unique equilibrium satisfying

\[
k (r^*; \alpha) = \frac{A_n}{A_s}
\]

The unique equilibrium is shown as below.

![Figure A.1: The Uniqueness of Equilibrium Cutoff Value of Workers’ Ability](image-url)
Proof. of Proposition 4: there can be three possible outcomes considering the relative location of SOE’s and non-SOE’s wage path: First, suppose that the coefficient in non-SOE is greater than that in SOE. In this case low-ability workers all crowd into SOE while high-ability workers all go to non-SOE. The cutoff point is in the range $[0, L]$.

![Figure A.2: Case 1: $A_n > A_s$](image1)

Second, suppose that the coefficient in non-SOE is less than that in SOE. Then low-ability workers and high-ability workers all merge into SOE while workers with abilities in the middle range prefer non-SOE.

![Figure A.3: Case 2: $A_s > A_n$](image2)

Third, suppose that the coefficient in non-SOE coincides with SOE’s. Now among all the workers with abilities greater than $L$, some will choose SOE and the other will choose non-SOE. For workers with the abilities lower than $r^*$, SOE is the better shelter, while for the workers with abilities in the middle range, non-SOE appears to be more
Furthermore, note that it cannot occur that the coefficient in non-SOE is so low that all workers choose to work in SOE, otherwise the non-SOE coefficient will expand to infinite. According to the three patterns shown above, the low-ability workers are all in SOE, the cutoff condition requires

$$A_n \left[ \int_{\Sigma_n} r dF (r) \right]^{\alpha-1} r^* = A_s \left[ \int_{\Sigma_s} r dF (r) \right]^{\alpha-1} \frac{\int_{r^*}^{\infty} r dF (r)}{F (r^*)}$$

While it is always true that $r^* F (r^*) > \int_{r^*}^{\infty} r dF (r)$, the following inequality holds:

$$A_n \left[ \int_{\Sigma_n} r dF (r) \right]^{\alpha-1} < A_s \left[ \int_{\Sigma_s} r dF (r) \right]^{\alpha-1}$$

**Proof.** Proposition 5: Rearrange the items, we get

$$\frac{r^* F (r^*)}{\int_{r^*}^{\infty} r dF (r)} = \frac{A_s}{A_n} \left[ \frac{\int_{r^*}^{L} r dF (r)}{\int_{r^*}^{\infty} r dF (r) + \int_{L}^{\infty} r dF (r)} \right]^{1-\alpha} \quad (A-1)$$

The left side increases in $r^*$ while the right side decreases in $r^*$. Therefore, if the equilibrium exists, it must be unique. And In the equation (A-1), when $r^*$ approaches $L$, the left side approaches $\frac{L F (L)}{\int_{r^*}^{\infty} r dF (r)}$ while the right side approaches 0. It suffices to show that when $r^*$ approaches $\varepsilon$, the left side is less than the right side for the existence of equilibrium. Use Lopita’s Rule, we have

$$\lim_{r^* \to \varepsilon} \frac{r^* F (r^*)}{\int_{r^*}^{\infty} r dF (r)} = \lim_{r^* \to \varepsilon} \frac{F (r^*) + r^* f (r^*)}{r^* f (r^*)} = 1 + \lim_{r^* \to \varepsilon} \frac{f (r^*)}{r^* f' (r^*)} = 1$$
So the existence condition is the following:

\[ 1 \leq \frac{A_s}{A_n} \left[ \frac{\int_{L}^{L} r dF(r)}{\int_{L_n}^{+\infty} r dF(r)} \right]^{1-\alpha} \]

\[ \frac{f_{\epsilon}^{L} r dF(r)}{f_{\epsilon}^{+\infty} r dF(r)} \]

**Proof.** of Lemma 6: In fact, consider the equation (A-1), rearrange the items and denote

\[ h(L, r^*; A_s, A_n) = \frac{A_n}{A_s} \left[ \frac{\int_{L}^{+\infty} r dF(r) - 1}{\int_{L_n}^{L} r dF(r)} \right]^{1-\alpha} - \frac{f_{\epsilon}^{r^*} r dF(r)}{r^* F(r^*)} = 0 \]

Since

\[ \partial \left( \frac{f_{\epsilon}^{r^*} r dF(r)}{r^* F(r^*)} \right) / \partial r^* < 0 \]

We have

\[ \frac{\partial h}{\partial r^*} > \frac{(1 - \alpha) A_n}{A_s} \left[ \frac{\int_{L}^{+\infty} r dF(r) - 1}{\int_{L_n}^{L} r dF(r)} \right]^{-\alpha} \frac{r^* f(r^*)}{\left( \int_{L_n}^{L} r dF(r) \right)^2} \]

Together with

\[ \frac{\partial h}{\partial L} = -\frac{(1 - \alpha) A_n}{A_s} \left[ \frac{\int_{L}^{+\infty} r dF(r) - 1}{\int_{L_n}^{L} r dF(r)} \right]^{-\alpha} \frac{Lf(L)}{\left( \int_{L_n}^{L} r dF(r) \right)^2} \]

We know that

\[ 0 < \frac{\partial r^*}{\partial L} < \frac{L f(L)}{r^* f(r^*)} \]

**Proof.** of Proposition 7: According to the expression of profit, we have

\[ \frac{\partial \pi_s}{\partial L} = (1 - \alpha) A_n \left[ \int_{L_n}^{r^*} r dF(r) + \int_{r^*}^{+\infty} r dF(r) \right]^{\alpha-1} \left( r^* f(r^*) \frac{\partial r^*}{\partial L} - L f(L) \right) < 0 \]

\[ \frac{\partial \pi_n}{\partial L} = (1 - \alpha) A_n \left[ \int_{L_n}^{L} r dF(r) \right]^{\alpha-1} \left( L f(L) - r^* f(r^*) \frac{\partial r^*}{\partial L} \right) > 0 \]

As for the social welfare

\[ \frac{\partial W}{\partial L} = A_n \left[ \int_{r^*}^{L} r dF(r) \right]^{\alpha-1} \left( L f(L) - r^* f(r^*) \frac{\partial r^*}{\partial L} \right) + \alpha A_n \left[ \int_{L_n}^{L} r dF(r) \right]^{\alpha-1} \left( r^* f(r^*) \frac{\partial r^*}{\partial L} - L f(L) \right) < 0 \]
Proof. of Proposition 9: According to the expression of profit, we have

\[
\frac{\partial \pi_s}{\partial A_s} = (1 - \alpha) \alpha A_s \left[ \int_{\xi}^{r^*} r^* F(r) + \int_{L}^{+\infty} F(r) \right]^{\alpha-1} r^* f(r^*) \frac{\partial r^*}{\partial A_s} \\
+ (1 - \alpha) \left[ \int_{\xi}^{r^*} r^* F(r) + \int_{L}^{+\infty} F(r) \right]^{\alpha-1} > 0
\]

\[
\frac{\partial \pi_n}{\partial A_s} = - (1 - \alpha) \alpha A_n \left[ \int_{r^*}^{L} F(r) \right]^{\alpha-1} r^* f(r^*) \frac{\partial r^*}{\partial A_n} < 0
\]

\[
\frac{\partial W}{\partial A_s} = - \alpha A_n \left[ \int_{r^*}^{L} F(r) \right]^{\alpha-1} r^* f(r^*) \frac{\partial r^*}{\partial A_n} + \left[ \int_{\xi}^{r^*} r^* F(r) + \int_{L}^{+\infty} F(r) \right]^{\alpha-1} > 0
\]

Proof. of Proposition 11: According to the expression of profit, we have

\[
\frac{\partial \pi_s}{\partial A_n} = (1 - \alpha) \alpha A_s \left[ \int_{\xi}^{r^*} r^* F(r) + \int_{L}^{+\infty} F(r) \right]^{\alpha-1} r^* f(r^*) \frac{\partial r^*}{\partial A_n} < 0
\]

\[
\frac{\partial \pi_n}{\partial A_n} = - (1 - \alpha) \alpha A_n \left[ \int_{r^*}^{L} F(r) \right]^{\alpha-1} r^* f(r^*) \frac{\partial r^*}{\partial A_n} + (1 - \alpha) \left[ \int_{r^*}^{L} F(r) \right]^{\alpha} > 0
\]

Proof. of lemma 12: If there is mixture, according to Lemma 8, for any worker in SOE with ability \( r_s < L \) and any worker in non-SOE with ability \( r_n < L \), we must have \( r_s \leq r_n \). Therefore, there is a cutoff \( r^* < L \) where workers will find it indifferent between working in SOE and working in non-SOE, i.e. \( A_n \left( \int_{\Sigma_n} F(r) \right) \alpha^{-1} r^* = A_s \left( \int_{\Sigma_s} F(r) \right) \alpha^{-1} \frac{\int_{r^*}^{L} F(r^*)}{F(r^*)} \), which implies \( A_n \left( \int_{\Sigma_n} F(r) \right) \alpha^{-1} < A_s \left( \int_{\Sigma_s} F(r) \right) \alpha^{-1} \).

Then for those workers who are fully screened, they will prefer working in SOE due to a higher proportional coefficient. However, we know it is impossible, since \( L \leq L^* \). Now assume all workers with abilities lower than \( L \) choose to work in SOE. Thus workers who are fully screened will weakly prefer non-SOE than SOE, for if not, the marginal entrance into non-SOE will lead to an infinitely high wage. This means that the proportional coefficient of non-SOE is at least as large as that of SOE: \( A_n \left( \int_{\Sigma_n} F(r) \right) \alpha^{-1} \geq A_s \left( \int_{\Sigma_s} F(r) \right) \alpha^{-1} \). However, considering the worker
with ability $L$ approaching from the left side, he now works in SOE, which means
$$A_n \left( \int_{\Sigma_n} r dF(r) \right)^{\alpha-1} L \leq A_s \left( \int_{\Sigma_s} r dF(r) \right)^{\alpha-1} \frac{r_0^{L \cdot r} F(r)}{F(L)}.$$ The two inequalities contradicts each other. ■

Proof. of Proposition 13: Now that the fully screened workers will weakly prefer
SOE than non-SOE, for if not, the marginal entrance into SOE will lead to infinite high wage, which means the proportional coefficient of SOE is at least as large as that of non-SOE: $A_n \left( \int_{\Sigma_n} r dF(r) \right)^{\alpha-1} \leq A_s \left( \int_{\Sigma_s} r dF(r) \right)^{\alpha-1}$. But considering the worker with ability lower than $L$, he now chooses to work in non-SOE, which means
$$A_n \left( \int_{\Sigma_n} r dF(r) \right)^{\alpha-1} \geq A_s \left( \int_{\Sigma_s} r dF(r) \right)^{\alpha-1}.$$ These two inequalities together imply
$$A_n \left( \int_{\Sigma_n} r dF(r) \right)^{\alpha-1} = A_s \left( \int_{\Sigma_s} r dF(r) \right)^{\alpha-1}.$$ ■

Proof. of lemma 14: Consider $\Sigma_s$ and $\Sigma_n$, and we have

$$\omega_s(r_s) = \alpha A_s \left( \int_{\Sigma_s+\Sigma_n} r dF(r) \right)^{\alpha-1} \tilde{r} \geq \omega_n(r_s) = \alpha A_n \left( \int_{\Sigma_n} r dF(r) \right)^{\alpha-1} r_s$$
$$\omega_s(r_n) = \alpha A_s \left( \int_{\Sigma_s+\Sigma_n} r dF(r) \right)^{\alpha-1} \tilde{r} \leq \omega_n(r_n) = \alpha A_n \left( \int_{\Sigma_n} r dF(r) \right)^{\alpha-1} r_n$$

which implies $r_s \leq r_n$. Consider $\Sigma_s$ and $\Sigma_{n,s}$, and we have

$$\omega_s(r_s) = \alpha A_s \left( \int_{\Sigma_s+\Sigma_{n,s}} r dF(r) \right)^{\alpha-1} \tilde{r} \geq \omega_{n,s}(r_s) = \alpha A_s \left( \int_{\Sigma_{s,s}} r dF(r) \right)^{\alpha-1} r_s - f$$
$$\omega_s(r_{n,s}) = \alpha A_s \left( \int_{\Sigma_s+\Sigma_{n,s}} r dF(r) \right)^{\alpha-1} \tilde{r} \leq \omega_{n,s}(r_{n,s}) = \alpha A_s \left( \int_{\Sigma_{s,s}} r dF(r) \right)^{\alpha-1} r_{n,s} - f$$

which implies $r_s \leq r_{n,s}$. Consider $\Sigma_n$ and $\Sigma_{n,s}$, and we have

$$\omega_n(r_n) = \alpha A_n \left( \int_{\Sigma_n} r dF(r) \right)^{\alpha-1} r_n \geq \omega_{n,s}(r_n) = \alpha A_s \left( \int_{\Sigma_{n,s}} r dF(r) \right)^{\alpha-1} r_n - f$$
$$\omega_n(r_{n,s}) = \alpha A_n \left( \int_{\Sigma_n} r dF(r) \right)^{\alpha-1} r_{n,s} \leq \omega_{n,s}(r_{n,s}) = \alpha A_s \left( \int_{\Sigma_{n,s}} r dF(r) \right)^{\alpha-1} r_{n,s} - f$$

which implies $r_n \leq r_{n,s}$. ■

Proof. of Proposition 15: Since (5.1) has the same form with Lemma 6, we know that
$$0 < \frac{\partial r^*_1}{\partial r^*_2} \leq \frac{r^*_2 f(r^*_2)}{r^*_1 f(r^*_1)}$$
and the existence of equilibrium requires
$$1 < \frac{A_s}{A_n} \left[ \frac{\int_L^r r dF(r)}{\int_L^{\infty} r dF(r)} \right]^{1-\alpha}$$
Now, the left side of (5.2) increases in $r_2^*$:

$$\frac{dh_2}{dr_2^*} = \frac{dh_2}{dr_2^*} + \frac{\partial h_2}{\partial r_2^*} \frac{\partial r_2^*}{\partial r_2^*} > 0$$

For the existence of equilibrium, we require that

$$f > \alpha A_n \left( \int_{\epsilon}^{r_1^*} rdF (r) \right)^{\alpha-1} \frac{r_2^*}{r_2^*}$$

where $r_2^*$ satisfies

$$1 = \frac{A_s}{A_n} \left[ \int_{\epsilon}^{r_2^*} rdF (r) \right]^{1-\alpha}$$

Therefore, a friction cost that is large enough will uniquely determine $r_2^*$ and $r_1^*$. ■

**Proof.** of Proposition 17: We want to prove the lemma by methods of contradiction.

Suppose $r_2^*$ also increases. From the equation (5.2) we know that $A_n \left( \int_{r_1^*}^{r_2^*} rdF (r) \right)^{\alpha-1} - A_n \left( \int_{\epsilon}^{r_1^*} rdF (r) \right)^{\alpha-1}$ must decline to offset the effect of increasing $r_2^*$. With the sum of $\int_{\epsilon}^{r_1^*} rdF (r) + \int_{r_2^*}^{r_1^*} rdF (r)$ and $\int_{r_1^*}^{r_2^*} rdF (r)$ being constant, $r_1^*$ has to increase to make $\left( \int_{r_1^*}^{r_2^*} rdF (r) \right)^{\alpha-1}$ increase. Now rewrite (5.2) as

$$\int_{r_1^*}^{r_2^*} rdF (r) \int_{r_1^*}^{r_2^*} rdF (r) \left( \frac{r_2^* F (r_1^*)}{\int_{r_1^*}^{r_2^*} rdF (r)} - 1 \right) = \frac{f}{\alpha} (A-3)$$

We can see the left side increases if $r_1^*$, $r_2^*$ and $\left( \int_{r_1^*}^{r_2^*} rdF (r) \right)^{\alpha-1}$ increase, contradicting the fact that is a constant. Obviously, $r_2^*$ cannot keep unchanged. Therefore, $r_2^*$ will decrease. Next, $\int_{r_1^*}^{r_2^*} rdF (r)$ must decrease, i.e. $r_2^* f (r_2^*) \frac{\partial r_2^*}{\partial A_s} - r_1^* f (r_1^*) \frac{\partial r_1^*}{\partial A_s} < 0$. Otherwise, $r_1^*$ has to decrease, which makes the left side of equation (A-3) decrease. Therefore, $\pi_n$ decreases, $\pi_s$ increases and $\frac{\partial W}{\partial A_s} > 0$. ■

**Proof.** of Proposition 18: We want to prove by methods of contradiction. Let $r_1^*$ increases also. Rewrite equation (5.1) as

$$\left( \frac{\int_{r_1^*}^{r_2^*} rdF (r) + \int_{r_2^*}^{r_1^*} rdF (r)}{\int_{\epsilon}^{r_2^*} rdF (r) + \int_{r_2^*}^{r_1^*} rdF (r)} \right)^{1-\alpha} = \frac{A_n}{A_s} \frac{r_1^* F (r_1^*)}{\int_{r_1^*}^{r_2^*} rdF (r)}$$

We know that $r_2^*$ and $\left( \int_{\epsilon}^{r_1^*} rdF (r) + \int_{r_2^*}^{r_1^*} rdF (r) \right)^{\alpha-1}$ both increase. Rewrite equation (5.2) as

$$A_s \left( \int_{\epsilon}^{r_1^*} rdF (r) + \int_{r_2^*}^{r_1^*} rdF (r) \right)^{\alpha-1} r_2^* \left( 1 - \frac{\int_{r_2^*}^{r_1^*} rdF (r)}{r_1^* F (r_1^*)} \right) = \frac{f}{\alpha} (5.4)$$
And it cannot be the case that the left side increases with $r_1^*$, $r_2^*$ and \( \left( \int_{x}^{r_1^*} r dF(r) + \int_{r_2^*}^{\infty} r dF(r) \right)^{\alpha-1} \), which contradicts the fact that the right side is a constant. Obviously, $r_1^*$ cannot keep unchanged. Therefore, $r_1^*$ has to decrease. Next, $\int_{r_1^*}^{r_2^*} r dF(r)$ must increase, i.e. $r_2^* f (r_2^*) \frac{\partial r_2^*}{\partial A_n} - r_1^* f (r_1^*) \frac{\partial r_1^*}{\partial A_n} > 0$. Otherwise, $r_2^*$ has to decreases, which makes the left side of (5.4) decrease. Therefore, $\pi_n$ increases, and $\pi_s$ decreases.

References


