Information Sharing in a Contest Game with Group Identity*

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ABSTRACT

In this paper, we study a private-information contest game with two stages. In stage 1 players simultaneously choose whether to announce their group identity, and in stage 2 each player simultaneously plays a within-group lottery contest and an across-group contest. Players' information sharing incentives are analyzed and all symmetric equilibria of the game are fully characterized. Our results show that (1) full disclosure by both types is always one of the equilibria; (2) full concealment by both types can be supported as an equilibrium information strategy when the relative magnitude of high and low valuations is large, and when such a relative magnitude is small, there is an equilibrium in which the high type randomizes and the low type fully conceals.

KEYWORDS: Tullock Contests, Information Sharing, Group Identity, Private Value

JEL Codes: D44, D82, D83, C72

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1 Introduction

In many settings of competition and competitiveness training, individuals do not know the characteristics of the competitors they will interact with. However, the organizer of the event or contest often knows the key features of all competitors. In order to ensure a rich and robust competitive environment, the organizer may deliberately arrange competitions which stack each individual against a competitor of their same group, and a competitor of a different group.

For example, in online video game events, the main features of contestants such as their subgroup membership, are easily made unobservable within the game. Within the game, there may be a special event arranged, such that each player needs to play against one other player of their own subgroup, and against one other player outside of their own subgroup. For instance, each relatively new player whose account is less than a year old must compete against both a fellow new player, as well as a more experienced player. Symmetrically, each experienced player must play against a fellow experienced player, and a new less experienced challenger. Such contest formats have also been prevalent as forms of training in educational and military settings in ancient and modern China, as well as in evaluation of treatment effects in randomized experiment designs.[1]

We study the above-mentioned phenomenon in a private-information contest game with players' type defined as their binary group identity over the valuation of the prize. In stage 1 players simultaneously choose whether to disclose their group identity, and in stage 2 each player simultaneously plays one lottery contest against a player from the same group and one contest against a player from the different group. Players maximize their average expected payoff from participating in these two contests. We show that there are only two symmetric equilibria in terms of players' incentive to share their own group identities, with one of them being full disclosure by both types. When the relative magnitude of high and low valuations is large, full concealment by both types is one of the other equilibrium information strategy profiles, while the other equilibrium is such that the high type randomizes between concealment and disclosure while the low type fully conceals when the relative magnitude of high and low valuations is small.

[1]For example, two groups of randomly selected subjects (human beings or animals) who have received different treatments may be tested on their performance by the treatment designer using a within-group comparison as well as a between-group comparison. Which treatment was received may be the private information of subjects while they could have an incentive to reveal or conceal it. Such a style of comparison by competition is usually for the purpose of testing fitness or survival rate.

The full disclosure equilibrium is straightforward since the information strategy becomes ineffective and types are automatically disclosed as long as one type reveals the information. The other equilibrium, where the low type always conceals and the high type may conceal or may randomize, depending on the relative magnitude of type valuations, is due to different incentives of disclosing by different types. We show that the overall effect of disclosing is negative except for the uninformed high type with small relative magnitude of type valuations.

We position our study within the literature on Tullock contests with information. Many studies in this area consider an exogenous informational environment (see Hurley and Shogren, 1998; Malueg and Yates, 2004; Fey, 2008; Wärneryd, 2003, 2013; Wasser, 2013; Einy et al., 2015; among others) or study the contest designer's information disclosure problem (for example, Qiang et al., 2014; Zhang and Zhou, 2016; Chen et al., 2017; Jiao et al., 2017; Serena, 2018; Lu et al., 2018; Chen, 2019; Cai et al., 2019). Our focus is more aligned with studies in which information is endogenized through players' incentives. Along these lines, Kovenock et al. (2015) considers information sharing in all-pay auction contests, Wu and Zheng (2017) and Ewerhart and Grünseis (2018), both assuming ex-ante commitment, study type-independent information sharing in lottery contests and type-dependent information sharing in non-deterministic contests of more general forms.[2] Departing from the existing studies, we examine the interim information sharing incentive by players of different types without the commitment assumption, in a lottery contest environment.

Our work also contributes to the literature by considering a new setup in which every player competes against both types of opponents whose types he/she may not know. This differs from the traditional setup where every player competes against only one opponent with an equally likely type.[3]

2 The Model

There are two groups of risk-neutral players, who are potential contestants in 2-person lottery contests. Groups, denoted by G_L and G_H, respectively, are of equal size. Players from different groups differ in their value of winning the contest, with $v_H > v_L > 0$. Every player participates in one within-group contest in which the opponent is randomly picked

[2] All-pay auction and Lottery contest are two special cases of Tullock Contests where the decisiveness parameter in the success function is infinity in the former setup and is equal to 1 in the latter setup.

[3] The belief updating mechanisms are different under these two setups with asymmetric information.

from the same group and one across-group contest in which the opponent is randomly picked from the different group, with no information feedback in-between. A player i's group identity, also known as his/her type, denoted by t_i, where $t_i \in \{L, H\}$, is privately known. We have $t_i = L$ and $v_i = v_L$ if $i \in G_L$, and we have $t_i = H$ and $v_i = v_H$ if $i \in G_H$, where H denotes high valuation and L denotes low valuation.

In a typical lottery contest, two players ($i = A, B$) compete by simultaneously exerting non-negative effort x_i. The success function for player i is given by $p_i(x_A, x_B) = \frac{x_i}{x_A + x_B}$ if $x_A + x_B > 0$ and $p_i(x_A, x_B) = \frac{1}{2}$ if $x_A = x_B = 0$. The expected payoff of player i with value of winning v_i under the effort profile (x_A, x_B) is thus $u_i(x_A, x_B; v_i) = p_i v_i - x_i$.

Before the contest stage starts, each player can simultaneously decide whether to reveal his/her group identity information to his/her opponents. We denote a typical player i's information sharing strategy as $s_i(t_i)$, with $s_i : \{H, L\} \longrightarrow [0, 1]$, representing the probability of disclosing i's group identity information. In the extreme cases, $s_i = 0$ refers to the full concealment decision, which is also denoted by C, and $s_i = 1$ refers to the full disclosure decision, which is also denoted by D.

Depending on players' information strategies, the contest in the second stage may be either with symmetric information (no information or full information) or with asymmetric information in terms of players' group identities. Each player i chooses his/her effort profile (x_{i1}, x_{i2}) to maximize his/her average payoff from the two lottery contests participated in.

The timeline of the game is as follows. Stage 1 (**Information Sharing**): Each player i simultaneously decides on s_i. Stage 2 (**Within-Group and Across-Group Contests**): Depending on s_i, players infer the group identity information of their opponents, and then participate in the two lottery contests described above, by choosing effort profile (x_{i1}, x_{i2}).

3 Analysis

We focus on symmetric equilibria where players of the same type play the same strategy, and we use backward induction to solve the equilibrium of the game. In the following analysis, we first characterize players' optimal effort strategies in stage 2, and then analyze players' information sharing incentives in stage 1.

3.1 Equilibrium Effort in Stage 2

Depending on players' information regarding the group identities of their opponents, there are 3 scenarios for each contest: (1) both players have information regarding each other's type; (2) both players have no information regarding each other's type; (3) Only one player has information regarding the other player's type.

Scenario 1: Full Information

If both players (i and $-i$)' types t_i and t_{-i} are publicly known, such a contest has been studied in Leininger (1993), and the unique equilibrium effort strategy profile $(x_i^F(t_i, t_{-i}), x_{-i}^F(t_i, t_{-i}))$ is given by

$$x_i^F(t_i, t_{-i}) = \frac{v_i v_{-i}}{(v_i + v_{-i})^2} v_i, \quad x_{-i}^F(t_i, t_{-i}) = \frac{v_i v_{-i}}{(v_i + v_{-i})^2} v_{-i},$$

with equilibrium payoff

$$u_i^F \equiv u_i(x_i^F, x_{-i}^F; v_i) = \frac{(v_i)^3}{(v_i + v_{-i})^2}, \quad u_{-i}^F \equiv x_{-i}(x_i^F, x_{-i}^F; v_{-i}) = \frac{(v_{-i})^3}{(v_i + v_{-i})^2}.$$

Scenario 2: No Information

If both players' types are private information, then each contest is equally likely to be a within-group contest or an across-group contest for both players. Such a setup is mathematically equivalent to a 2-person lottery contest with binary types of equal chance, which has been studied in Malueg and Yates (2004), and there is a unique equilibrium. Player i's equilibrium effort strategy given his/her type t_i, denoted as $x_i^N(t_i)$, has the following form:

$$x_H^N \equiv x_i^N(H) = \left(\frac{1}{8} + \frac{1}{2}\frac{v_H v_L}{(v_H + v_L)^2}\right) v_H, \quad x_L^N \equiv x_i^N(L) = \left(\frac{1}{8} + \frac{1}{2}\frac{v_H v_L}{(v_H + v_L)^2}\right) v_L,$$

with expected equilibrium payoff

$$u_H^N \equiv \frac{1}{2} u_i(x_H^N, x_H^N; v_H) + \frac{1}{2} u_i(x_H^N, x_L^N; v_H) = \frac{1}{2}\left(\frac{v_H^2}{(v_H + v_L)^2} + \frac{1}{4}\right) v_H,$$

$$u_L^N \equiv \frac{1}{2} u_i(x_L^N, x_H^N; v_H) + \frac{1}{2} u_i(x_L^N, x_L^N; v_H) = \frac{1}{2}\left(\frac{v_L^2}{(v_H + v_L)^2} + \frac{1}{4}\right) v_L.$$

Scenario 3: Asymmetric Information

If one player's type is publicly known while the other player's type is private information, such a contest is equivalent to one with asymmetric information, which was first studied in Zhang and Zhou (2016) for the interior solution and later fully characterized in Wu and Zheng (2017). Denote x_j^A as the equilibrium effort strategy of the player whose type $v_j(j = H, L)$ is public information. Similarly, the equilibrium effort strategy of the player whose type $v_k(k = H, L)$ is privately known, given the other player's type $v_j(j = H, L)$, is refereed to as x_{jk}^A. For convenience, define the relative magnitude of high and low valuations $\delta \equiv \frac{v_H}{v_L}$, with $\delta > 1$. Equilibrium effort strategies x_j^A and x_{jk}^A have the following forms:

$$x_L^A = \left(\frac{\sqrt{v_H} + \sqrt{v_L}}{3v_H + v_L}\right)^2 v_H v_L, \quad x_H^A = \begin{cases} \left(\frac{\sqrt{v_H}+\sqrt{v_L}}{3v_L+v_H}\right)^2 v_H v_L & if \quad \delta \leq 9 \\ \frac{v_H}{9} & if \quad \delta > 9. \end{cases}$$

$$x_{LH}^A = \sqrt{v_H}\sqrt{x_L^A} - x_L^A, \quad x_{LL}^A = \sqrt{v_L}\sqrt{x_L^A} - x_L^A, \quad x_{HH}^A = \sqrt{v_H}\sqrt{x_H^A} - x_H^A,$$

$$x_{HL}^A = \begin{cases} \sqrt{v_L}\sqrt{x_H^A} - x_H^A & if \quad \delta \leq 9 \\ 0 & if \quad \delta > 9. \end{cases}$$

3.2 Information Sharing in Stage 1

Now we examine players' information sharing incentives in Stage 1, by considering all possible cases and comparing players' payoffs under different information strategies in each case. For simplicity, we denote a typical player i's information strategy given his/her type is j ($j \in \{H, L, \}$), $s_i(t_i = j)$, by s_{ij}. By the symmetry assumption, in equilibrium we have $s_{ij} = s_{i'j}$ for any i and i' of the same type. Thus, it is without loss of generality to write the equilibrium information strategy profile as (s_L, s_H).

For equilibrium analysis, it is essential to consider the information sharing outcome given information sharing strategies. We use $P(s_L, s_H) \in [0, 1]$ to denote the probability that a player can infer the type of his/her opponents when the opponents' information strategies are s_L and s_H.[4] Given that different players' information strategies are played independently, we have

$$P(s_L, s_H) = 1 - (1 - s_L)(1 - s_H).$$

The relationship between the information sharing strategy profile (s_L, s_H) and information sharing outcome $P(s_L, s_H)$ implies that either $s_L = 1$ or $s_H = 1$ alone will lead to $P(s_L, s_H) = 1$. In other words, one type's information strategy will become ineffective if the

[4]Note that each player will always play against two opponents of different types, respectively. Thus, as long as one opponent discloses his/her type, the other opponent's type can be immediately inferred. Therefore, the probability that a L-type opponent is made known to a player is always equal to the probability that a H-type opponent player is made known to that player.

other type chooses to share information. Based on this observation, we immediately have the following result:

Proposition 1. *($s_L = 1, s_H = 1$) is an equilibrium information sharing strategy profile, resulting in information sharing outcome $P = 1$.*

Besides the above trivial equilibrium, we are more interested in equilibra where players' information strategies are effective. Before conducting further equilibrium analysis, we first establish three results regarding players' information sharing incentive. Due to space concerns, all proofs for these lemmas are relegated to the Appendix.

Lemma 1. *If the opponents' types are known to a player i while i's own type is unknown to the opponents, then i is better off by setting $s_i = 0$ regardless of his/her own type.*

Lemma 2. *If the opponents' types are unknown to an L-type player i and i's own type is unknown to the opponents, then i is better off by setting $s_{iL} = 0$.*

Let $\delta^* > 1$ be such that $\frac{1}{2}(\frac{\delta^{*2}}{(1+\delta^*)^2} + \frac{1}{4}) = \frac{5}{9}$. It is easy to obtain the unique value of δ^* which is $\delta^* \approx 12.88$.

Lemma 3. *If the opponents' types are unknown to a H-type player i and i's own type is unknown to the opponents, then $s_{iH} = 0$ when $\delta > \delta^* \approx 12.88$, $s_{iH} = 1$ when $1 < \delta < \delta^* \approx 12.88$, and $s_{iH} \in [0, 1]$ when $\delta = \delta^* \approx 12.88$.*

The above three lemmas provide us with a good understanding for players' incentive to share information under different informational environments. Lemma 1 implies that when a player can distinguish between the within-group contest and the across-group contest, both types will have incentive to conceal their own group identities. The intuition behind Lemma 1 is the following. By concealing instead of disclosing own type information, player i (she) makes her opponent (he) unsure about her type, which could lead to different effects depending on his type. When he is the same type as i, concealment will discourage him from competing intensively, and therefore benefits i; When he is the different type than i, concealment will encourage him to exert more effort, which is not desirable to i. The overall effect is positive by concealing information, if i knows the type of her opponent, resulting in Lemma 1.

Lemma 2 further tells us that an L-type player has incentive to conceal his/her group identity even when he/she cannot distinguish between the within-group contest and the across-group contest. By contrast, Lemma 3 shows that an H-type player's information sharing incentive when he/she has no information about the opponents' types, depends on the relative magnitude of high and low valuations. The intuitions behind Lemma 2 and Lemma 3 are again about the trade-off between two competing effects by disclosing type information when facing different types of opponents. If player i (she) chose to disclose her type, in case of facing a different type of opponent, the opponent (he) would have exerted less effort since he knew he was competing against a different type, which is beneficial to i; However, in case of facing a same type of opponent, he would have exerted more effort since he knew i was of the same type, which is not beneficial to i. The former (positive) effect is dominated by the latter (negative) effect for disclosing information when the uninformed i is low type and thus we have Lemma 2. When the uninformed i is high type, the former (positive) effect dominates the latter (negative) effect if the relative magnitude is small ($\delta \leq \delta^*$), so the uninformed high type i chooses to disclose; If the relative magnitude is large ($\delta > \delta^*$), the latter (negative) effect dominates the former (positive) effect, so the uninformed high type i chooses to conceal.

Combining the results in Lemmas 2 and 3, we can immediately see that when the ratio of high to low valuations exceeds a threshold ($\delta \geq \delta^*$), players who are uninformed about their opponents' types will always have incentive to conceal, regardless of their own types. Thus, such a full concealment information strategy can be supported in equilibrium, and we formally state this result in the following proposition.

Proposition 2. *When $\delta \geq \delta^* \approx 12.88$, $(s_L = 0, s_H = 0)$ is an equilibrium information sharing strategy profile, resulting in information sharing outcome $P = 0$.*

When the threshold of the relative magnitude is not reached ($1 < \delta < \delta^*$) however, players of different types will differ in their information sharing incentive. For such a ratio, Lemmas 1 and 2 show that an L-type player prefers concealment (if possible) whether or not he/she knows the types of his/her opponents, while Lemmas 1 and 3 show that an H-type player may have different information sharing incentives depending on his/her information about his/her opponents' types. This observation implies that a symmetric type-dependent information strategy profile where the low type conceals and the high type randomizes between concealment and disclosure can be supported in equilibrium. We characterize such an equilibrium in the following Proposition, and relegate the proof to the Appendix.

Proposition 3. *When $1 < \delta < \delta^* \approx 12.88$, $(s_L = 0, s_H = s^*)$ is an equilibrium information sharing strategy profile, resulting in information sharing outcome $(P = s^*)$, where $s^* \in (0, 1)$*

is uniquely determined by equation (1) if $1 < \delta \leq 9$ *and by equation (2) if* $9 \leq \delta < \delta^* \approx 12.88$.[5] *Equations (1) and (2) are given by*

$$\frac{s^*}{8}+\frac{s^*\delta^2}{2(\delta+1)^2}+(1-s^*)\left(\frac{1}{2}\frac{(\sqrt{\delta}+1)^2(\delta+1)}{(3+\delta)^2}\right)=\frac{s^*}{2}\left(1-\frac{1+\sqrt{\delta}}{3\delta+1}\right)^2+\frac{s^*}{2}\left(1-\frac{1+\sqrt{\delta}}{3+\delta}\right)^2+\frac{(1-s^*)}{2}\left(\frac{\delta^2}{(\delta+1)^2}+\frac{1}{4}\right) \quad (1)$$

$$\frac{s^*}{8}+\frac{s^*\delta^2}{2(\delta+1)^2}+(1-s^*)\frac{5}{9}=\frac{s^*}{2}\left(1-\frac{1+\sqrt{\delta}}{3\delta+1}\right)^2+\frac{s^*}{2}\left(\frac{4}{9}\right)+\frac{(1-s^*)}{2}\left(\frac{\delta^2}{(\delta+1)^2}+\frac{1}{4}\right) \quad (2)$$

3.3 Equilibrium Characterization

We now fully characterize the equilibrium information sharing decisions of players in the following theorem and the proof is again relegated to the Appendix.

Theorem 1. *If* $1 < \delta < \delta^* \approx 12.88$*, there are only 2 symmetric equilibrium information sharing strategy profiles, (i) full disclosure by both types (characterized in Proposition 1); (ii) full concealment by the L type and randomized information sharing by the H type (characterized in Proposition 3). If* $\delta \geq \delta^* \approx 12.88$*, there are only 2 symmetric equilibrium information sharing strategy profiles, (i) full disclosure by both types (characterized in Proposition 1); (ii) full concealment by both types (characterized in Proposition 2).*

Our results have implications for the incentives of players to reveal their group identities in inter/intra group competitions. Firstly, the analysis shows that full disclosure is always an equilibrium, since the information strategy becomes ineffective and types are automatically disclosed as long as one type reveals the information. Perhaps more interestingly, the other equilibrium strategy depends on the valuation ratio.

When the high type valuation sufficiently exceeds the low type valuation, all players regardless of type, choose to conceal their identities. The reason behind the full concealment equilibrium is that for both uninformed types the positive effect of disclosure when competing against a different type is dominated by the negative effect of disclosure when competing against the same type, when the relative magnitude is large.

However, when the high type valuation is relatively close to the low type valuation, a type-dependent equilibrium strategy emerges. The low valuation type still chooses to conceal, while the high valuation type sometimes discloses and sometimes conceals. The

[5]Note that when $\delta = 9$, equations (1) and (2) are exactly the same.

low type has incentive to conceal regardless of whether the low type is informed of the opponents' types, while the high type's incentive to conceal depends on whether the high type is informed of the opponents' types, resulting in a probabilistic equilibrium disclosure strategy when the high type opponent also randomizes.

Our findings share some similarities and some differences with the existing studies on players' information sharing incentive in contests. While Kovenock et al. (2015), under the setup of an all-pay auction contest, shows that information disclosure is always a strictly dominated strategy regardless of players' types, we find that in our setup a player's information sharing incentive may depend on one's own type, type valuation ratio, and the other player's information strategy, and furthermore full concealment by both types can only be one of the two possible equilibra in the situation where the type valuation ratio is high. Ewerhart and Grünseis (2018), on the other hand, in a non-deterministic contest environment, identifies an unfairness condition under which full disclosure is the unique equilibrium with ex-ante commitment, and we show that in our setup full disclosure is always a possible but never the unique equilibrium without any ex-ante commitment assumption. Our result also differs from Wu and Zheng (2017), which assumes ex-ante commitment and considers only type-independent information strategies. Wu and Zheng (2017) shows that full concealment is the unique equilibrium when the type valuation ratio is high and there are three equilibria (two asymmetric and one mixed-strategy) when the type valuation ratio is low. By contrast, our results indicate that there are always two symmetric equilibria regardless of the type valuation ratio, with one of them being full disclosure by both types. Furthermore, we show that when the type valuation ratio is high, full concealment by both types is the other equilibrium, and when the type valuation ratio is low, the other equilibrium is such that the high type randomizes and the low type fully conceals.[6]

4 Conclusion

In this paper, we study a 2-stage private-information contest game with group identity over the prize valuation. In the information sharing stage, players simultaneously choose whether to announce their group identity, and in the contest stage, each player simultaneously plays a within-group lottery contest as well as an across-group contest. We focus on symmetric equilibria and fully characterize all equilibria of the game.

[6]The cutoff value of the type distribution is different between our paper and Wu and Zheng (2017), with the former's cutoff being $\delta^* \approx 12.88$ and the latter's cutoff being $\bar{\delta} \approx 12.33$.

We show that full disclosure regardless of type is always an equilibrium information strategy. When the ratio of type valuations is low, a type-dependent information sharing strategy in which the low type fully conceals while the high type randomizes, is the only other possible information sharing strategy in equilibrium. When the ratio of type valuations is high, full concealment regardless of type is the only other possible information sharing strategy in equilibrium.

Directions for further study may include having more than 2 groups and allowing for different weights on within-group contests and across-group contests in players' payoff functions.

Proof for Lemma 1

Proof. First consider the case where player i is L-type.

Ⓒ If i conceals, each of the two contests will be with asymmetric information (Scenario 3), where i knows the opponent's type.

For the contest with the L-type opponent, the opponent's effort will be $x_L^A = \left(\frac{\sqrt{v_H}+\sqrt{v_L}}{3v_H+v_L}\right)^2 v_H v_L$, i's effort will be $x_{LL}^A = \sqrt{v_L}\sqrt{x_L^A} - x_L^A$, thus i's payoff will be $\pi_L^{A\to L} = \frac{x_{LL}^A}{x_L^A+x_{LL}^A}v_L - x_{LL}^A = \left(1-\frac{\delta+\sqrt{\delta}}{3\delta+1}\right)^2 v_L$.

When against the H-type opponent, we have $x_H^A = \begin{cases} \left(\frac{\sqrt{v_H}+\sqrt{v_L}}{3v_L+v_H}\right)^2 v_H v_L & if \quad \delta \leq 9 \\ \frac{v_H}{9} & if \quad \delta > 9, \end{cases}$

and $x_{HL}^A = \begin{cases} \sqrt{v_L}\sqrt{x_H^A} - x_H^A & if \quad \delta \leq 9 \\ 0 & if \quad \delta > 9. \end{cases}$, thus i's payoff will be $\pi_L^{A\to H} = \frac{x_{HL}^A}{x_H^A+x_{HL}^A}v_L - x_{HL}^A = \begin{cases} \left(1-\frac{\delta+\sqrt{\delta}}{3+\delta}\right)^2 v_L & if \quad \delta \leq 9 \\ 0 & if \quad \delta > 9. \end{cases}$

Thus, i's expected payoff equals $\pi_L^{Ⓒ} = \begin{cases} \left(\left(1-\frac{\delta+\sqrt{\delta}}{3\delta+1}\right)^2 + \left(1-\frac{\delta+\sqrt{\delta}}{3+\delta}\right)^2\right)\frac{v_L}{2} & if \quad \delta \leq 9 \\ \left(1-\frac{\delta+\sqrt{\delta}}{3\delta+1}\right)^2 \frac{v_L}{2} & if \quad \delta > 9. \end{cases}$.

Ⓓ If i discloses, each contest will be one with full information (Scenario 1).

When against the L-type opponent (the within-group contest), both players' efforts are equal to $\frac{v_L}{4}$. Thus, i's payoff will be $\pi_L^{F\to L} = \frac{v_L}{4}$.

When against the H-type opponent (the across-group contest), the opponent's effort equals $\frac{v_H v_L}{(v_H+v_L)^2}v_H$, and i's effort equals $\frac{v_H v_L}{(v_H+v_L)^2}v_L$. Thus, i's payoff will be $\pi_L^{F\to H} = \frac{v_L}{(\delta+1)^2}$.

Thus, i's expected payoff equals $\pi_L^{Ⓓ} = \frac{v_L}{8} + \frac{v_L}{2(\delta+1)^2}$.

Simple comparison shows $\pi_L^{Ⓒ} > \pi_L^{Ⓓ}$. Therefore, we have $s_{iL} = 0$.

Then, consider the case where player i is H-type.

Ⓒ If i conceals, each of the two contests will be with asymmetric information (Scenario 3).

When against the L-type opponent, we have $x_L^A = \left(\frac{\sqrt{v_H}+\sqrt{v_L}}{3v_H+v_L}\right)^2 v_H v_L$, and $x_{LH}^A = \sqrt{v_H}\sqrt{x_L^A} - x_L^A$. Thus, i's payoff will be $\pi_H^{A\to L} = \frac{x_{LH}^A}{x_L^A+x_{LH}^A}v_H - x_{LH}^A = \left(1-\frac{1+\sqrt{\delta}}{3\delta+1}\right)^2 v_H$.

When against the H-type opponent, we have $x_H^A = \begin{cases} \left(\frac{\sqrt{v_H}+\sqrt{v_L}}{3v_L+v_H}\right)^2 v_H v_L & if \quad \delta \leq 9 \\ \frac{v_H}{9} & if \quad \delta > 9, \end{cases}$

and $x_{HH}^A = \sqrt{v_H}\sqrt{x_H^A} - x_H^A$. Thus, i's payoff equals $\pi_H^{A\to H} = \frac{x_{HH}^A}{x_H^A+x_{HH}^A}v_H - x_{HH}^A$

$$= \begin{cases} \left(1 - \frac{1+\sqrt{\delta}}{3+\delta}\right)^2 v_H & if \quad \delta \le 9 \\ \frac{4}{9} v_H & if \quad \delta > 9. \end{cases}$$

Thus, i's expected payoff equals $\pi_H^{Ⓒ} = \begin{cases} \left(\left(1 - \frac{1+\sqrt{\delta}}{3\delta+1}\right)^2 + \left(1 - \frac{1+\sqrt{\delta}}{3+\delta}\right)^2\right) \frac{v_H}{2} & if \quad \delta \le 9 \\ \left(\left(1 - \frac{1+\sqrt{\delta}}{3\delta+1}\right)^2 + \frac{4}{9}\right) \frac{v_H}{2} & if \quad \delta > 9. \end{cases}$

Ⓓ If i discloses, the contest will be held in a full information scenario (Scenario 1).

When against the L-type opponent (the across-group contest), the opponent's effort equals $\frac{v_H v_L}{(v_H+v_L)^2} v_L$, and i's effort equals $\frac{v_H v_L}{(v_H+v_L)^2} v_H$. Thus, i's payoff will be $\pi_H^{F\to L} = \frac{\delta^2}{(\delta+1)^2} v_H$.

When against the H-type opponent (the within-group contest), both players' efforts are equal to $\frac{v_H}{4}$. Thus, i's payoff will be $\pi_H^{F\to H} = \frac{v_H}{4}$.

Thus, i's expected payoff equals $\pi_H^{Ⓓ} = \frac{v_H}{8} + \frac{\delta^2 v_H}{2(\delta+1)^2}$.

After comparison, we derive $\pi_H^{Ⓒ} > \pi_H^{Ⓓ}$. Therefore, we have $s_{iH} = 0$.

□

Proof for Lemma 2

Proof. Ⓒ If i conceals, each of the two contests will be one with no information (Scenario 2). i's effort in each contest is $x_L^N = \left(\frac{1}{8} + \frac{1}{2}\frac{v_H v_L}{(v_H+v_L)^2}\right) v_L$.

In the contest with the L-type opponent, the opponent's effort is $x_L^N = \left(\frac{1}{8} + \frac{1}{2}\frac{v_H v_L}{(v_H+v_L)^2}\right) v_L$, and i's payoff equals $\pi_L^{N\to L} = \frac{x_L^N}{x_L^N + x_L^N} v_L - x_L^N = \left(\frac{3}{8} - \frac{1}{2}\frac{v_H v_L}{(v_H+v_L)^2}\right) v_L$.

In the contest with the H-type opponent, the opponent's effort is $x_H^N = \left(\frac{1}{8} + \frac{1}{2}\frac{v_H v_L}{(v_H+v_L)^2}\right) v_H$, and i's payoff equals $\pi_L^{N\to H} = \frac{x_L^N}{x_H^N + x_L^N} v_L - x_L^N = \left(\frac{v_L}{v_H+v_L} - \frac{1}{2}\frac{v_H v_L}{(v_H+v_L)^2} - \frac{1}{8}\right) v_L$.

Thus, i's expected payoff will be $\pi_L^{Ⓒ} = \frac{1}{2}\pi_L^{N\to L} + \frac{1}{2}\pi_L^{N\to H} = \frac{1}{2}\left(\frac{v_L^2}{(v_H+v_L)^2} + \frac{1}{4}\right) v_L$.

② If i discloses, each of the two contests will be one with asymmetric information (Scenario 3). i's effort equals $x_L^A = \left(\frac{\sqrt{v_H}+\sqrt{v_L}}{3v_H+v_L}\right)^2 v_H v_L$.

In the contest with the L-type opponent, the opponent's effort is $x_{LL}^A = \sqrt{v_L}\sqrt{x_L^A} - x_L^A$, and i's payoff will be $\pi_L^{A\to L} = \frac{x_L^A}{x_{LL}^A + x_L^A} v_L - x_L^A = \frac{(\sqrt{v_H}+\sqrt{v_L})(2v_H+v_L-\sqrt{v_H}\sqrt{v_L})\sqrt{v_H}}{(3v_H+v_L)^2} v_L$.

In the contest with the H-type opponent, the opponent's effort is $x_{LH}^A = \sqrt{v_H}\sqrt{x_L^A} - x_L^A$, and i's payoff equals $\pi_L^{A\to H} = \frac{x_L^A}{x_{LH}^A + x_L^A} v_L - x_L^A = \frac{(\sqrt{v_H}+\sqrt{v_L})(2v_H\sqrt{v_L}+v_L\sqrt{v_L}-v_H\sqrt{v_H})}{(3v_H+v_L)^2} v_L$.

Thus, i's payoff equals $\pi_L^{Ⓓ} = \frac{1}{2}\pi_L^{A\to L} + \frac{1}{2}\pi_L^{A\to H} = \frac{1}{2}\frac{(\sqrt{v_H}+\sqrt{v_L})^2(v_H+v_L)}{(3v_H+v_L)^2} v_L$.

By comparison, we find $\pi_L^{Ⓒ} > \pi_L^{Ⓓ}$. Therefore, we have $s_{iL} = 0$.

Proof for Lemma 3

Proof. Ⓒ If i conceals, each of the two contests will be one with no information (Scenario 2). i's effort in each contest is $x_H^N = \left(\frac{1}{8} + \frac{1}{2}\frac{v_H v_L}{(v_H+v_L)^2}\right) v_H$.

In the contest with the L-type opponent, the opponent's effort is $x_L^N = \left(\frac{1}{8} + \frac{1}{2}\frac{v_H v_L}{(v_H+v_L)^2}\right) v_L$, and i's payoff equals $\pi_H^{N\to L} = \frac{x_B^N(v_H)}{x_A^N(v_L)+x_B^N(v_H)} v_H - x_B^N(v_H) = \left(\frac{\delta}{\delta+1} - \frac{1}{2}\frac{\delta}{(\delta+1)^2} - \frac{1}{8}\right) v_H$.

In the contest with the H-type opponent, the opponent's effort is $x_H^N = \left(\frac{1}{8} + \frac{1}{2}\frac{v_H v_L}{(v_H+v_L)^2}\right) v_H$, and i's payoff equals $\pi_H^{N\to H} = \frac{x_B^N(v_H)}{x_A^N(v_H)+x_B^N(v_H)} v_H - x_B^N(v_H) = \left(\frac{3}{8} - \frac{1}{2}\frac{\delta}{(\delta+1)^2}\right) v_H$;

Thus, i's expected payoff equals $\pi_H^{Ⓒ} = \frac{1}{2}\pi_H^{N\to L} + \frac{1}{2}\pi_H^{N\to H} = \frac{1}{2}\left(\frac{\delta^2}{(\delta+1)^2} + \frac{1}{4}\right) v_H$.

Ⓓ If i discloses, each of the two contests will be one with asymmetric information (Scenario 3). i's effort equals $x_H^A = \begin{cases} \left(\frac{\sqrt{v_H}+\sqrt{v_L}}{3v_L+v_H}\right)^2 v_H v_L & if \quad \delta \le 9 \\ \frac{v_H}{9} & if \quad \delta > 9. \end{cases}$

When against the L-type opponent, the opponent's effort is $x_{HL}^A = \begin{cases} \sqrt{v_L}\sqrt{x_H^A} - x_H^A & if \quad \delta \le 9 \\ 0 & if \quad \delta > 9, \end{cases}$, and i's payoff will be $\pi_H^{A\to L} = \frac{x_H^A}{x_{HL}^A + x_H^A} v_H - x_H^A = \begin{cases} \frac{(\sqrt{\delta}+1)(2\sqrt{\delta}+(\sqrt{\delta})^3-1)}{(3+\delta)^2} v_H & if \quad \delta \le 9 \\ \frac{8}{9} v_H & if \quad \delta > 9. \end{cases}$.

When against the H-type opponent, the opponent's effort will be $x_{HH}^A = \sqrt{v_H}\sqrt{x_H^A} - x_H^A$, and i's payoff will be $\pi_H^{A\to H} = \frac{x_H^A}{x_{HH}^A + x_H^A} v_H - x_H^A = \begin{cases} \frac{(\sqrt{\delta}+1)(2+\delta-\sqrt{\delta})\sqrt{v_L}}{(3+\delta)^2} v_H & if \quad \delta \le 9 \\ \frac{2}{9} v_H & if \quad \delta > 9; \end{cases}$

Thus, i's expected payoff equals $\pi_H^{Ⓓ} = \frac{1}{2}\pi_H^{A\to L} + \frac{1}{2}\pi_H^{A\to H} = \begin{cases} \frac{1}{2}\frac{(\sqrt{\delta}+1)^2(\delta+1)}{(3+\delta)^2} v_H & if \quad \delta \le 9 \\ \frac{5}{9} v_H & if \quad \delta > 9. \end{cases}$

By comparison, we find $\pi_H^{Ⓒ} > \pi_H^{Ⓓ}$ if $\delta > \delta^*$, and $\pi_H^{Ⓒ} \le \pi_H^{Ⓓ}$ if $1 < \delta \le \delta^*$. □

Proof for Proposition 3

Proof. Suppose $1 < \delta < \delta^*$. To show $(s_L = 0, s_H = s^*)$ is an equilibrium information sharing strategy profile, it suffices to check that a player of each type has no incentive to deviate given that all other players play according to $(s_L = 0, s_H = s^*)$.

First note that $(s_L = 0, s_H = s^*)$ results in an information sharing outcome where with probability $P = s^*$ a player knows his/her opponents' types and with probability $P = 1 - s^*$ he/she does not know his/her opponents' types.

By Lemmas 1 and 2, we know that an L-type player prefers concealment (if possible) whether he/she knows the types of his/her opponents or not. So any L-type player has no incentive to deviate from $s_L = 0$.

For an H-type player, his/her payoff by concealing when he/she knows the opponents' types is $\pi_H^{A\mathbb{C}} = \begin{cases} \left(\left(1 - \frac{1+\sqrt{\delta}}{3\delta+1}\right)^2 + \left(1 - \frac{1+\sqrt{\delta}}{3+\delta}\right)^2\right) \frac{v_H}{2} & if \quad \delta \leq 9 \\ \left(\left(1 - \frac{1+\sqrt{\delta}}{3\delta+1}\right)^2 + \frac{4}{9}\right) \frac{v_H}{2} & if \quad \delta > 9. \end{cases}$ H-type's payoff by concealing when he/she does not know the opponents' types is $\pi_H^{N\mathbb{C}} = \frac{1}{2}\left(\frac{\delta^2}{(\delta+1)^2} + \frac{1}{4}\right) v_H$. Assuming his/her opponents play the equilibrium strategy $(s_L = 0, s_H = s^*)$, H-type's expected payoff by concealing is thus a weighted sum of the above two terms, $\pi_H^{\mathbb{C}} = s^*\pi_H^{A\mathbb{C}} + (1 - s^*)\pi_H^{N\mathbb{C}}$. Similarly, we can write down H-type's payoff by disclosing when he/she knows the opponents' types as $\pi_H^{F\mathbb{D}} = \frac{v_H}{8} + \frac{\delta^2 v_H}{2(\delta+1)^2}$, and his/her payoff by disclosing when he/she does not know the opponents' types as $\pi_H^{A\mathbb{D}} = \begin{cases} \frac{1}{2}\frac{(\sqrt{\delta}+1)^2(\delta+1)}{(3+\delta)^2} v_H & if \quad \delta \leq 9 \\ \frac{5}{9} v_H & if \quad \delta > 9. \end{cases}$. Assuming his/her opponents play the equilibrium strategy, H-type's expected payoff by disclosing is thus a weighted sum of the above two terms, $\pi_H^{\mathbb{D}} = s^*\pi_H^{F\mathbb{D}} + (1 - s^*)\pi_H^{A\mathbb{D}}$.

Notice that if $1 < \delta \leq 9$, $\pi_H^{\mathbb{C}} = \pi_H^{\mathbb{D}}$ implies equation (1), and if $9 \leq \delta < \delta^*$, $\pi_H^{\mathbb{C}} = \pi_H^{\mathbb{D}}$ implies equation (2). Since s^* equalizes $\pi_H^{\mathbb{C}}$ and $\pi_H^{\mathbb{D}}$, any H-type player is indifferent between disclosing and concealing, and hence has no incentive to deviate from $s_H = s^*$.

Last we show that s^* is uniquely determined. Let $\Delta\pi(s) \equiv s\pi_H^{A\mathbb{C}} + (1 - s)\pi_H^{N\mathbb{C}} - s\pi_H^{F\mathbb{D}} - (1 - s)\pi_H^{A\mathbb{D}} = (\pi_H^{A\mathbb{C}} - \pi_H^{F\mathbb{D}})s + (\pi_H^{A\mathbb{D}} - \pi_H^{N\mathbb{C}})s - (\pi_H^{A\mathbb{D}} - \pi_H^{N\mathbb{C}})$. By Lemma 1, we have $\pi_H^{A\mathbb{C}} > \pi_H^{F\mathbb{D}}$ and by Lemma 3, we have $\pi_H^{A\mathbb{D}} > \pi_H^{N\mathbb{C}}$ since $1 < \delta < \delta^*$. Thus, $\Delta\pi(s)$ is increasing in s. Also note that $\Delta\pi(0) = -(\pi_H^{A\mathbb{D}} - \pi_H^{N\mathbb{C}}) < 0$ and $\Delta\pi(1) = \pi_H^{A\mathbb{C}} - \pi_H^{F\mathbb{D}} > 0$. Therefore, $\Delta\pi(s) = 0$ has a unique solution, which is defined as s^*.

□

Proof for Theorem 1

Proof. We show Theorem 1 in several steps.

Claim 1: $(s_L \in (0, 1], s_H \in [0, 1))$ cannot be an equilibrium information strategy profile.

By Lemmas 1 and 2, we know that an L-type player prefers concealment (if possible) whether he/she knows the types of his/her opponents or not. So $s_L > 0$ cannot be supported in any equilibrium such that $s_H < 1$. Thus, Claim 1 holds.

Claim 2: $(s_L \in [0, 1), s_H = 1)$ cannot be an equilibrium information strategy profile.

Suppose that $(s_L \in [0,1), s_H = 1)$ is an equilibrium. Consider an H-type player i's decision problem. Given the equilibrium information strategy $(s_L \in [0,1), s_H = 1)$, we have $P = 1$, implying that i knows the types of his/her opponents. By Lemma 1, i has incentive to conceal if possible. Since $s_L \in [0,1)$, i can indeed conceal effectively. This contradicts with $s_H = 1$. Thus Claim 2 holds.

Claim 3: The only possible equilibrium scenarios are $(s_L = 0, s_H \in [0,1))$ and $(s_L = 1, s_H = 1)$.

Claim 3 is immediate by Claims 1 and 2.

Claim 4: $(s_L = 0, s_H \in (0,1))$ cannot be an equilibrium information strategy profile when $\delta \geq \delta^* \approx 12.88$.

For $\delta \geq \delta^*$, by Lemmas 1 and 3, H-type players will always have incentive to conceal (if possible) whether they know their opponents' types or not, indicating that $s_H > 0$ cannot be supported in any equilibrium such that $s_L < 1$. Thus, Claim 4 holds.

Claim 5: $(s_L = 0, s_H = 0$ cannot be an equilibrium information strategy profile when $1 < \delta < \delta^* \approx 12.88$.

Suppose that $(s_L = 0, s_H = 0$ is an equilibrium. Consider an H-type player i's decision problem. Given the equilibrium information strategy $(s_L = 0, s_H = 0$, we have $P = 0$, implying that i does not know the types of his/her opponents. For $1 < \delta < \delta^* \approx 12.88$, by Lemma 3, i will have incentive to disclose in such an equilibrium. This contradicts with $s_H = 0$. Thus Claim 5 holds.

Combining Claims 3-5 and Propositions 1-3, we immediately have Theorem 1.

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