Optimal Subsidies in the Competition between Private and State-Owned Enterprises

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Abstract

Recent policy discussions have debated whether governments should adopt equal policies towards state-owned and private enterprises. We analyze this issue in a mixed oligopoly setting, in which the government can award different subsidies to these two types of firms. We show that the optimal subsidy policy is equal treatment, regardless of the relative weight on social welfare versus profits by the state-owned enterprise. This result is robust to the form of production, market demand, composition of firm types, and heterogeneity in the objectives of firms. However, heterogeneous cost structures among the firms yield a non-uniform optimal subsidy.

JEL Codes: D43, D61, H21, H25

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1 Introduction

State-owned enterprises have played a crucial role in China's economy and development, and they continue to hold substantial influence in a broad range of industries including financial, energy, metals, and transportation, among other sectors. While on an international level, foreign companies seek regulation of the government's support of state-owned firms out of concern for internationally competitive practices, another policy debate arises out of a domestic industrial concern.[1] Given that the government naturally tends to heavily subsidize state-owned enterprises during economic downturns, should the same economic aid be provided to domestic private enterprises?[2]

This question is currently an intensely debated one, while China's government seeks to reassure private firms that they too have official support during slowing economic times. However, the subsidizing of private firms not only expends government resources, but could distort the efficiency typically obtained in a competitive marketplace. Thus, it is unclear whether the best policy by the government is one of fairness across different types of firms, or of favorability to certain types of firms such as the state-owned enterprises.

Our main result demonstrates that even when the government has the choice to differentiate subsidies across firms in an industry, the optimal policy subsidizes the state-owned firm and all the private firms equally. The reason is that the efficiency gains from equalizing the playing field across firm types exceed the potential distortionary effect of the equally applied subsidy. This result suggests that even though a universal subsidy is costly to the government, it is theoretically preferable to a policy which targets only state-owned firms.

Our study contributes to the theoretical literature on mixed oligopoly and optimal policies (see DeFraja and Delbono, 1990 for a survey). One of commonly addressed topics is about the timing of privatization of mixed oligopolies and related subsidy policies. White (1996) examines the timing of subsidies in the privatization process. Poyago-Theotoky (2001) and Myles (2002) extend analysis of this question, finding an identical optimal subsidy under simultaneous or sequential moves, and Kato and Tomaru (2007) extends analysis to the case of different objectives of private firms. Fjell and Heywood (2004) examines the case in which privatization leads to either sequential or simultaneous move oligopoly, and derive the optimal

[1] For example, the G20 international business lobby has pressured the Chinese government to moderate favorable policies such as subsidy, debt relief and advantageous loans to state-owned enterprises; https://www.scmp.com/economy/china-economy/article/2167475/china-clashes-g20-business-lobby-group-over-support-state

[2] A recent example is the favorable loan given to state-owned automobile manufacturer FAW; https://www.scmp.com/economy/china-economy/article/2170253/chinas-state-owned-carmaker-gets-huge-lifeline-what-about

subsidy and welfare results.

Other notable studies examine the mixed oligopoly question in different contexts. Pal and White (1998) examines the effects of privatization of mixed oligopoly in an international trade setting. Heywood and Ye (2009) examines mixed oligopoly in a spatial price discrimination setting. Kato (2008) examines the government's privatization decision based on preferences over social welfare and tax revenue. Another line of studies examines the subsidy policy in a research and development setting. Lee and Tomaru (2017) find that the degree of privatization influences the optimal R&D tax, but not the optimal output subsidy. Haruna and Goel (2017) examine the optimal output subsidy in the case of research spillovers, finding that such subsidies may not attain efficiency.

Compared to these previously mentioned studies, our model focuses on the static setting, and differs crucially in the ability of the government to assign heterogeneous subsidies across different firms. Prior models have assumed a uniform subsidy across the entire industry, regardless of the ownership structure of the firm. Kato and Tomaru (2007) show that the optimal uniform subsidy is robust to heterogeneity in the weight of profit maximization in firms' objective functions. However, their analysis, like much of the prior literature, assumes a uniform subsidy exogenously. Our model by contrast, endogenizes the choice of subsidy levels across firms, and shows that the uniform one is in fact optimal. In this sense, the theoretical contribution of our work can be understood as a generalization of the settings presented by Kato and Tomaru (2007), as well as the preceding studies Myles (2002), Povago-Theotoky (2001), and White (1996). However, in contrast to these studies, our paper does not focus on a privatization process.

We begin by analyzing a benchmark model with linear demand, quadratic cost functions of firms, and $n+1$ firms, n of which are fully private and profit maximizing, and one of which is at least partially state-owned and therefore at least partially welfare-maximizing. Our main result in the benchmark case shows that the optimal subsidy policy by the government is a uniform per-unit production cost subsidy across the two types of firms.

We then demonstrate the robustness of this result to generalizations in the model features, firstly showing that the main result generalizes to any downward sloping demand function and secondly, showing that it generalizes to any increasing and convex cost function. Additionally, in terms of market structure, we show that the uniform subsidy result is robust to the number of partially state-owned firms, as well as being robust to heterogeneity in the objectives of those state-owned firms. Therefore, we show that the optimal uniform subsidy result is quite general and robust.

We also show however, that the optimality of the uniform subsidy is not robust to heterogeneity in firms' cost structures. The reason is that in the case of heterogeneous costs, the socially efficient production level differs across firms depending on their specific cost function, and therefore the optimal per-unit production subsidy is not the same across firms.

The remainder of the paper is organized as follows: Section 2 describes the benchmark model set-up; Section 3 describes the benchmark analysis and main results; Section 4 provides robustness results; Section 5 considers the case of heterogeneous production technology or costs; Section 6 concludes and discusses.

2 Benchmark Model

There are $n(\geq 1)$ identical private firms ($i = 1, \cdots, n$) and 1 firm that is jointly owned by the public and private sectors (indexed by $i = 0$). Private firm i ($i = 1, \cdots, n$) maximizes its own profit π_i while firm 0 maximizes a weighted average of social welfare W and its own profit π_0, denoted as $u_0 = \alpha W + (1-\alpha)\pi_0$, where $\alpha \in [0, 1]$.[3] It is common for state-owned enterprises to have some degree of a profitability objective, which is represented by this weighted objective function. The public sector share of the state-owned enterprise is owned by the government, and maximizes social welfare W. Note that in the extreme cases when $\alpha = 0$, firm 0 is simply a private firm, and when $\alpha = 1$, firm 0 is a public firm fully owned by the government.

Firms compete in the market for a homogeneous good. Demand is linear, denoted by $Q = a - p$, where Q is total output and p is the market price. Note that $Q = \sum_{i=0}^{n} q_i$, where q_i is the output of firm i, $i = 0, \cdots, n$.

Following the literature (Poyago-Theotoky, 2001; Fjell and Heywood, 2004), we assume here that all firms share the same producton technology with increasing marginal cost, denoted by $C(q_i) = c + \frac{1}{2}kq_i^2$, where $c \geq 0$, $k > 0$ and $i = 0, \cdots, n$. Since we do not focus the issue of firms entering the market, we set $c = 0$ without loss of generality.

We consider a two-stage game as follows: In stage 1, the government chooses the optimal output subsidy levels s_0 and s_1, where s_0 is the subsidy to firm 0 and s_1 is the subsidy to firm i, $i = 1, \cdots, n$. In stage 2, given the government's subsidy, firms compete simultaneously by chosing their own output level q_i, $i = 1, \cdots, n$.

Private firm i's profit is given by

$$\pi_i = q_i \left[a - \textstyle\sum_{i=0}^{n} q_i\right] - \frac{1}{2}kq_i^2 + s_1 q_i, \;\; i = 1, \cdots, n. \tag{1}$$

Firm 0's profit is given by

$$\pi_0 = q_0 \left[a - \textstyle\sum_{i=0}^{n} q_i\right] - \frac{1}{2}kq_0^2 + s_0 q_0. \tag{2}$$

Social welfare, defined as the sum of firms' profits and consumer surplus, is given by

[3]This weighting function also reflects the status of many state-owned enterprises in China, being partially state-owned and partially publicly traded.

$$
\begin{aligned}
W &= \frac{1}{2}\left[\sum_{i=0}^{n} q_i\right]^2 + \sum_{i=0}^{n} \pi_i - s_0 q_0 - s_1 \sum_{i=1}^{n} q_i & (3)\\
&= a\left[\sum_{i=0}^{n} q_i\right] - \frac{1}{2}\left[\sum_{i=0}^{n} q_i\right]^2 - \frac{1}{2} k \sum_{i=0}^{n} q_i^2. & (4)
\end{aligned}
$$

3 Analysis and Results

The two-stage game is solved by backward induction. In stage 2, given the optimal subsidy levels s_0 and s_1, all firms ($i = 0, \cdots, n$) simultaneously choose their output levels q_i to maximize their respective objectives. The first order conditions are given by

$$
\begin{aligned}
\frac{d\pi_i}{dq_i} &= a - \sum_{j=0}^{n} q_j - (k+1)q_i + s_1 = 0, \;\; i = 1, \cdots, n. & (5)\\
\frac{du_0}{dq_0} &= \alpha\left[a - \sum_{j=0}^{n} q_j - kq_0\right] a + (1-\alpha)\left[a - \sum_{j=0}^{n} q_j - (k+1)q_0 + s_0\right] = 0 & (6)
\end{aligned}
$$

Solving the above $n+1$ equations for q_i's, we obtain firms' outputs for the given subsidy levels (s_0, s_1):

$$
\begin{aligned}
q_0(s_0, s_1) &= \frac{(k+1)a + (1-\alpha)(n+k+1)s_0 - ns_1}{(k+1)^2 + nk + (1-\alpha)(n+k+1)} & (7)\\
q_i(s_0, s_1) &= \frac{(k+1-\alpha)a - (1-\alpha)s_0 + (k+2-\alpha)s_1}{(k+1)^2 + nk + (1-\alpha)(n+k+1)}, \;\; i = 1, \cdots, n. & (8)
\end{aligned}
$$

Proposition 1 *Firm 0's output $q_0(s_0, s_1)$ is increasing in its subsidy s_0 and decreasing in private firms' subsidy s_1. Private firm i's output $q_i(s_0, s_1)$ is increasing in its subsidy s_1 and decreasing in firm 0' subsidy s_0.*

Note that when setting $s_0 = s_1 = 0$ in $q_i(s_0, s_1)$ $(i = 0, \cdots, n)$ we obtain the standard result that firm 0's output exceeds the private firm's output, as $q_0(0,0) = \frac{(k+1)a}{(k+1)^2+nk+(1-\alpha)(n+k+1)} \geq \frac{(k+1-\alpha)a}{(k+1)^2+nk+(1-\alpha)(n+k+1)} = q_i(0,0)$.

In stage 1, taking into account the firms' optimal output as a function of the subsidy, that is $q_i(s_0, s_1)$ $(i = 0, \cdots, n)$, the government maximizes social welfare by choosing the optimal subsidy (s_0, s_1). The first order conditions imply the following two equations:

$$
\begin{aligned}
kq_0(s_0, s_1) + (1-\alpha)q_1(s_0, s_1) &= (k+1-\alpha)s_1 & (9)\\
-k(n+k+1)q_0(s_0, s_1) + \left[(k+1)^2 + nk\right] q_1(s_0, s_1) &= (k+1)s_1 & (10)
\end{aligned}
$$

Note that both $q_0(s_0, s_1)$ and $q_1(s_0, s_1)$ are linear functions of s_0 and s_1, so we can solve for s_0 and s_1 by using the above two equations. The results are characterized by the following proposition.

Proposition 2 *When firm* 0 *is at least partially privatized* ($\alpha \in [0, 1)$)*, the optimal subsidy levels for all firms are the same:* $s_0^* = s_1^* = s^* \equiv \frac{a}{n+k+1}$*; When firm* 0 *is fully owned by the government* ($\alpha = 1$)*, the optimal subsidy level for private firms is* $s_1^* = \frac{a}{n+k+1}$ *and the optimal subsidy level for firm* 0 *can be any amount, that is* $s_0^* \in R$*.*

Note that although the equilibrium strategies of the firms are identical to those in Poyago-Theotoky (2001), the key difference is that we endogenize the uniformity of the subsidy strategy, allowing for the government to potentially set different subsidy levels between firm 0 and the other firms. Surprisingly, the optimal subsidy turns out to be the same for all firms, as long as the state-owned firm has even a minimal weight on profitability in its objective function. In the special case that the state-owned firm is fully owned by the government and has no direct profit motive, the subsidy for private firms is the same as in the general case, while any subsidy optimally holds for the state-owned firm. The equilibrium result for output, price, profit and social welfare is given in the following corollary.

Corollary 1 *In equilibrium, all firms have the same output level* $q^* = \frac{a}{n+k+1}$*, the equilibrium price is* $p^* = \frac{ka}{n+k+1}$*, each firm's profit level is* $\pi^* = \frac{(k+2)a^2}{2(n+k+1)^2}$*, and the social welfare is* $W^* = \frac{(n+1)a^2}{2(n+k+1)}$*.*

A specific example of the equilibrium result for output, price, profit and social welfare in the case of duopoly is provided below.

Example 1 *Suppose* $k = 1, n = 1, a = 1$*. The demand function becomes* $Q = 1 - p$*, and the cost function becomes* $C(q_i) = \frac{1}{2}q_i^2$*,* $i = 0, 1$*. The government's optimal subsidy will be* $s^* = \frac{1}{3}$*, the firm's output will be* $q^* = \frac{1}{3}$*, the market price will be* $p^* = \frac{1}{3}$*, the firm's profit will be* $\pi^* = \frac{1}{6}$*, and the social welfare will be* $W^* = \frac{1}{3}$*.*

That is, the case of duopoly in which market size and marginal cost parameters are unitary, corresponds to the situation that in equilibrium each firm's output, the optimal subsidy, firm profit, and social welfare coincide, while each firm's profit (taking into account the government's subsidy) is half that. The comparative statics results of the main equilibrium variables of interest with respect to market characteristics are provided in the following corollary.

Corollary 2 *The equilibrium subsidy* s_0^**, output* q^**, and profit* π^* *are increasing in market size* a*, and decreasing in number of private firms* n *and cost parameter* k*; The equilibrium price* p^* *is increasing in market size* a *and cost parameter* k*, and decreasing in number of private firms* n*; The equilibrium social welfare* W^* *is increasing in market size* a *and number of firms* n*, and decreasing in cost parameter* k*.*

The corollary states that market size has an increasing effect on the subsidy amount, while the number of firms and marginal cost parameters affect the subsidy amount negatively. Output and profits bear the same direction of comparative statics to these market characteristic variables compared to the optimal subsidy. Equilibrium price follows the intuitive comparative statics, increasing in market size and marginal cost parameter, while decreasing in the number of private firms. The social welfare result also follows the intuitive comparative statics, increasing with respect to market size and number of private firms, while decreasing in the marginal cost parameter.

4 Robust Extensions

4.1 General Production Technology

We may wonder whether the equal treatment result for optimal subsidies is due to the specific assumption of quadratic functional form of the production technology. In this subsection we show that this is not the case. Assuming a general convex production function for all firms, denoted by $C(q_i)$, where $C'(q_i) > 0$, $C''(q_i) \geq 0$, and $C'(0) < a$, $i = 0, \cdots, n$, we can rewrite profit and social welfare as follows.

$$\pi_i = q_i \left[a - \textstyle\sum_{j=0}^{n} q_j\right] - C(q_i) + s_1 q_i, \;\; i = 1, \cdots, n. \tag{11}$$

$$\pi_0 = q_0 \left[a - \textstyle\sum_{j=0}^{n} q_j\right] - C(q_0) + s_0 q_0. \tag{12}$$

$$\begin{aligned} W &= \frac{1}{2}\left[\textstyle\sum_{i=0}^{n} q_i\right]^2 + \textstyle\sum_{i=0}^{n} \pi_i - s_0 q_0 - s_i \textstyle\sum_{i=1}^{n} q_i & (13)\\ &= a\left[\textstyle\sum_{i=0}^{n} q_i\right] - \frac{1}{2}\left[\textstyle\sum_{i=0}^{n} q_i\right]^2 - \textstyle\sum_{i=0}^{n} C(q_i). & (14) \end{aligned}$$

The two-stage game is once again solved by backward induction. In stage 2, given the optimal subsidy levels s_0 and s_1, all firms simultaneously choose their output levels q_i to maximize their respective objectives. The first order conditions are given by

$$\begin{aligned} \frac{d\pi_i}{dq_i} &= a - \textstyle\sum_{j=0}^{n} q_j - q_i - C'(q_i) + s_1 = 0, \;\; i = 1, \cdots, n. & (15)\\ \frac{du_0}{dq_0} &= \alpha\left[a - \textstyle\sum_{j=0}^{n} q_j - C'(q_0)\right] + (1-\alpha)\left[a - \textstyle\sum_{j=0}^{n} q_j - q_0 - C'(q_0) + s_0\right] = 0 & (16) \end{aligned}$$

By rearranging the above $n + 1$ equations, we obtain the following conditions:

$$[a - \sum_{j=0}^{n} q_j - C'(q_i)] + (s_1 - q_i) = 0, \;\; i = 1, \cdots, n. \tag{17}$$

$$[a - \sum_{j=0}^{n} q_j - C'(q_0)] + (1-\alpha)(s_0 - q_0) = 0 \tag{18}$$

Rather than following the standard backward induction method to solve for government's optimal subsidies and firms' equilibrium outputs, we now construct the equilibrium strategy directly.

Firstly, note that $\frac{\partial W(q_0,\cdots,q_n)}{\partial q_i} = a - \sum_{j=0}^{n} q_j - C'(q_i)$ and $\frac{\partial^2 W(q_0,\cdots,q_n)}{\partial q_i^2} = -1 - C''(q_i) < 0$, for $i = 0, \cdots, n$. Therefore, the necessary and sufficient conditions for the socially optimal output profile $(q_0, \cdots, q_n)$ are such that $a - \sum_{j=0}^{n} q_j - C'(q_i) = 0, i = 0, \cdots, n$, implying $q_i = q^{O_1}, \forall i$, where q^{O_1} is uniquely determined by $a - (n+1)q^{O_1} - C'(q^{O_1}) = 0$.[4]

Secondly, note that if we set $s_0 = s_1 = q^{O_1}$, then $q_i = q^{O_1}, \forall i$ is a solution to the system of equations (17)-(18). This means when the government chooses the uniform subsidy policy $s_0 = s_1 = q^{O_1}$, the firms' equilibrium outputs are socially optimal. Since the government's objective is to maximize social welfare, the best outcome that the government can achieve is the socially optimal outputs, therefore $s_0 = s_1 = q^{O_1}$ is an optimal subsidy policy for the government.

4.2 Generalized Demand

In the benchmark model, we assumed that the linear demand function is such that $Q = a - p$. We now allow for demand to take a more general form, and we denote the inverse demand function by $p(Q)$, where $p'(Q) < 0$, $p(0) > C'(0)$, and $\lim_{Q \to +\infty} p(Q) = 0$, $i = 0, \cdots, n$. Assuming a convex production function $C(\cdot)$, we can express profit, consumer surplus, and social welfare as follows.

$$\pi_i = q_i p(\sum_{j=0}^{n} q_j) - C(q_i) + s_1 q_i, \;\; i = 1, \cdots, n. \tag{19}$$

$$\pi_0 = q_0 p(\sum_{j=0}^{n} q_j) - C(q_0) + s_0 q_0. \tag{20}$$

$$CS = \int_{t=0}^{\sum_{i=0}^{n} q_i} p\,(t)\,dt - (\sum_{i=0}^{n} q_i) p(\sum_{i=0}^{n} q_i). \tag{21}$$

$$\begin{aligned} W &= CS + \sum_{i=0}^{n} \pi_i - s_0 q_0 - s_1 \sum_{i=1}^{n} q_i & (22) \\ &= \int_{t=0}^{\sum_{i=0}^{n} q_i} p\,(t)\,dt - \sum_{i=0}^{n} C(q_i). & (23) \end{aligned}$$

[4]Note that $a - (n+1)q$ is strictly decreasing in q and $C'(q)$ is weakly increasing in q, thus $f(q) \equiv a - (n+1)q - C'(q)$ is strictly decreasing in q. Since by assumption $f(0) = a - C'(0) > 0$ and $f(\frac{a}{n+1}) < 0$, we know by continuity of $f(\cdot)$ that $f(q) = 0$ has a unique solution $q^{O_1} \in (0, \frac{a}{n+1})$.

As before, the two-stage game is solved by backward induction. In stage 2, given the optimal subsidy levels s_0 and s_1, all firms simultaneously choose their output levels q_i to maximize their respective objectives. The first order conditions are given by

$$\frac{d\pi_i}{dq_i} = p(\textstyle\sum_{j=0}^n q_j) + q_i p'(\sum_{j=0}^n q_j) - C'(q_i) + s_1 = 0, \;\; i = 1, \cdots, n. \tag{24}$$

$$\frac{du_0}{dq_0} = \alpha \left[p(\textstyle\sum_{j=0}^n q_j) - C'(q_0)\right] + (1-\alpha)\left[p(\sum_{j=0}^n q_j) + q_0 p'(\sum_{j=0}^n q_j) - C'(q_0) + s_0\right] = 0 \tag{25}$$

By rearranging the above $n+1$ equations, we have the following conditions:

$$[p(\textstyle\sum_{j=0}^n q_j) - C'(q_i)] + (s_1 + q_i p'(\sum_{j=0}^n q_j)) = 0, \;\; i = 1, \cdots, n. \tag{26}$$

$$[p(\textstyle\sum_{j=0}^n q_j) - C'(q_0)] + (1-\alpha)(s_0 + q_0 p'(\sum_{j=0}^n q_j)) = 0 \tag{27}$$

Similar to the analysis in the previous subsection, note that the necessary and sufficient conditions for the socially optimal output profile $(q_0, \cdots, q_n)$ are such that $p(\sum_{j=0}^n q_j) - C'(q_i) = 0, \;\; i = 0, \cdots, n.$, implying $q_i = q^{O_2}, \forall i$, where q^{O_2} is uniquely determined by $p((n+1)q^{O_2}) - C'(q^{O_2}) = 0$.[5]

Also note that if we set $s_0 = s_1 = -q^{O_2}p'((n+1)q^{O_2})$, then $q_i = q^{O_2}, \forall i$ is a solution to the system of equations (26)-(27). This means when the government chooses the uniform subsidy policy $s_0 = s_1 = -q^{O_2}p'((n+1)q^{O_2})$, the firms' equilibrium outputs are socially optimal. Since the government's objective is to maximize social welfare, the best that the government can achieve are the socially optimal outputs, therefore $s_0 = s_1 = -q^{O_2}p'((n+1)q^{O_2})$ is an optimal subsidy policy for the government.

4.3 Generalized Heterogeneous Objectives

In the benchmark model, we assume there are only two types of firms: private firms and a (partially privatized) public firm. In this subsection, we further extend our model to allow for more than 2 types of firms. To be more specific, we allow for each firm i to have a different degree of privatization, $\alpha_i \in [0, 1]$, where $i = 0, \cdots, n$. The optimal subsidy profile will be $(s_0, \cdots, s_n)$, where s_i is the subsidy for firm i, $i = 0, \cdots, n$.

Assuming convex production function $C(\cdot)$ and general inverse demand function $p(\cdot)$, we can write firms' profit, consumer surplus, and social welfare as follows.

$$\pi_i = q_i p(\textstyle\sum_{j=0}^n q_j) - C(q_i) + s_i q_i, \;\; i = 0, \cdots, n. \tag{28}$$

[5]Note that $p((n+1)q)$ is strictly decreasing in q and $C'(q)$ is weakly increasing in q, thus $g(q) \equiv p((n+1)q) - C'(q)$ is strictly decreasing in q. Since by assumption $g(0) = p(0) - C'(0) > 0$ and $\lim_{q\to+\infty} p((n+1)q) - C'(q) < 0$, we know by continuity of $g(\cdot)$ that $g(q) = 0$ has a unique solution $q^{O_2} \in (0, +\infty)$.

$$CS = \int_{t=0}^{\sum_{i=0}^{n} q_i} p\,(t)\,dt - (\sum_{i=0}^{n} q_i) p(\sum_{i=0}^{n} q_i). \tag{29}$$

$$\begin{aligned} W &= CS + \sum_{i=0}^{n} \pi_i - \sum_{i=0}^{n} s_i q_i & (30)\\ &= \int_{t=0}^{\sum_{i=0}^{n} q_i} p\,(t)\,dt - \sum_{i=0}^{n} C(q_i). & (31) \end{aligned}$$

$$u_i = \alpha_i W + (1-\alpha_i)\pi_i \tag{32}$$

$$= \alpha_i \int_{t=0}^{\sum_{j=0}^{n} q_j} p\,(t)\,dt - \alpha_i \sum_{j=0}^{n} C(q_i) + (1-\alpha_i) q_i p(\sum_{j=0}^{n} q_j) - (1-\alpha_i)C(q_i) + (1-\alpha_i)s_i q_i,\ \ i = 0, \cdots, n. \tag{33}$$

The two-stage game is again solved by backward induction. In stage 2, given the optimal subsidy levels $(s_0, \cdots, s_n)$, all firms simultaneously choose their output levels q_i to maximize their respective objectives. The first order conditions are given by

$$\frac{du_i}{dq_i} = [p(\sum_{j=0}^{n} q_j) - C'(q_i)] + (1-\alpha_i)(s_i + q_i p'(\sum_{j=0}^{n} q_j)) = 0,\ \ i = 0, \cdots, n. \tag{34}$$

Note that the necessary and sufficient conditions for the socially optimal output profile $(q_0, \cdots, q_n)$ are $q_i = q^{O_2}, \forall i$, regardless of the objective functions of different firms.

Furthermore, it is easy to see that if we set $s_i = -q_i p'(\sum_{j=0}^{n} q_j) > 0$ $(i = 0, \cdots, n)$, the first order conditions for Cournot competition will coincide with the first order conditions under the social optimum. Therefore, an optimal subsidy should be uniform such that $s_i^* = -q^{O_2} p'((n+1)q^{O_2}), \forall i$, and social optimailty is achieved under such a subsidy policy with output $q_i = q^{O_2}, \forall i$. Also note that if $\alpha_i = 1$, s_i^* can be any real number.

Theorem 1 *With homogeneous production technology $C(q_i)$ and inverse demand function $p(\cdot)$, assuming $C'(q_i) > 0$, $C''(q_i) \geq 0$, $p'(\cdot) < 0$, $p(0) > C'(0)$, $\lim_{Q \to +\infty} p(Q) = 0$, regardless of firms' heterogeneous publicization levels $(\alpha_0, \cdots, \alpha_n) \in [0,1]^{n+1}$, $i = 0, \cdots, n$, under the optimal subsidy policy every firm has the same output level $q_i^* = q^{O_2}$, and the optimal subsidy is uniform such that*

$$s_i^* \begin{cases} = -q^{O_2} p'((n+1)q^{O_2}) & \text{if } \alpha_i \in [0,1) \\ \in R & \text{if } \alpha_i = 1 \end{cases}.$$

5 Optimal Subsidy under Heterogeneous Costs

Based on the analyses from previous sections, we observe that the uniform optimal subsidy result relies in essence, on the assumption of homogeneous production technology among all the firms. If the firms have different cost functions, the socially optimal output levels for different firms can be different, which will result in different optimal subsidy levels for different firms. We summarize this result in the following theorem and relegate the proof to the Appendix.

Theorem 2 *With heterogeneous publicization levels $\alpha_i \in [0,1]$, heterogeneous production technologies $C_i(q_i)$ and inverse demand function $p(\cdot)$, assuming $C_i'(q_i) > 0$, $C_i''(q_i) \geq 0$, $p'(\cdot) < 0$, $p(0) > C_i'(0)$, $\lim_{Q\to+\infty} p(Q) = 0$, $i = 0, \cdots, n$, firms' equilibrium outputs $(q_0^*, \cdots, q_n^*)$ are uniquely determined by*

$$p(\textstyle\sum_{j=0}^{n} q_j^*) - C_i'(q_i^*) = 0, \;\; i = 0, \cdots, n \;\;;$$

and the government's optimal subsidies $(s_0^*, \cdots, s_n^*)$ are such that

$$s_i^* \begin{cases} = -q_i^* p'(\sum_{j=0}^{n} q_j^*) & \text{if } \alpha_i \in [0,1) \\ \in R & \text{if } \alpha_i = 1 \end{cases} .$$

As the theorem shows, the optimal subsidy is not uniform across firms. In particular, the subsidy for each firm is dependent on the socially optimal level of production for that particular firm, which in turn depends on that firm's cost function. This result is consistent with the intuition of socially optimal production allocations across firms of differing marginal cost structures. A key insight of Theorem 1 and Theorem 2 is that a policy of uniform subsidization across state-owned and private firms is socially optimal under any degree of heterogenous profit motives across the firms, but in the case of heterogenous cost functions, socially efficient allocation of subsidies is driven by firms' relative cost advantages.

6 Concluding Remarks

During the current period of relative slowdown in economic growth compared to the previous decades in China, the government's policy of favorable subsidization of state-owned enterprises has been publicly challenged by both foreign and domestic private firms. Our study solves for the optimal subsidy distribution between private and state-owned firms, in a setting in which government can choose to differentiate the subsidies across firms. Generalizing various aspects of previous studies on optimal subsidy for mixed oligopoly, specifically by endogenizing the government's choice of relative subsidy, we show that in fact the socially optimal policy is a uniform per-unit production subsidy across all firms.

We show that this result is robust to non-linear demand functions, any increasing and convex cost function, the composition of state-owned and private firms in the market, and heterogeneity in the state-owned firms' emphasis on welfare maximization. We also analyze the case of heterogeneous production technologies and show that under this situation the optimal subsidy is not uniform, but depends on individual firms' cost functions.

Although the main result may be counterintuitive to the conventional wisdom on minimizing government interference in competitive markets, the efficiency result can be understood through the influence that the subsidies have on the competitive incentives between state-owned and private firms. In other words, if the government is to provide a subsidy to the state-owned firm to assist in its social welfare maximizing objective, the optimal policy involves an equal subsidy to the private firms, in order to maintain a competitive market environment. Since the government's objective is at least partly aligned with that of the state-owned enterprise, it will find this policy ideal for its own social welfare maximizing goals.

Our analysis raises an important insight in the management of economies with state-owned industrial components, which to our knowledge may not have been raised before. In particular, conditional that the government would like to maintain a state-owned sector of the economy, it should also carefully consider the subsidy policy to the private sector in order to maintain proper strategic incentives between the different types of firms.

There are several directions for extension of this work. For one, our current analysis has assumed simultaneous Cournot competition among the firms. However, in some realistic settings, either the state-owned firm or one of the private firms may be the Stackelberg leader. Future work may conduct equilibrium analysis under different types of market compositions (ex. Li and Zheng, 2004) and network demand structures (ex. Kuang et al., 2019). In addition, here we have assumed that the government is not budget constrained in its allocation of subsidies, but is willing to implement the optimal subsidy scheme on behalf of social efficiency. Future work can consider some of the practical constraints that government may face in implementing the optimal policy, as well as other channels for achieving social efficiency such as investment policy (Lien et al., 2016), labor market policies (Feng et al., forthcoming), and information policy (Wu and Zheng, 2017; Jiao et al., 2018).

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References

[1] De Fraja, Giovanni, and Flavio Delbono, "Game Theoretic Models of Mixed Oligopoly", Journal of Economic Surveys, Vol. 4 (1990), No. 1, p. 1 – 17.

[2] Eckaus, Richard S., "China's exports, subsidies to state owned enterprises and the WTO", China Economic Review, Vol. 17 (2006), p. 1 – 13.

[3] Feng, Zhenhua, Jaimie W. Lien, and Jie Zheng, "Flexible or Mandatory Retirement? Welfare Implications of Retirement Policies for a Population with Heterogeneous Health Conditions," International Review of Economics and Finance, forthcoming

[4] Fjell, Kenneth, and John S. Heywood, "Mixed oligopoly, subsidization and the order of firm's moves: the relevance of privatization", Economics Letters, Vol. 83 (2004), p. 411 – 416.

[5] Haruna, Shoji and Rajeev K. Goel, "Output subsidies in mixed oligopoly with research spillovers", Journal of Economics and Finance, Vol. 41 (2017), Issue 2, p. 235 – 256.

[6] Heywood, John S., and Guangliang Ye, "Mixed oligopoly, sequential entry, and spatial price discrimination", Economic Inquiry, Vol. 47 (2009), No. 3, p. 589 – 597.

[7] Jiao, Qian, Jaimie W. Lien, and Jie Zheng, "Managing Competitions Using Information: Information Disclosure Policy in Contests with Heterogeneous Players", Working Paper, 2018

[8] Kato, Hideya, "Privatization and government preference", Economics Bulletin, Vol. 12 (2008), No. 40, p. 1 – 7.

[9] Kato, Kazuhiko and Yoshihiro Tomaru, "Mixed oligopoly, privatization, subsidization and the order of firms' moves: Several types of objectives", Economics Letters, Vol. 96 (2007), p. 287 – 292.

[10] Kuang, Zhonghong, Zeng Lian, Jaimie Lien, and Jie Zheng, "Serial and Parallel Duopoly Competition in Two-Part Transportation Routes", Working Paper, 2019

[11] Lee, Sang-Ho, and Yoshihiro Tomaru, "Output and R&D subsidies in a mixed oligopoly", Operations Research Letters, Vol. 45 (2017), p. 238 – 241.

[12] Li, Mingzhi, and Jie Zheng, "Open Source Software Movement: A Challenging Opportunity for the Development of China's Software Industry," Journal of Electronic Science and Technology of China, 2004, 2(3): 47-52

[13] Lien, Jaimie W., Wei Wang, and Jie Zheng, "A Model of Capital Allocation, Education and Job Choice in China," with , The Chinese Economy, 2016, 49(5): 307-326

[14] Myles, Gareth, "Mixed oligopoly, subsidization and the order of firms' moves: an irrelevance result for the general case." Economics Bulletin, Vol. 12 (2002), No. 1 pp. 1-6.

[15] Pal, Debashis, and Mark D. White, "Mixed Oligopoly, Privatization, and Strategic Trade Policy", Southern Economic Journal, Vol. 65 (1998), No. 2, p. 264 – 281.

[16] Payago-Theotoky, J., "Mixed oligopoly, subsidization and the order of firms' moves: an irrelevance result", Economics Bulletin, Vol. 12 (2001), No. 3, p. 1 – 5.

[17] White, Mark D., "Mixed oligopoly, privatization and subsidization", Economics Letters, Vol. 53 (1996), p. 189 – 195.

[18] Wu, Zenan, and Jie Zheng, "Information Sharing in Private Value Lottery Contest," Economics Letters, August 2017, 157: 36-40

Appendix

Proof of Proposition 1:

Proposition 1: Firm 0's output $q_0(s_0, s_1)$ is increasing in its subsidy s_0 and decreasing in private firms' subsidy s_1. Private firm i's output $q_i(s_0, s_1)$ is increasing in its subsidy s_1 and decreasing in firm 0' subsidy s_0.

Proof. Given $q_0(s_0, s_1) = \frac{(k+1)a+(1-\alpha)(n+k+1)s_0-ns_1}{(k+1)^2+nk+(1-\alpha)(n+k+1)}$, we have

$$\begin{aligned}
\frac{\partial q_0(s_0, s_1)}{\partial s_0} &= \frac{(1-\alpha)(n+k+1)}{(k+1)^2+nk+(1-\alpha)(n+k+1)} > 0, \\
\frac{\partial q_0(s_0, s_1)}{\partial s_1} &= \frac{-n}{(k+1)^2+nk+(1-\alpha)(n+k+1)} < 0.
\end{aligned}$$

Given $q_i(s_0, s_1) = \frac{(k+1-\alpha)a-(1-\alpha)s_0+(k+2-\alpha)s_1}{(k+1)^2+nk+(1-\alpha)(n+k+1)}$, $i = 1, \cdots, n$, we have

$$\begin{aligned}
\frac{\partial q_i(s_0, s_1)}{\partial s_0} &= \frac{-(1-\alpha)}{(k+1)^2+nk+(1-\alpha)(n+k+1)} < 0, \\
\frac{\partial q_i(s_0, s_1)}{\partial s_1} &= \frac{(k+2-\alpha)}{(k+1)^2+nk+(1-\alpha)(n+k+1)} > 0.
\end{aligned}$$

■

Proof of Proposition 2:

Proposition 2: When firm 0 is at least partially privatized ($\alpha \in [0, 1)$), the optimal subsidy levels for all firms are the same: $s_0^* = s_1^* = s^* \equiv \frac{a}{n+k+1}$; When firm 0 is fully owned by the government ($\alpha = 1$), the optimal subsidy level for private firms is $s_1^* = \frac{a}{n+k+1}$ and the optimal subsidy level for firm 0 can be any number, that is $s_0^* \in R$.

Proof. Comibing equations (7)-(10), we obtain the following two equations:

$$\begin{aligned}
Bs_1 - Cs_0 &= Da, \\
Es_0 - Fs_1 &= Ga,
\end{aligned}$$

where

$$\begin{aligned}
B &= (k+1-\alpha)\left[(k+1)^2+nk+(1-\alpha)(n+k+1)\right] - (1-\alpha)(k+2-\alpha) + nk, \\
C &= (1-\alpha)\left[(n+k+1)k-(1-\alpha)\right], \\
D &= (1-\alpha)(k+1-\alpha) + k(k+1), \\
E &= (1-\alpha)\left[(n+k+1)^2k + (k+1)^2 + nk\right], \\
F &= n\left[(n+k+1)k-(1-\alpha)\right], \\
G &= (1-\alpha)\left[(k+1)^2+nk\right] - nk.
\end{aligned}$$

Note that $B > 0, D > 0, F > 0, C \geq 0, E \geq 0$ and G can be positive or negative depending on the value of α.

When $\alpha = 1$, we have

$$\begin{aligned}
B &= k\left[(k+1)^2 + nk\right] + nk, \\
C &= 0, \\
D &= k(k+1), \\
E &= 0, \\
F &= n(n+k+1)k, \\
G &= -nk.
\end{aligned}$$

Thus, the optimal value of s_0 can be any number and we can solve for the optimal value of s_1 as follows:

$$s_1^* = \frac{Da}{B} = -\frac{G}{F} = \frac{a}{n+k+1}.$$

When $\alpha \in [0, 1)$, the optimal values of s_0 and s_1 are uniquely determined by the following two equations:

$$\begin{aligned}
s_0^* &= \frac{DF + BG}{BE - CF} a, \\
s_1^* &= \frac{DE + CG}{BE - CF} a.
\end{aligned}$$

It is easy to show that

$$DF + BG = DE + CG = \frac{BE - CF}{n+k+1}.$$

Thus we have

$$s_0^* = s_1^* = \frac{a}{n+k+1}.$$

■

Proof of Theorem 2:

We can write firms' profit, consumer surplus, and social welfare as follows.

$$\pi_i = q_i p(\textstyle\sum_{j=0}^{n} q_j) - C_i(q_i) + s_i q_i, \;\; i = 0, \cdots, n. \tag{35}$$

$$CS = \int_{t=0}^{\sum_{i=0}^{n} q_i} p\,(t)\,dt - (\textstyle\sum_{i=0}^{n} q_i) p(\sum_{i=0}^{n} q_i). \tag{36}$$

$$\begin{aligned}
W &= CS + \textstyle\sum_{i=0}^{n} \pi_i - \sum_{i=0}^{n} s_i q_i && (37) \\
&= \int_{t=0}^{\sum_{i=0}^{n} q_i} p\,(t)\,dt - \textstyle\sum_{i=0}^{n} C_i(q_i). && (38)
\end{aligned}$$

$$u_i = \alpha_i W + (1-\alpha_i)\pi_i \tag{39}$$

$$= \alpha_i \int_{t=0}^{\sum_{j=0}^n q_j} p\,(t)\,dt - \alpha_i \sum_{j=0}^n C_i(q_i) + (1-\alpha_i) q_i p(\sum_{j=0}^n q_j) - (1-\alpha_i) C_i(q_i) + (1-\alpha_i) s_i q_i,\ \ i = 0, \cdots, n. \tag{40}$$

The two-stage game is solved by backward induction. In stage 2, given the optimal subsidy levels $(s_0, \cdots, s_n)$, all firms simultaneously choose their output levels q_i to maximize their respective objectives. The first order conditions are given by

$$\frac{du_i}{dq_i} = [p(\sum_{j=0}^n q_j) - C_i'(q_i)] + (1-\alpha_i)(s_i + q_i p'(\sum_{j=0}^n q_j)) = 0,\ \ i = 0, \cdots, n. \tag{41}$$

Note that $\frac{\partial W(q_0, \cdots, q_n)}{\partial q_i} = p(\sum_{j=0}^n q_j) - C_i'(q_i)$ and $\frac{\partial^2 W(q_0, \cdots, q_n)}{\partial q_i^2} = p'(\sum_{j=0}^n q_j) - C_i''(q_i) < 0$, for $i = 0, \cdots, n$. Therefore, the necessary and sufficient conditions for the socially optimal output profile are

$$p(\sum_{j=0}^n q_j) - C_i'(q_i) = 0,\ \ i = 0, \cdots, n\ \ ,$$

from which the socially optimal output profile $(q_0^*, \cdots, q_n^*)$ is uniquely determined.

Also, it is easy to see that if we set $s_i = -q_i^* p'(\sum_{j=0}^n q_i^*) > 0$ $(i = 0, \cdots, n)$, then $(q_0^*, \cdots, q_n^*)$ is a solution to the system of equations (41). This means when the government chooses the uniform subsidy policy $s_i = -q_i^* p'(\sum_{j=0}^n q_i^*)$, the firms' equilibrium outputs are socially optimal. Since the government's objective is to maximize the social welfare, the best that the government can achieve are the socially optimal outputs, therefore $s_i = -q_i^* p'(\sum_{j=0}^n q_i^*)$ is an optimal subsidy policy for the government.

Obviously if $\alpha_i = 1$, s_i^* can be any real number.