# Optimal Joint Design of Timing and Information Disclosure in Tullock Contests

5th China Conference on Organizational Economics, SUIBE, Shanghai, China

Jian Chen[1] Zhonghong Kuang[1] Jie Zheng[2]

1. Department of Management Science and Engineering, School of Economics and Management, Tsinghua University
2. Department of Economics, School of Economics and Management, Tsinghua University

August 12th, 2017

# Agenda of Presentation

- Background and Literature
- Model and Results
  - Type-Independent Timing and Type-Independent Information Disclosure
  - Type-Independent Timing and Type-**Dependent** Information Disclosure
  - Type-**Dependent** Timing and Type-Independent Information Disclosure
  - Type-**Dependent** Timing and Type-**Dependent** Information Disclosure
- Summary and Discussion

# Background

- Organizations are often interested in how to motivate individuals using information policy or using timing policy or using both.
- Real World Examples:
  - A sales manager wants to maximize his employees' total efforts in a sales contest, and needs to decide under what conditions he will disclose the information about employees' abilities, and also needs to decide whether to make the contest simultaneous-move or sequential-move
  - Why are some sports games designed as a simultaneous-move contest while the others designed as a sequential-move contest, each of which may have different information structures?
- In a competing environment with heterogeneous competitors, both information and timing play important roles.

# What We Do in this Project

- The problem we consider in this paper:
  - Two competitors with private information on binary distributed types will compete against each other in a Tullock contest game to maximize their own payoff
  - The designer, who can both design the contest timing structure and the contest information structure, aims to maximize the total effort of competitors.
- Questions to answer:
  - How do competitors rationally behave under different timing policies and information disclosure policies?
  - How to jointly design the timing policy and the information disclosure policy to maximize the total effort of competitors?

# Main Contribution to the Literature

- Characterizing **Optimal Symmetric Information Disclosure Policy** for Sequential Tullock Contests with Incomplete Information
- Characterizing **Optimal Timing Policy** for Tullock Contests with Incomplete Information
- Characterizing **Optimal Joint Design of Symmetric Information Disclosure Policy and Timing Policy**

# Related Literature

Information Disclosure in (Simultaneous-Move) Contests:
**All-pay Auction:**

- Morath and Munster (2013): General setting, player's (type-independent) information acquisition incentive
- Fu, Jiao and Lu (2014): Symmetric setting, designer's type-independent disclosure policy
- Kovenock, Morath, and Munster (2015): Symmetric setting, (type-independent) information sharing incentive
- Jiao, Lien, and Zheng (2016): Asymmetric setting, uniform distribution, designer's (type-dependent) disclosure policy
- Lu, Ma, and Wang (2016): Symmetric setting, binary distribution, designer's type-dependent symmetric disclosure policy

# Related Literature

Information Disclosure in (Simultaneous-Move) Contests:
**Tullock Contest:**

- Zhang and Zhou (2015): Asymmetric setting, Discrete type distribution, type-dependent optimal disclosure policy (signal)
- Serena (2016): Symmetric setting, binary distribution, designer's type-dependent symmetric disclosure policy
- Wu and Zheng (2017): Symmetric setting, binary distribution, player's disclosure strategy and designer's type-independent disclosure policy

# Related Literature

Timing in Contests:

**All-pay Auction:**

- Segev and Sela [2014]: Private Information, Timing exogenously given

**Tullock Contest:**

- Morgan (2003): Binary type distribution, complete information
- Linster (1993): Binary type distribution, complete information; (also solving for unique PBE for incomplete information setting)

# Contest Setting

- 2 risk neutral players ($i = 1, 2$) participate in a single-prize Tullock contest
- Each player's value of winning $v_i \in \{v_l, v_h\}$ is private information, iid, $\Pr(v_i = v_l) = \Pr(v_i = v_h) = \frac{1}{2}$
- Player $i$ chooses his bid $b_i \geq 0$, and his success function is $p_i(b_1, b_2) = \frac{b_i}{b_1+b_2}$.
- Player $i$'s payoff is $u_i(b_1, b_2|v_i) = \frac{b_i}{b_1+b_2} v_i - b_i$.
- Defnine $\delta \equiv \frac{v_h}{v_l} \in [1, +\infty)$

# Contest Setting

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- The designer chooses a timing policy and an information disclosure policy jointly to maximize the total effort level.
- Given the timing policy and the information disclosure policy, the players play the Tullock contest accordingly

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  - When the timing policy is type-independent, $T(v_1, v_2) = Seq$ $\forall v_1, v_2 \in \{v_l, v_h\}$ or $T(v_1, v_2) = S$ $\forall v_1, v_2 \in \{v_l, v_h\}$, where $Seq$ denotes the sequential-move contest setup and $S$ denotes the simultaneous-move contest policy.

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  - When the timing policy is type-dependent, $T(v_h, v_l), T(v_l, v_h) \in \{S, H, L\}$, $T(v_h, v_h) \in \{S, H\}$, $T(v_l, v_l) \in \{S, L\}$, and we allow for $T(v_1, v_2) \neq T(v_1', v_2')$ if $(v_1, v_2) \neq (v_1', v_2')$, $\forall v_1, v_2, v_1', v_2' \in \{v_l, v_h\}$, where $H$ denotes the sequential-move contest setup where a high type player moves first and $L$ denotes the sequential-move contest setup where a low type player moves first.

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  - When the disclosure policy is type-independent, $P(v_1, v_2) = D$ $\forall v_1, v_2 \in \{v_l, v_h\}$ or $P(v_1, v_2) = C$ $\forall v_1, v_2 \in \{v_l, v_h\}$, where $D$ denotes the disclosure policy and $C$ denotes the concealment policy

# Symmetric Information Disclosure Policies

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  - When the disclosure policy is type-independent, $P(v_1, v_2) = D$ $\forall v_1, v_2 \in \{v_l, v_h\}$ or $P(v_1, v_2) = C$ $\forall v_1, v_2 \in \{v_l, v_h\}$, where $D$ denotes the disclosure policy and $C$ denotes the concealment policy
  - When the disclosure policy is type-dependent, $P(v_1, v_2) \in \{C, D\}$ and we allow for $P(v_1, v_2) \neq P(v_1', v_2')$ if $(v_1, v_2) \neq (v_1', v_2')$, $\forall v_1, v_2, v_1', v_2' \in \{v_l, v_h\}$.

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- Stage 1, the designer chooses a timing policy $\mathbb{T}$ and a disclosure policy $\mathbb{P}$, and announces both of them.

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  - Players 1 and 2 compete in either a simultaneous-move contest or a sequential-move contest, specified by timing policy $\mathbb{T}$.

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  - $v_1$ and $v_2$ are disclosed iff $P(v_1, v_2) = D$.
  - Players 1 and 2 compete in either a simultaneous-move contest or a sequential-move contest, specified by timing policy $\mathbb{T}$.
  - The prize is given to the winner and the payoffs are realized.

# Type-Independent Timing and Type-Independent Disclosure (1)

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- Leininger (1993) showed the above setup has the following equilibrium:

$$b_1^*(v_1, v_2) = \frac{v_1 v_2}{(v_1 + v_2)^2} v_1$$
$$b_2^*(v_1, v_2) = \frac{v_1 v_2}{(v_1 + v_2)^2} v_2$$

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- The designer's expected revenue is

$$\prod_{SD}^{II} = \frac{v_l}{16} \frac{2\delta^2 + 12\delta + 2}{1 + \delta}$$

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$$
\begin{aligned}
b_i^*(v_h) &= \left(\frac{1}{8} + \frac{1}{2}\frac{v_h v_l}{(v_h + v_l)^2}\right) v_h \\
b_i^*(v_l) &= \left(\frac{1}{8} + \frac{1}{2}\frac{v_h v_l}{(v_h + v_l)^2}\right) v_l
\end{aligned}
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$$b_i^*(v_h) = \left(\frac{1}{8} + \frac{1}{2}\frac{v_h v_l}{(v_h + v_l)^2}\right) v_h$$
$$b_i^*(v_l) = \left(\frac{1}{8} + \frac{1}{2}\frac{v_h v_l}{(v_h + v_l)^2}\right) v_l$$

- The designer's expected revenue is

$$\prod_{SC}^{II} = \frac{v_l}{16}\frac{2\delta^2 + 12\delta + 2}{1 + \delta}$$

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- Morgan (2003) and we (independently) showed the above setup has the following equilibrium:

$$
\begin{aligned}
b_1^*(v_1, v_2) &= \min\left(\frac{v_1^2}{4v_2}, v_2\right) \\
b_2^*(b_1, v_2) &= \max\left(\sqrt{b_1 v_2} - b_1, 0\right)
\end{aligned}
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- The designer's expected revenue is

$$\prod_{SeqD}^{II} = \begin{cases} \frac{v_l}{16}\left(4\delta + 4\right) & \delta \in (1, 2] \\ \frac{v_l}{16}\left(2\delta + 8\right) & \delta \in (2, +\infty) \end{cases}$$

# Type-Independent Timing and Type-Independent Disclosure (4)

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- $\mathbb{T} = Seq$ and $\mathbb{P} = C$. This is the case of sequential-move Tullock contests with private information.
- Linster (1993) and we (independently) showed the above setup has the following equilibrium:

$$b_1^*(v_h) = \begin{cases} \frac{v_h}{16}(1+\sqrt{\delta})^2 & \delta \in [1, 2.44) \\ v_l & \delta \in [2.44, 16] \\ \frac{v_h}{16} & \delta \in (16, +\infty) \end{cases}$$

$$b_1^*(v_l) = \frac{v_h}{16\delta^2}(1+\sqrt{\delta})^2$$

$$b_2^*(b_1, v_h) = \sqrt{b_1 v_h} - b_1$$

$$b_2^*(b_1, v_l) = \max(\sqrt{b_1 v_l} - b_1, 0)$$

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- $\mathbb{T} = Seq$ and $\mathbb{P} = C$. This is the case of sequential-move Tullock contests with private information.
- The designer's expected revenue is

$$\prod_{SeqC}^{II} = \begin{cases} \frac{v_l}{16}\left(\delta^{1.5} + 2\delta + 2\delta^{0.5} + 2 + \delta^{-0.5}\right) & \delta \in (1, 2.44) \\ \frac{v_l}{16}\left(5\delta^{0.5} + 6 + \delta^{-0.5}\right) & \delta \in [2.44, 16] \\ \frac{v_l}{16}\left(\delta + \delta^{0.5} + 6 + \delta^{-0.5}\right) & \delta \in (16, \infty) \end{cases}$$

# Type-Independent Timing and Type-Independent Disclosure - Revenue Comparison

- When $\mathbb{T} = S$, $\prod_{SD}^{II} = \prod_{SC}^{II}$ (Lemma 4.3)

# Type-Independent Timing and Type-Independent Disclosure - Revenue Comparison

- When $\mathbb{T} = S$, $\prod^{II}_{SD} = \prod^{II}_{SC}$ (Lemma 4.3)
- When $\mathbb{T} = Seq$, $\prod^{II}_{SeqC} \geq \prod^{II}_{SeqD} \iff \delta \in [1, 4.59]$ (Theorem 4.2)

# Type-Independent Timing and Type-Independent Disclosure - Revenue Comparison

- When $\mathbb{T} = S$, $\prod_{SD}^{II} = \prod_{SC}^{II}$ (Lemma 4.3)
- When $\mathbb{T} = Seq$, $\prod_{SeqC}^{II} \geq \prod_{SeqD}^{II} \iff \delta \in [1, 4.59]$ (Theorem 4.2)
- When $\mathbb{P} = D$, $\prod_{SeqD}^{II} \geq \prod_{SD}^{II} \iff \delta \in [1, 3]$ (Theorem 4.4)

# Type-Independent Timing and Type-Independent Disclosure - Revenue Comparison

- When $\mathbb{T} = S$, $\prod_{SD}^{II} = \prod_{SC}^{II}$ (Lemma 4.3)
- When $\mathbb{T} = Seq$, $\prod_{SeqC}^{II} \geq \prod_{SeqD}^{II} \iff \delta \in [1, 4.59]$ (Theorem 4.2)
- When $\mathbb{P} = D$, $\prod_{SeqD}^{II} \geq \prod_{SD}^{II} \iff \delta \in [1, 3]$ (Theorem 4.4)
- When $\mathbb{P} = C$, $\prod_{SeqC}^{II} \geq \prod_{SC}^{II} \iff \delta \in [1, 4.09]$ (Theorem 4.5)

# Type-Independent Timing and Type-Independent Disclosure - Optimal Policy

**Theorem**

*The optimal type-independent timing and type-independent disclosure policy is the following:*

$$\begin{aligned} \mathbb{T} &= S, \mathbb{P} = D \text{ or } C \text{ if and only if } \delta \in [4.09, +\infty) \\ \mathbb{T} &= Seq, \mathbb{P} = C \quad \text{ if and only if } \delta \in [1, 4.09] \end{aligned}$$

Note that $\mathbb{T} = Seq, \mathbb{P} = D$ is a strictly dominated policy!!

# Type-Dependent Information Structure in Sequential Contests

- A type-dependent information disclosure policy is a vector of 4 binary variables, each taking value $C$ or $D$, where 1st (respectively, 2nd, 3rd and 4th) element corresponds to the disclosure policy choice in case of realization $(v_h, v_h)$ (respectively, $(v_h, v_l)$, $(v_l, v_h)$ and $(v_l, v_l)$).

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- There are 16 combinations. However, some of them are not effective. By merging them together, we can divide those 16 policy into 4 different classes.

# Type-Dependent Information Structure in Sequential Contests

## Proposition

*There are 4 classes of information disclosure policy,*

1. $(\mathcal{C},\mathcal{C},\mathcal{C},\mathcal{C})$, *expected revenue denoted as* $\prod_{SeqCC}^{ID}$
2. $(\mathcal{C},\mathcal{C},\mathcal{C},\mathcal{D}),(\mathcal{C},\mathcal{C},\mathcal{D},\mathcal{C}),(\mathcal{C},\mathcal{C},\mathcal{D},\mathcal{D})$, *expected revenue denoted as* $\prod_{SeqCD}^{ID}$
3. $(\mathcal{C},\mathcal{D},\mathcal{C},\mathcal{C}),(\mathcal{D},\mathcal{C},\mathcal{C},\mathcal{C}),(\mathcal{D},\mathcal{D},\mathcal{C},\mathcal{C})$, *expected revenue denoted as* $\prod_{SeqDC}^{ID}$
4. $(\mathcal{D},\mathcal{D},\mathcal{D},\mathcal{D})$ *(all other 8 policies are equivalent to this policy), expected revenue denoted as* $\prod_{SeqDD}^{ID}$

# Type-Dependent Information Structure in Sequential Contests - Revenue Comparison

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# Type-Dependent Information Structure in Sequential Contests - Revenue Comparison

- $\prod_{SeqCC}^{ID} > \prod_{SeqCD}^{ID}$ $\forall \delta \in [1, +\infty)$
- $\prod_{SeqDC}^{ID} > \prod_{SeqDD}^{ID}$ $\forall \delta \in [1, +\infty)$
- In sequential setup, $(\mathcal{C}, \mathcal{C}, \mathcal{C}, \mathcal{C})$ is the optimal disclosure policy when $\delta \in [1, 4]$, and $(\mathcal{D}, \mathcal{D}, \mathcal{C}, \mathcal{C})$ (or an equivalent policy) is the optimal disclosure policy when $\delta \in [4, +\infty)$,

# Type-Dependent Information Structure in Simultaneous Contests

- A type-dependent information disclosure policy is a vector of 3 binary variables, each taking value $C$ or $D$, where 1st (respectively, 2nd, and 3rd) element corresponds to the disclosure policy choice in case of realization $(v_h, v_h)$ (respectively, $(v_h, v_l)$ or $(v_l, v_h)$, and $(v_l, v_l)$).

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- There are 9 combinations. However, some of them are not effective.

# Type-Dependent Information Structure in Simultaneous Contests

- A type-dependent information disclosure policy is a vector of 3 binary variables, each taking value $C$ or $D$, where 1st (respectively, 2nd, and 3rd) element corresponds to the disclosure policy choice in case of realization $(v_h, v_h)$ (respectively, $(v_h, v_l)$ or $(v_l, v_h)$, and $(v_l, v_l)$).
- There are 9 combinations. However, some of them are not effective.
- In simultaneous setup, Serena (2016) showed $(\mathcal{D}, \mathcal{C}, \mathcal{C})$ is the optimal disclosure policy $\forall \delta \in [1, +\infty)$!!

# Type-Independent Timing and Type-Dependent Disclosure - Optimal Policy

**Theorem**

*The optimal type-independent timing and type-dependent disclosure policy is the following:*

$$\begin{aligned} \mathbb{T} &= S, \quad \mathbb{P} = (\mathcal{D}, \mathcal{C}, \mathcal{C}) \quad \textit{if and only if } \delta \in [3.59, +\infty) \\ \mathbb{T} &= Seq, \mathbb{P} = (\mathcal{C}, \mathcal{C}, \mathcal{C}, \mathcal{C}) \quad \textit{if and only if } \delta \in [1, 3.59] \end{aligned}$$

Note that $\mathbb{T} = Seq, \mathbb{P} = (\mathcal{D}, \mathcal{D}, \mathcal{C}, \mathcal{C})$ is a strictly dominated policy!!

# Type-Dependent Timing Policy with Complete Information

- The policy $\mathbb{P}$ should consist of 3 parts, corresponding to 3 different combinations of bidding value, $(v_h, v_h)$, $(v_h, v_l)$ or $(v_l, v_h)$, $(v_l, v_l)$.
- Note that both sequential and simultaneous schemes generate identical revenue when bidders have the same realized value.
- Therefore the only part we need to consider about is what to do in $(v_h, v_l)$ or $(v_l, v_h)$ case.
- By Morgan (2003), we should let the bidder of $v_h$ move first.
- Thus, $(\mathcal{S}, \mathcal{H}, \mathcal{S})$ (or an equivalent policy $(\mathcal{S}, \mathcal{H}, \mathcal{L})/(\mathcal{H}, \mathcal{H}, \mathcal{L})/(\mathcal{H}, \mathcal{H}, \mathcal{S})$) is the optimal timing policy with complete information $\forall \delta \in [1, +\infty)$.

# Type-Dependent Timing Policy with Private Information

- There are in total $2 \times 3 \times 2 = 12$ policies
-

$$(\mathcal{S}, \mathcal{L}, \mathcal{S}), (\mathcal{S}, \mathcal{H}, \mathcal{S}), (\mathcal{H}, \mathcal{S}, \mathcal{L})$$
$$(\mathcal{S}, \mathcal{S}, \mathcal{S}), (\mathcal{S}, \mathcal{S}, \mathcal{L}), (\mathcal{S}, \mathcal{H}, \mathcal{L})$$
$$(\mathcal{S}, \mathcal{L}, \mathcal{L}), (\mathcal{H}, \mathcal{S}, \mathcal{S}), (\mathcal{H}, \mathcal{H}, \mathcal{S})$$
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# Type-Dependent Timing Policy with Private Information

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# Type-Dependent Timing Policy with Private Information

- There are in total $2 \times 3 \times 2 = 12$ policies
- $$(\mathcal{S},\mathcal{H},\mathcal{S})$$
$$(\mathcal{S},\mathcal{S},\mathcal{S}),(\mathcal{S},\mathcal{H},\mathcal{L})$$
$$(\mathcal{S},\mathcal{L},\mathcal{L}),(\mathcal{H},\mathcal{S},\mathcal{S}),(\mathcal{H},\mathcal{H},\mathcal{S})$$
$$(\mathcal{H},\mathcal{H},\mathcal{L}),(\mathcal{H},\mathcal{L},\mathcal{S}),(\mathcal{H},\mathcal{L},\mathcal{L})$$

# Type-Dependent Timing Policy and Type-Independent Disclosure - Optimal Policy

**Theorem**

*The following is an optimal type-dependent timing and type-independent disclosure policy :*

$$\begin{cases} \mathbb{T} = (\mathcal{H}, \mathcal{H}, \mathcal{S}),\ \mathbb{P} = C & \delta \in (1, 4] \\ \mathbb{T} = (\mathcal{S}, \mathcal{H}, \mathcal{S}),\ \mathbb{P} = D & \delta \in (4, 5.39) \\ \mathbb{T} = (\mathcal{H}, \mathcal{S}, \mathcal{S}),\ \mathbb{P} = C & \delta \in [5.39, +\infty) \end{cases}$$

# Type-Dependent Timing Policy and Type-Dependent Disclosure - Optimal Policy

**Theorem**

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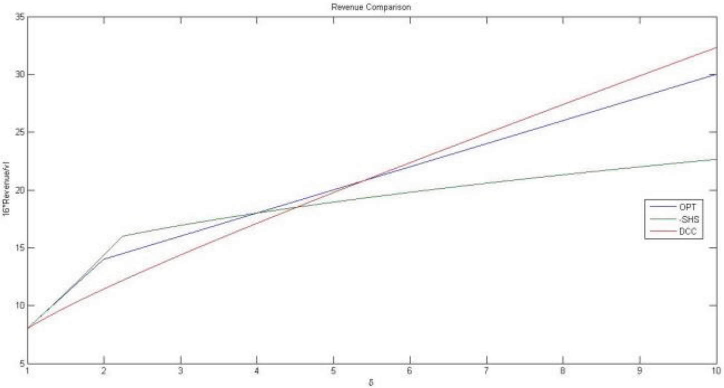
$$\begin{cases} \mathbb{T} = (\mathcal{H}, \mathcal{H}, \mathcal{S}),\ \mathbb{P} = C & \delta \in (1, 4] \\ \mathbb{T} = (\mathcal{S}, \mathcal{H}, \mathcal{S}),\ \mathbb{P} = D & \delta \in (4, 5.39) \\ \mathbb{T} = (\mathcal{H}, \mathcal{S}, \mathcal{S}),\ \mathbb{P} = C & \delta \in [5.39, +\infty) \end{cases}$$

Note that $\mathbb{T} = (\mathcal{H}, \mathcal{S}, \mathcal{S}), \mathbb{P} = C$ generates the same revenue as $\mathbb{T} = S, \mathbb{P} = (\mathcal{D}, \mathcal{C}, \mathcal{C})$.

# A Graphic Illustration



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# Summary

- We fully characterize the optimal joint design of timing policy and symmetric information disclosure policy in a Tullock contest of 2 players with a binary type distribution, considering the following scenarios:
  - Type-Independent Timing and Type-Independent Information Disclosure
  - Type-Independent Timing and Type-**Dependent** Information Disclosure
  - Type-**Dependent** Timing and Type-Independent Information Disclosure
  - Type-**Dependent** Timing and Type-**Dependent** Information Disclosure
- Our work generalizes the optimal timing results in Morgan (2003) and the optimal information disclosure results in Zhang and Zhou (2015), Wu and Zheng (2017) and Serena (2016).

# Work in Progress

- Individual Players' Incentive Analysis
- Social Welfare Comparison
- Optimal Design for Generalized (Asymmetric) Information Disclosure Policy (and Timing Policy)
- General Binary Type Distribution (Unequal Probabilities for Different Types)

# Thank You!