International Trade with Social Comparisons*

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ABSTRACT:
As consumers in countries around the world become increasingly aware of and sensitive to the products that their foreign counterparts consume, a natural question is what predictions do classic trade frameworks hold when incorporating social comparison-based preferences? We analyze this question in a general equilibrium framework for a two-country, two-good world, in which the gap between domestic and foreign consumption of a product can enter into the representative consumer’s utility function. We consider nine exhaustive social comparison scenarios, which differ based upon the combination and origin of products that consumers of each country hold a social comparison over. We show that Home Comparison preference (social comparison over the home-produced good) unilaterally brings consumption and welfare benefits to the home country, and global welfare is enhanced when both countries maintain such preferences. On the other hand, Foreign Envy (social comparison over the foreign-produced good) is disadvantageous to the home country, contributing negatively to welfare when the other country either prefers its own produced good or has no particular social preference. Mutual Foreign Envy however, tends to contribute positively to global welfare. Our analysis helps to explain the social welfare incentives of policymakers in promoting cross-country comparisons of domestic goods among their own consumers, while advocating domestically-produced goods as status symbols abroad.

KEYWORDS: International Trade, Social Comparison, Consumption Comparison, Social Reference Effects, General Equilibrium

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1 Introduction

Consumers around the world are increasingly aware of and sensitive to the goods and services that other consumers are purchasing. Exchange of information across national boundaries has grown easier due to increased international travel and world-wide internet access. For example, China’s consumers observe first-hand through experiences abroad or word-of-mouth, the quality and varieties of products available in the United States and in Europe. US consumers traveling abroad may admire the high-speed railway systems in China, Japan and Europe, while lamenting that this technology has so far not been implemented in the United States. The influence of the products used by consumers around the world is heightened by the prevalence of photos and discussions on social media about various products. As a result, the product demand by consumers in a country may be substantially influenced by the products that individuals in other countries consume.

We capture this phenomenon in a two-country, two-good general equilibrium model of international trade with potential social comparisons of consumption across countries. We allow residents of a country to have utility with a socially-based comparison of consumption over a basket of products produced at home and abroad. If consumers abroad consume more of a socially compared good than those consumers in the home country, home consumers experience a drop in utility compared with their utility of the absolute consumption of that good. On the other hand, if consumers abroad consume less of that socially compared good than those consumers in the home country, a positive additional utility ensues. We abstract away from the production of goods and directly consider the case where goods are assigned to countries as initial endowments. This facilitates our examination of the price effects of social comparison preferences across the initial distribution of consumption goods, without the potentially more complicated effects of production technologies and inputs to production.

The literature on international trade has primarily utilized classical specifications of consumer preferences which preclude the possibility of a regard for other individuals in the utility function. This may be surprising given that many political debates about appropriate trade policy center not only on efficiency concerns but emphasize fairness and distributive issues. Davidson, Matusz and Nelson (2006) comprehensively discuss the importance of fairness concerns in trade models and trade policy, and provide an example of merging fairness considerations into trade models by incorporating the inequity aversion preference of Fehr and Schmidt (1999) into the endogenous tariff model of Mayer (1984). Brown...
and Stern (2006) discuss international trade agreements from the perspective of fairness considerations, including equality of opportunity and distributive equity between countries, arguing that these fairness concerns may be important for maintaining long-run trade relationships. From the perspective of social preferences towards global laborers, Dragusanu, Giovannucci and Nunn (2014) discuss the market for Fair Trade products, which aims to provide socially acceptable wages and working conditions to manufacturing workers. Compared to our current study, Brown and Stern (2006) focuses mostly on trade policy negotiations through WTO/GATT and fairness concerns among countries, while Dragusanu, Giovannucci and Nunn (2014) focus on the issue of workers working and living conditions. By contrast, our model focuses on the consumers side of the market, consider the outcome in an endowment economy where consumers of either country may have a consumption-based social preference.

A related growing literature examines the relationship between international trade and income distribution. Fajgelbaum, Grossman and Helpman (2011) provide a model with income distributions in each country, and derive the production that richer countries net export high quality goods and net import lower quality goods. This approach differs from ours in that although it models income inequality through the income distribution, it avoids specifying any other-regarding preference in the modeling approach. Finally, in a paper whose approach may be closer to ours, Hosseini (2005) uses concepts in behavioral economics to explain foreign direct investment decisions.

Our paper contributes to the literature which seeks to apply behavioral economics concepts grounded in psychological regularities to practical macroeconomic problems of policy relevance. A classic example of the application of behavioral features in macroeconomics is the seminal paper Laibson (1997) which applies a version of hyperbolic discounting (i.e. quasi-hyperbolic) to the dynamic asset allocation and consumption problem, showing that hyperbolic discounting households find a valuable commitment device in illiquid assets. In the behavioral economics domain, our study draws from the large literature on social preferences. Fehr and Schmidt (1999) famously proposed that individuals have an intrinsic preference for evenly distributed outcomes, known as inequity aversion. Subsequent models of social preferences proposed alternative specifications of how decision-makers value the distribution of payoffs and procedures for distributing payoffs among individuals. A survey of modeling approaches in the social preference literature is provided by Sobel (2005).

Our approach proposes an additional gain/loss term appended to the comparison good in question in the consumption utility function. This loss term provides an additional premium.
when home consumption of a good exceeds foreign consumption of the good, and provides a penalty when foreign consumption exceeds home consumption. The premium/penalty term serves as an effective bonus/reduction in the actualized quantity of the good actually enjoyed. In this sense, our model is also related to the gain-loss utility proposed by Koszegi and Rabin (2006)\textsuperscript{10} which appends a Prospect Theory (Kahneman and Tversky, 1979\textsuperscript{11}) style loss aversion utility term to the standard consumption utility.\textsuperscript{1} Our model differs from theirs in that the gain-loss term is embedded in the consumption term for the good in question, and maintains a symmetric valuations of losses and gains relative to the other country's consumption which serves as the reference point. Our preference specification follows Ghiglino and Goyal (2010)\textsuperscript{12}, Alexeev and Chih (2015)\textsuperscript{13}, Immorlica, Kranton, Manea, and Stoddard (2017)\textsuperscript{14} and Feng, Lien and Zheng (2018)\textsuperscript{15}, which is a commonly used functional form in the network and social comparison literature. Finally, we note that our paper utilizes a behavioral model which lies at the intersection of the social preference and reference-dependence models, and is thus related to a small but growing behavioral economics literature at this intersection, including Kuziemko, Buell, Reich and Norton (2014)\textsuperscript{16}, Schwerter (2015)\textsuperscript{17}, and Lien and Zheng (2015b)\textsuperscript{18}.

We analyze several cases that vary based on which countries have social comparison preferences over which good(s), and compare it to the standard model in the absence of social comparisons. An introduction to the cases we consider is as follows:

First, we consider the case that we refer to as Common Comparison, in which consumers in both countries have a social comparison of a product in common, while the other product in the global market does not induce any social comparison for either country’s consumers. This captures situations where products are asymmetric in terms of their ability to induce social comparisons. For example, other countries’ consumption of socks may not inspire social comparison, while the consumption of cellular phones may more naturally inspire such comparisons.

Next, we consider the case we refer to as Home Comparison, in which a single country has a social comparison preference over the good that they hold the higher initial endowment in, while the other country has no social comparison preferences. In other words, residents of one country are treating their home produced good as a type of status symbol which they prefer to consume more of compared to other countries, while the other country does not share this view. An example might be the Chinese hard liquor Moutai, which is currently...
considered prestigious and high quality domestically, but has yet to become a very popular drink globally.

Continuing with the case of only a single country holding a social comparison preference, we consider the case of Foreign Envy, in which a country has a social comparison preference on the good that the other country has the higher initial endowment of, while the other country has no social comparison preference over either good. An example may be decades ago in China, when technological products from the US such as automobiles and home appliances were admired and desirable due to awareness and prevalence of these technologies in the consumption bundle of the typical US household.

Dual Home Comparison is the case in which each country has a social comparison preference over their own home endowed good. In other words, consumers in China want to have more Moutai than Americans do, while US consumers want to consume more of the US liquor Bourbon whiskey than Chinese consumers do. The opposite case of Mutual Foreign Envy is when each country has a social comparison preference over the other country’s home endowed good. For example, people in China want to consume at least as much Bourbon Whiskey as consumers in the US do, while US consumers want to consume equal or more Moutai than their Chinese counterparts. A potentially more intuitive example of Mutual Foreign Envy is that Americans want to have high-speed rail like China does, while people in China want to have spacious houses with backyards like many American consumers do.

One-sided Comparison preference is the case where a single country holds a social comparison on both goods in the global market, while the other country has no social comparison preference. For example, as a developing country, consumers in China may care a great deal about how their consumption basket compares to a developed country such as the US, while US consumers may not naturally make any comparisons between themselves and consumers in China.

We refer to the next two cases as Asymmetric Dual Home Comparison and Asymmetric Mutual Foreign Envy. In each of these cases, one country has a comparison preference over both goods in the global market, while the other country holds a comparison preference only over one particular good. In the case of Asymmetric Dual Home Comparison, that good is the one which that country holds the higher endowment in. In the case of Asymmetric Mutual Foreign Envy, that good is the one which the other country has the higher endowment in. For example, in the case of Asymmetric Dual Home Comparison, consumers in China hold comparison preferences over both goods in the global market, while US consumers hold a comparison preference over home-produced Hollywood movies. An example of Asymmet-
ric Mutual Foreign Envy, is that consumers in China may hold comparison preferences over both goods in the global market, while US consumers hold a comparison preference over native Chinese cuisine.

The final case we consider is Ubiquitous Comparison. In this case, both countries have comparison preferences over both goods in the global market; in other words, each country is comparing the entire basket of goods consumed with the basket consumed in the other country.

To briefly summarize the results, we find that for individual countries, holding a social comparison preference over their home produced good can unilaterally improve welfare in that country. In fact, the improvement in welfare holds for each country even if both countries are to hold such social comparison preference over their home-produced good. By contrast, holding a social comparison preference over a foreign produced good unilaterally reduces welfare in a given country. However, if both countries have such a preference over foreign goods, the welfare of both countries is enhanced. This asymmetry in social comparison effects between home produced and foreign produced goods highlights the relative incentives that policy-makers have to cultivate a home-based relative consumption preference compared to a foreign-based relative consumption preference.

The remainder of the paper proceeds as follows: In Section 2, we introduce the general modeling framework without social comparisons and establish the baseline results; In Section 3, we consider the nine cases of international trade with social comparisons as discussed above; In Section 4 we conduct comparisons across the different cases and summarize our main findings; In Section 5 we conclude and discuss the policy implications.

## 2 Model Setup

Suppose there are two countries in the world: $L$ and $Z$. We have two goods $a$ and $b$, say apples and bananas. The respective consumption quantities of each good are denoted by $a_L$ and $b_L$ for country $L$, and $a_Z$ and $b_Z$ for country $Z$. The two countries each have different endowments $(e^L_a, e^L_b)$ and $(e^Z_a, e^Z_b)$. We suppose that the amount of endowment of $a$ in country $L$ is more than their endowment of good $b$, meaning $e^L_a > e^L_b$. On the other hand, we suppose that $e^Z_a < e^Z_b$ for country $Z$. We refer to good $a$ as the home good of country $L$, and good $b$ as the home good of country $Z$. Good $a$ is thus the foreign good of country $Z$, and good $b$ is the foreign good of country $L$.

For countries $L$ and $Z$, we assume that the representative consumer’s utility function,
and thus the aggregate utility of consumers as a whole are represented by:

\[ U_L(a_L, b_L, a_Z, b_Z) \] (1)
\[ U_Z(a_L, b_L, a_Z, b_Z) \] (2)

For country \( L \), consumption is subject to the constraint:

\[ P_a a_L + P_b b_L = P_a e_a^L + P_b e_b^L \] (3)

Similarly, for country \( Z \), consumption is subject to the constraint:

\[ P_a a_Z + P_b b_Z = P_a e_a^Z + P_b e_b^Z \] (4)

where \( P_a \) and \( P_b \) are the prices of the two goods. To simplify our derivation process, we normalize \( P_b = 1 \), and the constraint expressions now become:

\[ P_a a_L + b_L = P_a e_a^L + e_b^L \] (5)
\[ P_a a_Z + b_Z = P_a e_a^Z + e_b^Z \] (6)

For simplification, we invoke the following normalizations: \( e_a^L + e_a^Z = 1; \ e_b^L + e_b^Z = 1. \)

### 2.1 Baseline Case

First we consider the simple case where both countries consume without any social comparisons of consumption goods. To optimize the representative consumer’s Cobb-Douglas utility for country \( L \), we have

\[ \max u_L = a_L b_L \]
\[ \text{s.t. } P_a a_L + b_L = P_a e_a^L + e_b^L \] (7)

Likewise, for country \( Z \),

\[ \max u_Z = a_Z b_Z \]
\[ \text{s.t. } P_a a_Z + b_Z = P_a e_a^Z + e_b^Z \] (8)

Using the first-order conditions, we obtain the results for quantities and price in the
baseline case as follows:

\[
\begin{align*}
    a_L &= \frac{e_a^L e_b^Z - e_a^Z e_b^L + 2e_b^L}{2} \\
    a_Z &= \frac{e_a^Z e_b^L - e_a^L e_b^Z + 2e_b^Z}{2} \\
    b_L &= \frac{e_a^Z e_b^L - e_a^L e_b^Z + 2e_a^L}{2} \\
    b_Z &= \frac{e_a^L e_b^Z - e_a^Z e_b^L + 2e_a^Z}{2} \\
    P_a &= 1
\end{align*}
\]

In subsequent cases with social comparison preferences, we will consider a scenario of extreme initial distribution of endowments for welfare analysis comparison purposes. In other words, we will consider the scenario that country \( L \) is solely endowed with good \( a \) and country \( Z \) is solely endowed with good \( b \). In the baseline model, by setting \( e_a^L = 1, e_b^L = 0, e_a^Z = 1, e_b^Z = 0 \), we obtain \( a_L = \frac{1}{2}, a_Z = \frac{1}{2}, b_L = \frac{1}{2}, b_Z = \frac{1}{2}, P_a = 1 \).

In other words, in this scenario, the prices of both goods are the same. In addition, both countries consume the same amounts of each good and are equally well off. Finally, we note that each country’s welfare is given by the realized utility of consumptions of each good, \( \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \). We use this baseline result as a comparison for our subsequent results when consumers have consumption comparisons with other countries’ consumers.

### 2.2 Social Comparisons

We now turn to our main analysis of interest, which allows consumers to have social comparison-based preferences with respect to other countries’ consumption. In our framework, the fundamental difference between countries is in how their endowment of traded goods compares to the other country’s endowment. Simultaneously, consumers in those countries may have social comparison preferences over one or both goods traded on the global market. This yields nine combinations for us to consider in terms of goods endowments and preference specifications, without loss of generality. These cases are summarized for convenience in Table 1.

\[\text{Note that in theory there should be in total } 2^4 - 1 = 15 \text{ cases in addition to the baseline case. That is because for each of the cases } 1, 2, 3, 6, 7, 8, \text{ there exists another symmetric case. Without loss of generality, for those cases that have symmetric cases, we only consider one of them.}\]
Table 1: Preference Comparisons

<table>
<thead>
<tr>
<th>Case Description</th>
<th>Country L’s comparison</th>
<th>Country Z’s comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline Case: No Comparison</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Case 1: Common Comparison</td>
<td>Good a</td>
<td>Good a</td>
</tr>
<tr>
<td>Case 2: Home Comparison</td>
<td>Good a</td>
<td>None</td>
</tr>
<tr>
<td>Case 3: Foreign Envy/Admiration</td>
<td>Good b</td>
<td>None</td>
</tr>
<tr>
<td>Case 4: Dual Home Comparison</td>
<td>Good a</td>
<td>Good b</td>
</tr>
<tr>
<td>Case 5: Mutual Foreign Envy/Admiration</td>
<td>Good b</td>
<td>Good a</td>
</tr>
<tr>
<td>Case 6: One-sided Comparison</td>
<td>Goods a and b</td>
<td>None</td>
</tr>
<tr>
<td>Case 7: Asymmetric Mutual Foreign Envy</td>
<td>Goods a and b</td>
<td>Good a</td>
</tr>
<tr>
<td>Case 8: Asymmetric Dual Home Comparison</td>
<td>Goods a and b</td>
<td>Good b</td>
</tr>
<tr>
<td>Case 9: Ubiquitous Comparison</td>
<td>Goods a and b</td>
<td>Goods a and b</td>
</tr>
</tbody>
</table>

3 Cases with Social Comparison

We introduce the notion of cross-country social comparisons using a slight modification of the standard utility function in the baseline model. Let the parameter $\lambda_1$ represent the social comparison weight of good $a$ in the two countries and $\lambda_2$ represent the social comparison weight over good $b$. For example, in the utility function with social comparison, the country which holds such preference over good $a$ (without loss of generality) will evaluate the gap between home consumption of good $a$ and foreign consumption of good $a$. This gap, weighted by parameter $\lambda_1$ is added from the actual consumption term for good $a$. This modeling approach for social comparison borrows exactly from Ghiglino and Goyal (2010) [15], Alexeev and Chih (2015) [18], Immorlica, Kranton, Manea, and Stoddard (2017) [17], and Feng, Lien and Zheng (2018) [18], and is a widely accepted and tractable approach for modeling such preferences.

We now introduce the analysis for the nine scenarios summarized in Table 1.

3.1 Case 1: Common Comparison

The case of common comparison refers to the scenario where consumers of different countries have preferences with a social comparison on a common good, which can be either good $a$ or good $b$. Without loss of generality, suppose representative consumers in countries $L$ and $Z$ have preferences with a social comparison on good $a$. We apply the modified social comparison utility specification to describe this situation.
For country $L$, 
\[
\text{max } u_L = (a_L + \lambda_1(a_L - a_Z))b_L \\
\text{s.t. } P_a a_L + b_L = P_a e_a^L + e_b^L 
\] (9)

For country $Z$, 
\[
\text{max } u_Z = (a_Z + \lambda_1(a_Z - a_L))b_Z \\
\text{s.t. } P_a a_Z + b_Z = P_a e_a^Z + e_b^Z 
\] (10)

Due to the lengthy algebraic expressions for prices and consumption, for each case going forward we relegate these precise derived results to Appendix A. Instead, we focus on providing discussion of the results for particular numerical endowment scenarios to understand how the mechanics of the social comparison preference operates in the model.

Recall that our general model set up specified merely which country has the greater endowment of any particular good, compared to the other country. To simplify expressions and obtain concrete results, as in the baseline model above, we consider the extreme cases where each country is solely endowed with their home good. In other words, we focus on the scenario where country $L$ has full endowment of good $a$ while country $Z$ has no endowment of good $a$, and country $Z$ has full endowment of good $b$ while country $L$ has no endowment of good $b$. In addition, for simplicity, we assume that the intensity of social comparison, represented by parameter $\lambda$ (if such comparison exists) is equal across the two goods.

Setting $e_a^L = 1$, $e_b^L = 0$, $e_b^Z = 1$, $e_a^Z = 0$, and $\lambda_1 = \lambda$, we obtain the following expressions for consumption and relative price (compared on the right hand side to the baseline case without social comparison):

\[
a_L = \frac{1 + 2\lambda}{2 + 3\lambda} > \frac{1}{2} \\
b_L = \frac{(1 + \lambda)^2}{2 + 3\lambda} > \frac{1}{2} \\
a_Z = \frac{1 + \lambda}{2 + 3\lambda} < \frac{1}{2} \\
b_Z = \frac{1 + \lambda - \lambda^2}{2 + 3\lambda} < \frac{1}{2} \\
P_a = 1 + \lambda > 1
\]

In this case of Common Comparison, we have reached three main findings when compar-
ing to the baseline case of no social comparisons. As may be expected, the good which both countries have a social comparison over becomes more expensive in the global market. In addition, in terms of consumption outcomes, the country which is fortunate to be endowed with that common socially compared good consumes more of both goods, while the other country consumes less of both goods. In terms of the welfare outcome, it immediately follows that the country with the endowment of the common socially compared good becomes better off compared to the baseline case, while the other country becomes worse off. These results are summarized as follows:

**Observation 1** Compared with the Baseline case, the Common Comparison case exhibits the following patterns in equilibrium:

1. Prices: The common socially compared good becomes more expensive.
2. Consumption: The country whose home (foreign) good is the common socially compared good consumes more (less) of both goods.
3. Welfare: The country whose home (foreign) good is the common socially compared good is better (worse) off.

The above results imply that countries in the world economy have incentive to market and develop their home endowed product as a social comparison good around the world. In the real world, in practice, it can be very difficult to cleanly observe or quantify such effects without interference from various factors outside of our model, such as government stability, trade policies, and so on. Our analysis indicates that if all else were held equal, countries enjoy consumption and welfare gains from producing an international status product. For example, through the common comparison phenomenon, the US enjoys such welfare gains for Apple products, Italy and France for their home produced designer brand items, as well as Germany and Japan for their renowned home appliances.

### 3.2 Case 2: Home Comparison

While the previous case considered the scenario that both countries have social comparisons over a particular good, we now consider the case that a single country has a social comparison preference over its own endowed good, while the other country has no social comparison, deriving utility only from the actual amount of products consumed. Note that in Case 4, we
consider the scenario that both countries have social comparisons over their home endowed good, but we first consider the current asymmetric case.

Suppose consumers in country $L$ have preferences with social comparison on good $a$ while country $Z$ does not have any such preference.

For each country, the consumer’s utility maximization problem is described as follows:

For country $L$,

$$\max u_L = (a_L + \lambda_1 (a_L - a_Z)) b_L$$

$$s.t. \ P_a a_L + b_L = P_a e_a^L + e_b^L$$

For country $Z$,

$$\max u_Z = a_Z b_Z$$

$$s.t. \ P_a a_Z + b_Z = P_a e_a^Z + e_b^Z$$

The exact closed-form expressions for quantities and prices are again relegated to the Appendix A due to length concerns. Here, we once again consider the extreme allocation of initial endowments and equal intensity of social comparison across the two goods to obtain the intuition of the results.

Setting $e_a^L = 1, e_b^L = 0, e_a^Z = 1, e_b^Z = 0, \lambda_1 = \lambda$, we obtain the following consumption and price values, with comparison to the baseline case on the right hand side.

$$a_L = \frac{2\lambda + 1}{3\lambda + 2} > \frac{1}{2}$$

$$b_L = \frac{1}{2}$$

$$a_Z = \frac{\lambda + 1}{3\lambda + 2} < \frac{1}{2}$$

$$b_Z = \frac{1}{2}$$

$$P_a = \frac{3\lambda + 2}{2(\lambda + 1)} > 1$$

The results show that good $a$, the home good of the country with comparison preferences over that good, becomes more expensive. In terms of consumption, the home country consumes more of that socially compared home good, while the foreign country consumes less of it. Consumption of the non-socially compared good remains the same as in the

\[3\text{Note that for the case of Home Comparison there also exists an analogous scenario where consumers in country Z have social comparison preferences on good b while country L does not have any such preference.}\]
baseline case for both countries. As a result, the country whose endowed good drives its own social comparison preference experiences a welfare increase, while the other country experiences a welfare decrease. The findings are summarized as follows:

**Observation 2** Compared with the Baseline case, the Home Comparison case exhibits the following patterns in equilibrium:

1. **Prices:** The socially compared home good becomes more expensive.

2. **Consumption:** The home (foreign) country consumes more (less) of the socially compared home good, and both countries consumption of the other good is unchanged.

3. **Welfare:** The home (foreign) country is better (worse) off.

The results for Home Comparison raise an interesting policy insight. That is, welfare in the home country can be unilaterally increased if a social comparison preference over the home good can simply be cultivated domestically. In other words, if US consumers develop a global comparison preference over the consumption of Hollywood movies due to marketing and advertising, they can actually become better off in the world market, even if other countries’ consumers are not having any social comparisons over movies consumed. Governments may thus find it advantageous to highlight the benefits of their domestically supplied product, especially if this product is not as readily available in other countries. As another example, China may highlight the widespread availability of fast home delivery and shared bicycle service in its major cities, services which are not currently as abundant in countries such as the US. By encouraging a comparison with consumers abroad, consumption and welfare of the consumers in China can be raised.

### 3.3 Case 3: Foreign Envy/Admiration

Continuing the analysis of scenarios in which only one country has a social comparison preference, we now analyze the case of Foreign Envy/Admiration, which is when that country has a social comparison preference on the foreign good while the other country has no social comparison preference. Note that later on in Case 5 we will consider the case of Mutual Foreign Envy.

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*In our analysis, we do not differentiate the source of such a preference of social comparison, which can be due to either a negative feeling like envy, a positive feeling like admiration, or some combination of the two. In the remainder of the paper, for convenience we simply refer to the word ”envy” as a representation of this social comparison.
Suppose country $L$ has preferences with social comparison on good $b$ and country $Z$ does not have any such social comparison preference. The representative consumer’s problem is as follows:

For country $L$,

$$\max u_L = a_L(b_L + \lambda_2(b_L - b_Z))$$

s.t. $P_a a_L + b_L = P_a e_a^L + e_b^L$ \hspace{1cm} (13)

For country $Z$,

$$\max u_Z = a_Z b_Z$$

s.t. $P_a a_Z + b_Z = P_a e_a^Z + e_b^Z$ \hspace{1cm} (14)

The exact expressions for quantities and prices can be found in Appendix A, due to length concerns. We once again consider the simplified case of extreme endowment allocations and homogeneous intensity of social comparison.

Setting $e_a^L = 1$, $e_b^L = 0$, $e_b^Z = 1$, $e_a^Z = 0$, $\lambda_2 = \lambda$, we obtain the following expressions:

$$a_L = \frac{1}{\lambda + 2} < \frac{1}{2}$$

$$b_L = \frac{1}{2}$$

$$a_Z = \frac{\lambda + 1}{\lambda + 2} > \frac{1}{2}$$

$$b_Z = \frac{1}{2}$$

$$P_a = \frac{\lambda + 2}{2(\lambda + 1)} < 1$$

The results show that Foreign Envy is in some sense the opposite of Home Comparison. In this case, it is the socially compared foreign good which becomes more expensive, and the consumption and welfare loss is at the expense of the country with the social comparison preference over that foreign good. Despite the fact that the consumption of the socially compared good remains the same as in the baseline case, the home country is made worse.

\footnote{Note that for the case of Foreign Envy there also exists an analogous scenario where consumers in country $Z$ have preferences with social comparison on good $a$ while country $L$ does not have any such preference.}

\footnote{It is worth mentioning that Foreign Envy is not the mirror image of Home Comparison, because they are opposite only in terms of direction of the results with respect to the Baseline case, not in terms of magnitude.}
off, while the foreign country benefits. The findings are summarized as follows:

**Observation 3** Compared with the Baseline case, the Foreign Envy case exhibits the following patterns in equilibrium:

1. **Prices:** The socially compared foreign good becomes more expensive.

2. **Consumption:** The home (foreign) country consumes less (more) of the non-socially compared home good, and both countries consumption of the other good is unchanged.

3. **Welfare:** The home (foreign) country is worse (better) off.

This result highlights the incentive that domestic policy makers have to reduce admiration of foreign products. Social comparison of products originating in foreign countries alone serves to increase the relative price of that good, bestowing greater purchasing power upon the foreign country. Correspondingly, the foreign country consumes more of the good produced by the admiring country, while the amount of the admired good consumed has no change compared to the baseline result. The home country, which admired the product originating from abroad, is worse off. In other words, fear of social comparison preference over a foreign produced good could lead to the encouragement of inward-looking attitudes and skepticism about foreign products.

### 3.4 Case 4: Dual Home Comparison

We now revisit the concept of Home Comparison as in Case 2, but apply it to both countries. In Dual Home Comparison, each country has a comparison preference over its own home good, but holds no comparison preference over the other country’s endowed good.

Suppose country $L$ has preferences with social comparison over good $a$, and country $Z$ has preference with social comparison on good $b$. The maximization problems of the two countries are as follows:

For country $L$,

$$\max u_L = (a_L + \lambda_1(a_L - a_Z))b_L$$

subject to 

$$P_a a_L + b_L = P_a e^L_a + e^L_b \quad (15)$$

For country $Z$,

$$\max u_Z = a_Z(b_Z + \lambda_2(b_Z - b_L))$$
\[ s.t. \ P_a a_Z + b_Z = P_a e^Z_a + e^Z_b \] 

While the exact expressions for quantities consumed and prices can be found in the Appendix A, we again highlight the result for the scenario of extreme endowments and homogeneous intensity of social comparison, and expect that qualitatively similar results can also be found under less extreme allocations.

Setting \( e^L_a = 1, \ e^L_b = 0, \ e^Z_b = 1, \ e^Z_a = 0, \ \lambda_1 = \lambda_2 = \lambda \), we obtain the following expressions:

\[
\begin{align*}
a_L &= \frac{2\lambda + 1}{3\lambda + 2} > \frac{1}{2} \\
b_L &= \frac{\lambda + 1}{3\lambda + 2} < \frac{1}{2} \\
a_Z &= \frac{\lambda + 1}{3\lambda + 2} < \frac{1}{2} \\
b_Z &= \frac{2\lambda + 1}{3\lambda + 2} > \frac{1}{2} \\
P_a &= 1
\end{align*}
\]

Compared to the baseline case with no social comparisons, the case of Dual Home Comparison has no net effect on the relative prices of the goods. This is intuitive since our example under this case leaves the economy symmetric in endowments and corresponding preferences. In terms of consumption outcomes, each country consumes more of their home good, which is serving as the socially compared product, and less of the other country’s endowed good. However, compared to the baseline case of no social comparison preferences, each country is better off, because each country consumes more of the good with social comparison and the utility gain from social comparison dominates the utility loss from uneven consumption of the two goods. The proof of the welfare analysis is included in the Appendix B1. In summary, for the case of Dual Home Comparison, we have the following findings.

**Observation 4** Assuming \( \lambda_1 = \lambda_2 \), compared with the Baseline case, the Dual Home Comparison case exhibits the following patterns in equilibrium:

1. Prices: The prices of both goods are unaffected.
2. Consumption: Both countries consume more of their home good and less of the foreign good.
3. Welfare: Both countries are better off.

The case of Dual Home Comparison raises the policy implication that cultivating preferences which focus only on the domestically produced good brings welfare benefits. The analysis of Home Comparison indicated that governments have incentive to unilaterally encourage a social comparison over the home good, and the current case shows that if both countries implement this approach, each country also benefits compared to the Baseline case. Thus, from the perspective of global policy makers, cultivating domestically-produced goods as status symbols in their home country can be arguably interpreted as a type of dominant strategy.

3.5 Case 5: Mutual Foreign Envy/Admiration

Our next case is the exact opposite of Dual Home Comparison. Each country holds a comparison preference over the good that the other country is heavily endowed with, which is the symmetric version of Foreign Envy.

We suppose that country $L$ has preference with social comparison on good $b$ while country $Z$ has preference with social comparison on good $a$. The utility maximization problems are specified as follows:

For country $L$,

$$\text{max } u_L = a_L(b_L + \lambda_2(b_L - b_Z))$$
$$s.t. \ P_a a_L + b_L = P_a e_a^L + e_b^L$$

(17)

For country $Z$,

$$\text{max } u_Z = (a_Z + \lambda_1(a_Z - a_L))b_Z$$
$$s.t. \ P_a a_Z + b_Z = P_a e_a^Z + e_b^Z$$

(18)

We examine the case of extreme endowments and homogeneous intensity of social comparison, while the general solution is provided in Appendix A.

Setting $e_a^L = 1, e_b^L = 0, e_b^Z = 1, e_a^Z = 0, \lambda_1 = \lambda_2 = \lambda$ we obtain the following expressions:

$$a_L = \frac{\lambda + 1}{3\lambda + 2} < \frac{1}{2}$$

$$b_L = \frac{2\lambda + 1}{3\lambda + 2} > \frac{1}{2}$$

17
\[ a_Z = \frac{2\lambda + 1}{3\lambda + 2} > \frac{1}{2} \]
\[ b_Z = \frac{\lambda + 1}{3\lambda + 2} < \frac{1}{2} \]

\[ P_a = 1 \]

The result bears some similarity to the case of Dual Home Comparison. Since the world economy is again symmetric with respect to preferences and endowments, prices remain unaffected by the social comparison preference compared to the baseline case. Differing from the case of Dual Home Comparison, each country consumes less of their home endowed good, and more of the other country’s endowed good. Finally, both countries are left better off than in the baseline case, similarly to the case of Dual Home Comparison, and for similar reasons.

The results can be summarized as follows.

**Observation 5** Assuming \( \lambda_1 = \lambda_2 \), compared with the Baseline case, the Mutual Foreign Envy case exhibits the following patterns in equilibrium:

1. **Prices:** The price of both goods are unaffected.

2. **Consumption:** Both countries consume less of their home good and more of the foreign good.

3. **Welfare:** Both countries are better off.

The analysis demonstrates that while one-sided Foreign Envy reduces the welfare of the home country and enhances the welfare of the foreign country compared to the baseline case, in the case of Mutual Foreign Envy, both countries experience welfare gains compared to the Baseline case. Each country consumes less of their home-produced good and more of the foreign-produced good, and there is no conflict of interest in the welfare implication of this consumption allocation due to the comparison preference being placed on different goods by each country.

### 3.6 Case 6: One-sided Comparison

After observing so far that symmetric forms of social comparison preferences tend to increase welfare, we now turn to examining the other asymmetric cases. Here, we consider the
scenario in which one country has social comparison preferences over both goods (the home good and the foreign good), while the other country simply does not care about social comparisons over any good. This may correspond to a case where one country is sensitive to global comparisons, while the other country has a more domestically focused outlook.

Suppose country \( L \) has preferences with social comparison on both of the goods but country \( Z \) has preferences with no social comparisons. Each country’s maximization problem as follows:

For country \( L \),

\[
\max u_L = (a_L + \lambda_1(a_L - a_Z))(b_L + \lambda_2(b_L - b_Z))
\]
\[\text{s.t. } P_a a_L + b_L = P_a e^L_a + e^L_b \tag{19}\]

For country \( Z \),

\[
\max u_Z = a_Z b_Z
\]
\[\text{s.t. } P_a a_Z + b_Z = P_a e^Z_a + e^Z_b \tag{20}\]

Once again setting \( e^L_a = 1, e^L_b = 0, e^Z_b = 1, e^Z_a = 0, \lambda_1 = \lambda_2 = \lambda \), to address the case of extreme endowments and symmetric social comparison weights, we obtain the following expressions:

\[
a_L = \frac{1}{2}
\]
\[
b_L = \frac{1}{2}
\]
\[
a_Z = \frac{1}{2}
\]
\[
b_Z = \frac{1}{2}
\]
\[
P_a = 1
\]

The result is identical to the baseline case without social comparisons. What this case demonstrates is that it is ineffectual in the global marketplace for only one country to be symmetrically socially sensitive to both goods, while the other country only cares about its own actual consumption. In this case, the social comparison preference tends to cancel

\[\text{Note that for the case of One-sided Comparison there also exists another scenario where consumers in country } Z \text{ have preferences with social comparison on both of the good while country } L \text{ has preference with no social comparison.}\]
itself through its identical weighting on both goods, since the country with such preference is budget constrained and can do no better for itself by reallocating its purchases.

The implications in this case are summarized as follows.

**Observation 6** Assuming \( \lambda_1 = \lambda_2 \), compared with the Baseline case, the One-sided Comparison case exhibits the following patterns in equilibrium:

1. **Prices**: The prices of goods are unaffected.

2. **Consumption**: Countries consume the same amounts of each good as in the baseline case.

3. **Welfare**: Welfare is unaffected compared to the baseline case.

### 3.7 Case 7: Asymmetric Mutual Foreign Envy

Another form of potential asymmetry is when both countries have some form of social comparison preference, but this preference is not symmetric across countries. In this case, one country has a social comparison over both goods on the global market, but the other country only has a social comparison over its foreign endowed good. This corresponds to a case that one country is especially globally aware of other countries’ consumption baskets, but the other country is mainly concerned about how their consumption compares with respect to a product from abroad.

When country \( L \) has a preference with social comparison on both goods and country \( Z \) has a preference with social comparison only on good \( a \) the maximization problems are as follows\(^9\)

For country \( L \),

\[
\text{max } u_L = (a_L + \lambda_1(a_L - a_Z))(b_L + \lambda_2(b_L - b_Z)) \\
\text{s.t. } P_a a_L + b_L = P_a e^L_a + e^L_b 
\]

(21)

For country \( Z \),

\[
\text{max } u_Z = (a_Z + \lambda_1(a_Z - a_L))b_Z 
\]

\(^9\)Note that for the case of Asymmetric Mutual Foreign Envy there also exists another scenario where consumers in country \( Z \) have preferences with social comparison on both of the good while country \( L \) has preference with social comparison only on good \( b \).
\[ s.t. \ P_a a_Z + b_Z = P_a e_a^Z + e_b^Z \] 

Under the assumptions of extreme endowments and symmetric social comparison weights across goods, we set \( e_a^L = 1, e_b^L = 0, e_a^Z = 1, e_b^Z = 0, \lambda_1 = \lambda_2 = \lambda \), and obtain the following expressions\(^\text{[10]}\):

\[
\begin{align*}
a_L &= \frac{(\lambda + 1)^2}{\lambda^2 + 4\lambda + 2} > \frac{1}{2} \\
 b_L &= \frac{2\lambda + 1}{-\lambda^2 + 3\lambda + 2} > \frac{1}{2} \\
 a_Z &= \frac{2\lambda + 1}{\lambda^2 + 4\lambda + 2} < \frac{1}{2} \\
 b_Z &= \frac{-\lambda^2 + \lambda + 1}{-\lambda^2 + 3\lambda + 2} < \frac{1}{2} \\
 P_a &= \frac{\lambda^2 + 4\lambda + 2}{-\lambda^2 + 3\lambda + 2} > 1
\end{align*}
\]

Note that \( b_L > 0, b_Z > 0, P_a > 0 \) require that \( \lambda < \frac{1}{2} (\sqrt{5} + 1) \). In other words, to ensure a solution with positive quantities and prices, the degree of social comparison cannot be too large.

Here, we refer to good \( a \) as the commonly compared good, since both countries have socially comparative preferences over it. As in the case of Common Comparison (Case 1), the price of the common socially compared good increases. The result also indicates that as may be expected, the consumption levels increase for both goods by the country with comparative preferences over both goods (country \( L \)), which in this case is also the country with the endowment of the commonly compared good. Here, country \( L \) benefits via higher income due to its endowment of the universally socially valued good in the market, enabling purchase of more goods overall. At the same time, this country has social comparisons over both goods in the market, driving greater consumption of each, accompanied by higher welfare. The other country, which has purely Foreign Envy preferences consumes less of both goods and is worse off in terms of realized welfare. The above findings can be summarized as follows:

**Observation 7** Assuming \( \lambda_1 = \lambda_2 \), compared with the Baseline case, the case of Asymmetric Mutual Foreign Envy exhibits the following patterns in equilibrium:

1. **Prices:** The commonly compared good becomes more expensive.

\(^\text{[10]}\)We focus on equilibrium with positive price and positive consumption on both goods by restricting the range of \( \lambda \) as \((0, \frac{1 + \sqrt{5}}{2})\).
2. Consumption: The country whose home (foreign) good is the commonly compared good consumes more (less) of both goods.

3. Welfare: The country whose home (foreign) good is the commonly compared good is better (worse) off.

The result highlights the disadvantage suffered by country Z due to not being endowed with the commonly compared good, similar to the previous case of Common Comparison (Case 1), although this disadvantage is somewhat diminished in terms of welfare loss in the current case by country L’s foreign envy for the product of country Z. From a policy perspective, promotion of domestic products as a socially-compared good again yields higher welfare for the home country. Introducing two-sided social comparison in consumption is not harmful to a country relative to the baseline case, so long as the commonly compared good is home-endowed. Under such a circumstance, the other country is comparatively harmed by the presence of its own purely foreign envy.

3.8 Case 8: Asymmetric Dual Home Comparison

The previous case leads naturally to the analogous situation for Asymmetric Dual Home Comparison. In this case, while country L has a comparison preference on both goods as in the previous scenario, country Z has a social comparison only over its own home-endowed good.

Country L has a social comparison preference on both goods while country Z has preference with social comparison on good b, leading to the following maximization problems:  

For country L,

\[
\max u_L = (a_L + \lambda_1(a_L - a_Z))(b_L + \lambda_2(b_L - b_Z))
\]

s.t. \( P_a a_L + b_L = P_a e^L_a + e^L_b \)  

For country Z,

\[
\max u_Z = a_Z(b_Z + \lambda_2(b_Z - b_L))
\]

s.t. \( P_a a_Z + b_Z = P_a e^Z_a + e^Z_b \)  

Note that for the case of Asymmetric Dual Home Comparison there also exists another scenario where consumers in country Z have preferences with social comparison on both of the good while country L has preference with social comparison on good a.
Once again using the example of extreme endowments and symmetric intensity of social comparison, we set $e_a^L = 1$, $e_b^L = 0$, $e_b^Z = 1$, $e_a^Z = 0$, $\lambda_1 = \lambda_2 = \lambda$, obtaining

$$a_L = \frac{2\lambda + 1}{\lambda^2 + 4\lambda + 2} < \frac{1}{2}$$

$$b_L = \frac{\lambda + 1}{3\lambda + 2} < \frac{1}{2}$$

$$a_Z = \frac{(\lambda + 1)^2}{\lambda^2 + 4\lambda + 2} > \frac{1}{2}$$

$$b_Z = \frac{2\lambda + 1}{3\lambda + 2} > \frac{1}{2}$$

$$P_a = \frac{\lambda^2 + 4\lambda + 2}{3\lambda^2 + 5\lambda + 2} < 1$$

The main difference between this case and the immediately previous case is that the country with comparison preference over just one good has a home comparison instead of foreign envy. This shifts the common comparison across countries to that country’s own endowed good, and once again this good increases in price compared to the baseline case. The consumption and welfare ranks switch relative to Asymmetric Mutual Foreign Envy, and the advantage is now with country $Z$. Similarly to the case of Asymmetric Mutual Foreign Envy, we have the following findings:

**Observation 8** Assuming $\lambda_1 = \lambda_2$, compared with the Baseline case, the Asymmetric Dual Home Comparison case exhibits the following patterns in equilibrium:

1. **Prices:** The commonly compared good becomes more expensive.
2. **Consumption:** The country whose home (foreign) good is the commonly compared good consumes more (less) of both goods.
3. **Welfare:** The country whose home (foreign) good is the commonly compared good is better (worse) off.

This again highlights the advantage of unilaterally encouraging home comparison rather than foreign envy.

### 3.9 Case 9: Ubiquitous Comparison

Finally, we arrive at our last case, in which both countries have social comparisons over both goods in the global market.
The maximization problems are as follows:

For country $L$,

$$\max u_L = (a_L + \lambda_1(a_L - a_Z))(b_L + \lambda_2(b_L - b_Z))$$

s.t. $P_a a_L + b_L = P_a e_a^L + e_b^L$  \hspace{1cm} (25)

For country $Z$,

$$\max u_Z = (a_Z + \lambda_1(a_Z - a_L))(b_Z + \lambda_2(b_Z - b_L))$$

s.t. $P_a a_Z + b_Z = P_a e_a^Z + e_b^Z$  \hspace{1cm} (26)

For the case of extreme endowments and symmetric intensity of social comparison, we can obtain our usual easily interpretable results.

Setting $e_a^L = 1$, $e_b^L = 0$, $e_b^Z = 1$, $e_a^Z = 0$, $\lambda_1 = \lambda_2 = \lambda$, we obtain the following expressions:

$$a_L = \frac{1}{2}$$

$$b_L = \frac{1}{2}$$

$$a_Z = \frac{1}{2}$$

$$b_Z = \frac{1}{2}$$

$$P_a = 1$$

The equilibrium outcome is identical to the baseline case, and furthermore identical to the case of One-Sided Comparison (Case 6).

**Observation 9** Assuming $\lambda_1 = \lambda_2$, compared with the Baseline case, the Ubiquitous Comparison case exhibits the following patterns in equilibrium:

1. **Prices**: The prices of goods are unaffected.

2. **Consumption**: Countries consume the same amounts of both goods as in the baseline case.

3. **Welfare**: Welfare is unaffected compared to the baseline case.
By applying the social comparison preference uniformly to both goods and for both countries, the solution in the case of symmetric extreme endowments and symmetric intensity of social comparisons simplifies to the baseline case. Intuitively, when both countries have such preferences, of equal weighting and equally across goods, there is no excess motivation of any particular country over the other to increase their relative consumption status. It implies that as long as these social comparison preferences can be applied uniformly across all countries and goods, their forces balance or cancel, and we are back where we started with the baseline case. It is mainly for those cases in which some asymmetry exists that prices, consumption and welfare are impacted.

4 Comparison Across All Cases

To summarize all the results across cases, we expand Table 1 to Table 2 incorporating the findings:

<table>
<thead>
<tr>
<th>Country L’s comparison (endowed with good a)</th>
<th>Country Z’s comparison (endowed with good b)</th>
<th>Price</th>
<th>Quantity</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline Case: None</td>
<td>None</td>
<td>$P_a = 1$</td>
<td>$a_L = \frac{1}{2}, b_L = \frac{1}{2}$</td>
<td>Equally well off</td>
</tr>
<tr>
<td>Case 1: Good a</td>
<td>Good a</td>
<td>$P_a &gt; 1$</td>
<td>$a_L &gt; \frac{1}{2}, b_L &gt; \frac{1}{2}$</td>
<td>Country L better off</td>
</tr>
<tr>
<td>Case 2: Good a</td>
<td>None</td>
<td>$P_a &gt; 1$</td>
<td>$a_L &gt; \frac{1}{2}, b_L = \frac{1}{2}$</td>
<td>Country L better off</td>
</tr>
<tr>
<td>Case 3: Good b</td>
<td>None</td>
<td>$P_a &lt; 1$</td>
<td>$a_L &lt; \frac{1}{2}, b_L = \frac{1}{2}$</td>
<td>Country Z worse off</td>
</tr>
<tr>
<td>Case 4: Good a</td>
<td>Good b</td>
<td>$P_a = 1$</td>
<td>$a_L &gt; \frac{1}{2}, b_L &lt; \frac{1}{2}$</td>
<td>Both better off</td>
</tr>
<tr>
<td>Case 5: Good b</td>
<td>Good a</td>
<td>$P_a = 1$</td>
<td>$a_L &lt; \frac{1}{2}, b_L &gt; \frac{1}{2}$</td>
<td>Both better off</td>
</tr>
<tr>
<td>Case 6: Goods a and b</td>
<td>None</td>
<td>$P_a = 1$</td>
<td>$a_L = \frac{1}{2}, b_L = \frac{1}{2}$</td>
<td>Equally well off</td>
</tr>
<tr>
<td>Case 7: Goods a and b</td>
<td>Good a</td>
<td>$P_a &gt; 1$</td>
<td>$a_L &gt; \frac{1}{2}, b_L &gt; \frac{1}{2}$</td>
<td>Country L better off</td>
</tr>
<tr>
<td>Case 8: Goods a and b</td>
<td>Good b</td>
<td>$P_a &lt; 1$</td>
<td>$a_L &lt; \frac{1}{2}, b_L &lt; \frac{1}{2}$</td>
<td>Country Z worse off</td>
</tr>
<tr>
<td>Case 9: Goods a and b</td>
<td>Goods a and b</td>
<td>$P_a = 1$</td>
<td>$a_L = \frac{1}{2}, b_L = \frac{1}{2}$</td>
<td>Equally well off</td>
</tr>
</tbody>
</table>

Here we set $P_b = 1, e_a^L = 1, e_b^L = 0, e_b^Z = 1, e_a^Z = 0, \lambda_1 = \lambda_2 = \lambda$ where applicable to simplify the results.
4.1 Comparison with the Baseline Case

Comparing price, consumption, and welfare between the baseline case and each of the 9 social comparison cases, we summarize our main findings in the following three propositions.

Define the social comparison degree of good $i$, denoted by $d_i$, as the number of countries that have social comparison preference for good $i$, where $i = a, b$. Obviously, in our framework we have $d_a, d_b \in \{0, 1, 2\}$.

**Proposition 1 (Relative prices):** Assume symmetric endowments and homogeneous intensity of social comparisons. If $d_a = d_b$, the prices of the two goods are the same, that is, $P_a = P_b$. If $d_a > (\leq) d_b$, the price of good $a$ is higher (lower) than the price of good $b$, that is, $P_a > (\leq) P_b$.

It is easy to see that the Baseline Case, Case 4 (Dual Home Comparison), Case 5 (Mutual Foreign Envy), Case 6 (One-Sided Comparison), and Case 9 (Ubiquitous Comparison), all have $d_a = d_b$, and thus the prices of the two goods are the same. This result summarizes the idea that any asymmetry in total social comparison preferences between the goods in the global market will induce a demand-driven price effect. Socially compared goods invoke higher prices in the global marketplace, whether the primary origin of demand is at home or abroad.

For $j = L, Z$, define the home social comparison indicator of country $j$, denoted by $f^j$, as follows. $f^j = 1$ if country $j$ has a social comparison preference only for its own home good. $f^j = -1$ if country $j$ has social comparison preference only for the home good of the other country. $f^j = 0$ if country $j$ has no social comparison preference. If country $j$ has social comparison preferences for both goods: let $f^j = 1$ when the other country has a social comparison preference only for the home good of country $j$; let $f^j = -1$ when the other country has social comparison preference only for its own home good; and $f^j = 0$ otherwise.

In other words, the home social comparison indicator is a measure of the degree of relative social comparison preference over the home good. It straightforwardly measures the social comparison preference over a home good if a country only has a social comparison over that good and not their foreign good. However in the case that a country has social comparison preferences for both goods, it is difficult to determine the degree of preference without additional information.

\[12\text{Propositions 1-3 can be easily proved by combining the results from Observations 1-9, which are summarized in Table 2.}\]
comparison preferences over both goods, the other country’s social comparison status serves as a tie-breaker in determining the strength of that country’s home good social comparison strength.

**Proposition 2 (Consumption):** Assume symmetric endowments and homogeneous intensity of social comparisons. For \( j = L, Z \), if \( f^j = 1 \), country \( j \) consumes more than \( \frac{1}{2} \) of its home good; if \( f^j = 0 \), country \( j \) consumes exactly \( \frac{1}{2} \) of its home good; if \( f^j = -1 \), country \( j \) consumes less than \( \frac{1}{2} \) of its home good.

The proposition states that when the strength of home social comparison as defined by the indicator function is positive, a country consumes more of its home good than in the baseline case. By contrast, if the home social comparison indicator is negative, then a country consumes less of its home good than in the baseline case. The consumption effect of a positive indicator value due to a country’s own social comparison preference (the first part of the definition of \( f^j \)) can be understood based on the fact that a country with social comparison preference only over its home endowed good is simultaneously incentivized to consume more of it and export less of it to the global marketplace. The consumption effect of a positive indicator value due to the other country’s social comparison preference (the second part of the definition of \( f^j \), in the cases that country \( j \) has social comparison preference over both goods) can be understood by comparing Case 7 of Asymmetric Mutual Foreign Envy to Case 8 of Asymmetric Dual Home Comparison. In the former case, although one country has social comparisons over both goods, the other country only socially compares its consumption of the foreign good. As a result, that good obtains a higher price and given the increased revenue, is consumed more intensively by its domestic market. However, in the latter case, the country with comparison preference over just one good holds such preference over its own home endowed good. As a result, that particular good has the common comparison advantage and obtains a higher price. The country with a comparison preference over both goods then consumes less of its foreign good.

Define the social comparison degree of country \( j \), denoted by \( d^j \), as the number of countries that have social comparison preference for the home good of country \( j \), where \( j = L, Z \). Obviously, in our framework we have \( d^L, d^Z \in \{0, 1, 2\} \).

**Proposition 3 (Social welfare):** Assume symmetric endowments and homogeneous in-
ensity of social comparisons. If \( d^L = d^Z \), the two countries are equally well off, and at least weakly better off compared to the Baseline case. If \( d^L > (<)d^Z \), country L is better (worse) off and country Z is worse (better) off, both compared to the Baseline case.

The social welfare generalization follows straightforwardly from the results on relative prices, and highlights the income advantage a country receives from its home-endowed good holding a higher price in the global marketplace. Regardless of preferences, the country whose endowed good has the price advantage obtains the heightened ability to distribute consumption advantageously between the two available goods based on their utility function.

We note that throughout the paper, we have adhered to the traditional utility-based notion of welfare, and not a purely consumption-based measure. Thus, the results should be interpreted in utility terms and not purely in terms of quantities of products consumed. In our setting, this means that a country that has consumed quantitatively more under a particular scenario may not experience a proportional welfare gain by merely increasing its absolute consumption. Rather, the gain can be enhanced or diminished depending on the comparison preference component.

4.2 Comparison across Different Cases

In the remaining analysis that follows, we compare the consumption volumes and prices across different cases directly.

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 7</th>
<th>Case 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_L = \frac{2\lambda+1}{3\lambda+2} )</td>
<td>( a_L = \frac{2\lambda+1}{3\lambda+2} )</td>
<td>( a_Z = \frac{\lambda+1}{\lambda+2} )</td>
<td>( a_L = \frac{2\lambda+1}{3\lambda+2} )</td>
<td>( a_Z = \frac{2\lambda+1}{3\lambda+2} )</td>
<td>( a_L = \frac{(\lambda+1)^2}{\lambda^2+4\lambda+2} )</td>
<td>( a_Z = \frac{(\lambda+1)^2}{\lambda^2+4\lambda+2} )</td>
</tr>
</tbody>
</table>

Here we set \( p_b = 1, e_L = 1, e_L = 0, e_Z = 1, e_Z = 0, \lambda_1 = \lambda_2 = \lambda \) where applicable to simplify the results.

By comparing the consumption level of the good \( a \) in different cases based on Table 3, we find \( \frac{(\lambda+1)^2}{\lambda^2+4\lambda+2} < \frac{2\lambda+1}{3\lambda+2} < \frac{\lambda+1}{\lambda+2} \), when \( 0 < \lambda < \frac{1}{2} (\sqrt{5} + 1) \) holds. If \( \lambda > \frac{1}{2} (\sqrt{5} + 1) \), the results are \( \frac{2\lambda+1}{3\lambda+2} < \frac{(\lambda+1)^2}{\lambda^2+4\lambda+2} < \frac{\lambda+1}{\lambda+2} \). However, recall that in order for Case 7 to have a sensible equilibrium, we require that \( \lambda < \frac{1}{2} (\sqrt{5} + 1) \). Therefore, the relationship among these values should be \( \frac{(\lambda+1)^2}{\lambda^2+4\lambda+2} < \frac{2\lambda+1}{3\lambda+2} < \frac{\lambda+1}{\lambda+2} \). The result indicates that the case of Foreign

\[13\] The detailed proof of the results is provided in Appendix C1.
Envy has the highest consumption volume of good \( a \) in country \( Z \), where country \( L \) prefers good \( b \) while country \( Z \) has no social comparison preference in either good.

Table 4: Summary of good \( b \) consumption levels greater than \( \frac{1}{2} \) in country \( L \) and \( Z \)

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 7</th>
<th>Case 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_L = \frac{(\lambda + 1)^2}{3\lambda + 2} )</td>
<td>( b_L = \frac{2\lambda + 1}{3\lambda + 2} )</td>
<td>( b_L = \frac{2\lambda + 1}{3\lambda + 2} )</td>
<td>( b_L = \frac{2\lambda + 1}{-\lambda^2 + 3\lambda + 2} )</td>
<td>( b_L = \frac{2\lambda + 1}{3\lambda + 2} )</td>
</tr>
<tr>
<td>( b_Z = \frac{2\lambda + 1}{3\lambda + 2} )</td>
<td>( b_Z = \frac{2\lambda + 1}{3\lambda + 2} )</td>
<td>( b_Z = \frac{2\lambda + 1}{3\lambda + 2} )</td>
<td>( b_Z = \frac{2\lambda + 1}{3\lambda + 2} )</td>
<td>( b_Z = \frac{2\lambda + 1}{3\lambda + 2} )</td>
</tr>
</tbody>
</table>

Here we set \( P_b = 1, e_L^a = 1, e_L^b = 0, e_Z^b = 1, e_a^Z = 0, \lambda_1 = \lambda_2 = \lambda \) where applicable to simplify the results.

By comparing the consumption level of good \( b \) in different cases based on Table 4, we obtain \( \frac{2\lambda + 1}{3\lambda + 2} < \frac{2\lambda + 1}{3\lambda + 2} < \frac{(\lambda + 1)^2}{3\lambda + 2} \) when \( 0 < \lambda < \frac{1}{2} (\sqrt{5} + 1) \) holds. If \( \frac{1}{2} (\sqrt{5} + 1) < \lambda < \frac{1}{2} (\sqrt{17} + 3) \), the results are \( \frac{2\lambda + 1}{3\lambda + 2} < \frac{(\lambda + 1)^2}{3\lambda + 2} < \frac{2\lambda + 1}{-\lambda^2 + 3\lambda + 2} \) \( \frac{10}{12} \). However, again recall that in order for Case 7 to have sensible equilibrium, we require \( \lambda < \frac{1}{2} (\sqrt{5} + 1) \). Therefore, the relationship among these values should be \( \frac{2\lambda + 1}{3\lambda + 2} < \frac{2\lambda + 1}{-\lambda^2 + 3\lambda + 2} < \frac{(\lambda + 1)^2}{3\lambda + 2} \). The Common Comparison preference of Case 1 has the highest amount of good \( b \) consumption in country \( L \).

Table 5: Summary of price of good \( a \) greater than 1

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_a = \lambda + 1 )</td>
<td>( P_a = \frac{3\lambda + 1}{2\lambda + 2} )</td>
<td>( P_a = \frac{\lambda^2 + 4\lambda + 2}{-\lambda^2 + 3\lambda + 2} )</td>
</tr>
</tbody>
</table>

Here we set \( P_b = 1, e_a^L = 1, e_a^L = 0, e_b^Z = 1, e_a^Z = 0, \lambda_1 = \lambda_2 = \lambda \) where applicable to simplify the results.

Table 5 lists the cases where the relative price of good \( a \) is higher than 1. By comparing the good \( a \)’s price above, we find \( \frac{3\lambda + 1}{2\lambda + 2} < \frac{\lambda^2 + 4\lambda + 2}{-\lambda^2 + 3\lambda + 2} < \lambda + 1 \) when \( 0 < \lambda < \frac{1}{2} (\sqrt{5} + 1) \) holds. If \( \frac{1}{2} (\sqrt{5} + 1) < \lambda < \frac{1}{2} (\sqrt{17} + 3) \), the results are \( \frac{3\lambda + 1}{2\lambda + 2} < \lambda + 1 < \frac{\lambda^2 + 4\lambda + 2}{-\lambda^2 + 3\lambda + 2} \) \( \frac{13}{15} \). However, again recall that in order for Case 7 to have sensible equilibrium, we require \( \lambda < \frac{1}{2} (\sqrt{5} + 1) \). Therefore, the relationship among these values should be \( \frac{3\lambda + 1}{2\lambda + 2} < \frac{\lambda^2 + 4\lambda + 2}{-\lambda^2 + 3\lambda + 2} < \lambda + 1 \). Similar to Table 4, good \( a \) has the highest price in the Common Comparison preference of Case 1.

Table 6 displays the import and export volumes of each country, under each scenario. Assuming that \( 0 < \lambda < \frac{1}{2} (\sqrt{5} + 1) \), the relationship between the different export and import volumes in the table is as follows \( \frac{16}{16} \).

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\( ^{14} \)The detailed proof of the results is provided in Appendix C2.  
\( ^{15} \)The detailed proof of the results is provided in Appendix C3.  
\( ^{16} \)The detailed proof of the results is provided in Appendix C4.
\[
\frac{\lambda + 1}{3 \lambda + 2} < \frac{2 \lambda + 1}{\lambda^2 + 4 \lambda + 2} < \frac{1}{2} < \frac{\lambda^2 + 2 \lambda + 1}{3 \lambda + 2} < \frac{2 \lambda + 1}{\lambda^2 + 3 \lambda + 2} < \frac{\lambda + 1}{3 \lambda + 2} < \frac{2 \lambda + 1}{\lambda^2 + 3 \lambda + 2} < \frac{\lambda^2 + 2 \lambda + 1}{3 \lambda + 2}
\]

<table>
<thead>
<tr>
<th>Case</th>
<th>Country L</th>
<th>Export</th>
<th>Import</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Country L</td>
<td>$\frac{\lambda + 1}{3 \lambda + 2}$</td>
<td>$\frac{\lambda^2 + 2 \lambda + 1}{3 \lambda + 2}$</td>
<td></td>
</tr>
<tr>
<td>Country Z</td>
<td>$\frac{\lambda^2 + 2 \lambda + 1}{3 \lambda + 2}$</td>
<td>$\frac{\lambda + 1}{3 \lambda + 2}$</td>
<td></td>
</tr>
<tr>
<td>Case 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Country L</td>
<td>$\frac{\lambda + 1}{3 \lambda + 2}$</td>
<td>$\frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>Country Z</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{\lambda + 1}{3 \lambda + 2}$</td>
<td></td>
</tr>
<tr>
<td>Case 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Country L</td>
<td>$\frac{\lambda + 1}{\lambda + 2}$</td>
<td>$\frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>Country Z</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{\lambda + 1}{\lambda + 2}$</td>
<td></td>
</tr>
<tr>
<td>Case 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Country L</td>
<td>$\frac{\lambda + 1}{3 \lambda + 2}$</td>
<td>$\frac{\lambda + 1}{3 \lambda + 2}$</td>
<td></td>
</tr>
<tr>
<td>Country Z</td>
<td>$\frac{\lambda + 1}{3 \lambda + 2}$</td>
<td>$\frac{\lambda + 1}{3 \lambda + 2}$</td>
<td></td>
</tr>
<tr>
<td>Case 5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Country L</td>
<td>$\frac{2 \lambda + 1}{3 \lambda + 2}$</td>
<td>$\frac{2 \lambda + 1}{3 \lambda + 2}$</td>
<td></td>
</tr>
<tr>
<td>Country Z</td>
<td>$\frac{2 \lambda + 1}{3 \lambda + 2}$</td>
<td>$\frac{2 \lambda + 1}{3 \lambda + 2}$</td>
<td></td>
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<tr>
<td>Case 6</td>
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</tr>
<tr>
<td>Country L</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>Country Z</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>Case 7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Country L</td>
<td>$\frac{2 \lambda + 1}{\lambda^2 + 4 \lambda + 2}$</td>
<td>$\frac{2 \lambda + 1}{-\lambda^2 + 3 \lambda + 2}$</td>
<td></td>
</tr>
<tr>
<td>Country Z</td>
<td>$\frac{2 \lambda + 1}{-\lambda^2 + 3 \lambda + 2}$</td>
<td>$\frac{2 \lambda + 1}{\lambda^2 + 4 \lambda + 2}$</td>
<td></td>
</tr>
<tr>
<td>Case 8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Country L</td>
<td>$\frac{\lambda^2 + 2 \lambda + 1}{\lambda^2 + 4 \lambda + 2}$</td>
<td>$\frac{\lambda + 1}{3 \lambda + 2}$</td>
<td></td>
</tr>
<tr>
<td>Country Z</td>
<td>$\frac{\lambda + 1}{3 \lambda + 2}$</td>
<td>$\frac{\lambda^2 + 2 \lambda + 1}{\lambda^2 + 4 \lambda + 2}$</td>
<td></td>
</tr>
<tr>
<td>Case 9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Country L</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>Country Z</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td></td>
</tr>
</tbody>
</table>

Here we set $P_b = 1$, $e_a^L = 1$, $e_a^Z = 0$, $e_b^L = 1$, $e_b^Z = 0$, $\lambda_1 = \lambda_2 = \lambda$
where applicable to simplify the results.

The relationship implies that the Common Comparison Preference of Case 1 (country $L$ and country $Z$ both prefer good $a$) achieves the highest export volume of good $b$ by country $Z$. The lowest export volume is provided by Country $L$ also in the Common Comparison case, which is also its export volume in Case 2 (Home Comparison) and Case 4 (Dual Home Comparison), and country $Z$’s export volume in Cases 4 (Dual Home Comparison) and 8.
5 Conclusion

The current global economic climate facilitates social comparisons across countries through plentiful social media, internet exposure and global travel, and has also recently coincided with inward-focused trade policies by several world economic powers. By analyzing a simple global economy that is free from geopolitical concerns, cultural factors and other complications, our model can potentially help explain world leaders’ incentives to cultivate comparison-based preferences among citizens at home and abroad.

The analysis shows that each country benefits from a preference structure in which their own consumers compare global consumption of the good endowed domestically, and gain extra utility from higher consumption of that good. Such a preference might be cultivated in the media and in government policies by emphasizing the home produced product and how fortunate domestic consumers are to have the consumption advantage in it compared to consumers in other countries. In addition, an enhanced welfare can be obtained by each country compared to the baseline case, even if the other country decides to cultivate a similar preference in its own domestic market. In other words, domestically-inward focused framing of consumption, and a cultivation of the feeling that home-produced goods are more desirable, has welfare advantages which policy-makers may be motivated to pursue.

While encouragement of such Home Comparison preference seems relatively straightforward, the consequences of Foreign Envy preferences are slightly more complex. A country which is alone in its admiration and consumption comparison of foreign products experiences lower utility outcomes than it would otherwise. Such welfare losses can only be avoided (and welfare gains obtained) if the other country simultaneously adopts its own analogous Foreign Envy preference. In other words, our model shows that encouraging admiration and social comparison of foreign products can be a risky and possibly disadvantageous approach, unless a country is confident that consumers in other countries reciprocate by viewing their products as status items.

When comparing the welfare consequences of inward versus outward consumption comparison preferences, it provides a potential explanation for why countries go through periodic episodes of inward-focus, but very seldom encourage domestic citizens to idolize foreign products.

The model also provides insights about countries’ persuasion efforts towards consumers
in foreign countries. The analysis shows us that a country can benefit in terms of both wealth and utility by promoting its products as comparison symbols abroad. For products that could plausibly be interpreted by consumers as status or comparison items, the incentive to advertise is thus not only at the individual firm level, but at the national policy level.

In addition, if a country can select what kinds of products it specializes in producing, countries having a choice in the matter would like to produce a globally admired product that incites social comparisons and is highly visible in public. By contrast, countries would like to avoid specializing in the production of items which consumers have little opportunity to form strong comparisons over, such as mundane household or personal use items.

Clearly, our model and results are derived from a simplified setting that isolates the effect of consumer preferences on prices, consumption and welfare in general equilibrium. In reality, there are also production-based factors that determine relative prices, consumption and welfare in international trade, which we have not studied here. In addition, our utility functional form precludes some types of consumer preferences, notably preferences which represent a degree of necessary consumption of one or more goods, which could serve as a key motivation for countries to produce non-status items. Finally, although our analysis provides hints about policy-makers’ incentives to encourage comparison preferences, our analysis itself does not formally include policy-makers as players in the global trade game. Rather, our analysis takes preferences of consumers as given, which is standard in economic modeling. Indeed, despite their motives, policy-makers in some countries may have limited ability to shape their citizens’ preferences, and must take them as given. Our model is perhaps most straightforwardly suited to isolating insights on welfare comparisons in scenarios differing by consumers’ given social-comparison preferences, and leaves these other issues for possible future work.
Appendix A  General Results for Prices and Quantities

In this section, the closed-form expressions for prices and quantities are provided for each of the 9 cases. $a_L$ denotes consumption of good $a$ (apples) by country $L$, and $a_Z$ denotes the consumption of good $a$ by country $Z$. $b_L$ denotes consumption of good $b$ (bananas) by country $L$, and $b_Z$ denotes the consumption of good $b$ by country $Z$. $P_a$ is the price of good $a$, where recall that the price of good $b$ $P_b$ is normalized to 1.

Case 1: Common Comparison

$$a_L = \frac{e_b^Z((\lambda_1 + 1)e_a^L + \lambda_1) - (\lambda_1 + 1)(e_a^Z - 2)e_b^L}{3\lambda_1 + 2}$$
$$a_Z = \frac{e_b^L((\lambda_1 + 1)e_a^Z + \lambda_1) - (\lambda_1 + 1)(e_a^L - 2)e_b^Z}{3\lambda_1 + 2}$$
$$b_L = \frac{e_b^Z(1 + \lambda_1)(-\lambda_1 + e_a^L(1 + 2\lambda_1)) + e_b^L(2 + \lambda_1(4 + \lambda_1) - e_a^Z(1 + \lambda_1)(1 + 2\lambda_1))}{3\lambda_1 + 2}$$
$$b_Z = \frac{e_b^L(1 + \lambda_1)(-\lambda_1 + e_a^Z(1 + 2\lambda_1)) + e_b^Z(2 + \lambda_1(4 + \lambda_1) - e_a^L(1 + \lambda_1)(1 + 2\lambda_1))}{3\lambda_1 + 2}$$
$$P_a = \lambda_1 + 1$$

Case 2: Home Comparison

$$a_L = \frac{e_b^Z((\lambda_1 + 1)e_a^L + \lambda_1) - (\lambda_1 + 1)(e_a^Z - 2)e_b^L}{2(\lambda_1 + 1)e_b^L + (3\lambda_1 + 2)e_b^Z}$$
$$a_Z = \frac{(\lambda_1 + 1)(e_a^Z e_b^L - (e_a^L - 2) e_b^Z)}{2(\lambda_1 + 1)e_b^L + (3\lambda_1 + 2)e_b^Z}$$
$$b_L = \frac{e_b^L((2\lambda_1 + 1)e_a^Z - 2(\lambda_1 + 1)) + e_b^Z(\lambda_1 - (2\lambda_1 + 1)e_a^L)}{2(\lambda_1 + 1)(e_a^L - 2) + (3\lambda_1 + 2)e_a^Z}$$
$$b_Z = \frac{(\lambda_1 + 1)((e_a^L - 2)e_b^Z - e_a^Z e_b)}{2(\lambda_1 + 1)(e_a^L - 2) + (3\lambda_1 + 2)e_a^Z}$$
$$P_a = \frac{-2(\lambda_1 + 1)e_b^L + (3\lambda_1 + 2)e_b^Z}{2(\lambda_1 + 1)(e_b^Z - 2) + (3\lambda_1 + 2)e_b^Z}$$
Case 3: Foreign Envy/Admiration

\[ a_L = \frac{(\lambda_2 + 1)(2 - e^Z_a) e^L_b + e^Z_b ((\lambda_2 + 1)e^L_a - \lambda_2)}{2(\lambda_2 + 1)e^L_b + (\lambda_2 + 2)e^Z_b} \]

\[ a_Z = \frac{(\lambda_2 + 1)(e^Z_b(e^Z_b - 2(\lambda_2 + 1)) - e^Z_b(\lambda_2 - 2(\lambda_2 + 1)e^L_b))}{2(\lambda_2 + 1)e^L_b + (\lambda_2 + 2)e^Z_b} \]

\[ b_L = \frac{e^L_b - (\lambda_2 + 1)(\lambda_2 + 1)e^Z_a + 2(\lambda_2 + 1) + (\lambda_2 + 2)) - (\lambda_2 + 1)e^Z_a((\lambda_2 + 1)e^L_a + \lambda_1)}{2(\lambda_2 + 1) + (\lambda_2 + 2)e^Z_a - 4(\lambda_2 + 1)} \]

\[ b_Z = \frac{2(\lambda_2 + 1)(\lambda_2 + 2)e^L_a + (\lambda_2 + 2)e^Z_a - 4(\lambda_2 + 1)}{2(\lambda_2 + 1) + (\lambda_2 + 2)e^Z_a - 4(\lambda_2 + 1)} \]

\[ P_a = \frac{2(\lambda_2 + 1)(\lambda_2 + 2)e^L_a + (\lambda_2 + 2)e^Z_a - 4(\lambda_2 + 1)}{2(\lambda_2 + 1) + (\lambda_2 + 2)e^Z_a - 4(\lambda_2 + 1)} \]

Case 4: Dual Home Comparison

\[ a_L = \frac{e^L_b - (\lambda_1 + 1)(\lambda_2 + 1)e^Z_a + 2(\lambda_2 + 1) + (\lambda_2 + 2) + (\lambda_2 + 1)e^Z_a((\lambda_1 + 1)e^L_a + \lambda_1)}{2(\lambda_1 + \lambda_2 + 2)e^L_b + (3\lambda_1 + 2)(\lambda_2 + 1)e^Z_b} \]

\[ a_Z = \frac{e^Z_b(\lambda_1 + 1)((\lambda_2 + 1)e^L_a - e^Z_a) + (\lambda_2 + 1)(\lambda_2 + 1)e^Z_a + 2(\lambda_2 + 1)e^Z_b}{2(\lambda_1 + \lambda_2 + 2)e^L_b + (3\lambda_1 + 2)(\lambda_2 + 1)e^Z_b} \]

\[ b_L = \frac{(\lambda_1 + 1)((\lambda_2 + 1)e^L_a - e^Z_a) + (\lambda_2 + 1)(\lambda_2 + 1)e^Z_a + 2(\lambda_2 + 1)e^Z_b}{2(\lambda_1 + \lambda_2 + 2)e^L_b + (3\lambda_1 + 2)(\lambda_2 + 1)e^Z_b} \]

\[ b_Z = \frac{(\lambda_1 + 1)((\lambda_2 + 1)e^L_a - e^Z_a) + (\lambda_2 + 1)(\lambda_2 + 1)e^Z_a + 2(\lambda_2 + 1)e^Z_b}{2(\lambda_1 + \lambda_2 + 2)e^L_b + (3\lambda_1 + 2)(\lambda_2 + 1)e^Z_b} \]

\[ P_a = \frac{(\lambda_1 + 1)((\lambda_2 + 1)e^L_a - e^Z_a) + (\lambda_2 + 1)(\lambda_2 + 1)e^Z_a + 2(\lambda_2 + 1)e^Z_b}{2(\lambda_1 + \lambda_2 + 2)e^L_b + (3\lambda_1 + 2)(\lambda_2 + 1)e^Z_b} \]

Case 5: Mutual Foreign Envy/Admiration

\[ a_L = \frac{(\lambda_1 + 1)((\lambda_2 + 1)(e^Z_a - 2) e^L_b + e^Z_b ((\lambda_2 - 2(\lambda_2 + 1)e^L_a))}{3(\lambda_1 + 2)(\lambda_2 + 1)e^L_b + (2\lambda_1 + \lambda_2 + 2)e^Z_b} \]

\[ a_Z = \frac{(\lambda_1 + 1)((\lambda_2 + 1) e^Z_a - e^Z_a) + (\lambda_2 + 1)(\lambda_2 + 1)e^Z_a e^L_b + (\lambda_2 + 1) e^Z_b + (2(\lambda_2 + 1) + (\lambda_2 + 2)) e^Z_b}{3(\lambda_1 + 2)(\lambda_2 + 1)e^L_b + (2\lambda_1 + \lambda_2 + 2)e^Z_b} \]

\[ b_L = \frac{((\lambda_1 (\lambda_2 - 1) - 1) e^L_a - (\lambda_1 + 1) e^L_b) - e^L_b ((\lambda_1 (\lambda_2 - 1) - 1) e^Z_a + 2(\lambda_2 + 1) + (\lambda_2 + 2)) e^Z_b}{3(\lambda_1 + 2)(\lambda_2 + 1)e^L_b + (2\lambda_1 + \lambda_2 + 2)e^Z_b - 4(\lambda_2 + 1)} \]

\[ b_Z = \frac{(\lambda_2 + 1) + (\lambda_2 + 1) e^L_a - e^Z_a - (\lambda_1 + 1) e^L_b - (2(\lambda_2 + 1) + (\lambda_2 + 2)) e^Z_b}{3(\lambda_1 + 2)(\lambda_2 + 1)e^L_b + (2\lambda_1 + \lambda_2 + 2)e^Z_b - 4(\lambda_2 + 1) - \lambda_1 (3\lambda_2 + 4)} \]

\[ P_a = \frac{(\lambda_1 + 1)((\lambda_2 + 1)e^L_a - e^Z_a) + (\lambda_2 + 1)(\lambda_2 + 1)e^Z_a + 2(\lambda_2 + 1)e^Z_b}{3(\lambda_1 + 2)(\lambda_2 + 1)e^L_b + (2\lambda_1 + \lambda_2 + 2)e^Z_b - 4(\lambda_2 + 1) - \lambda_1 (3\lambda_2 + 4)} \]
Case 6: One-sided Comparison

\[ a_L = \frac{e^Z_b((\lambda_1 + 1)(\lambda_2 + 1)e^L_a + \lambda_1 - \lambda_2) - (\lambda_1 + 1)(\lambda_2 + 1)(e^Z_a - 2)e^L_b}{2(\lambda_1 + 1)(\lambda_2 + 1)e^L_b + (\lambda_2 + \lambda_1(2\lambda_2 + 3) + 2)e^Z_b} \]

\[ a_Z = \frac{(\lambda_1 + 1)(\lambda_2 + 1)(e^L_a(-e^Z_b) + e^Z_a e^L_b + 2e^Z_b)}{2(\lambda_1 + 1)(\lambda_2 + 1)e^L_b + (\lambda_2 + \lambda_1(2\lambda_2 + 3) + 2)e^Z_b} \]

\[ b_L = \frac{e^L_b((\lambda_1 + 1)(\lambda_2 + 2) + 1)e^Z_b - 2(\lambda_1 + 1)(\lambda_2 + 1)) - e^Z_b((\lambda_1(\lambda_2 + 2) + 1)e^L_a - \lambda_1 + \lambda_2)}{2(\lambda_1 + 1)(\lambda_2 + 1)e^L_b + (\lambda_2 + \lambda_1(2\lambda_2 + 3) + 2)e^Z_b - 4(\lambda_1 + 1)(\lambda_2 + 1)} \]

\[ b_Z = \frac{-(\lambda_1 + 1)(\lambda_2 + 1)e^L_b + (\lambda_2 + \lambda_1(2\lambda_2 + 3) + 2)e^Z_b}{2(\lambda_1 + 1)(\lambda_2 + 1)e^L_b + (\lambda_2 + \lambda_1(2\lambda_2 + 3) + 2)e^Z_b - 4(\lambda_1 + 1)(\lambda_2 + 1)} \]

\[ P_a = -\frac{2(\lambda_1 + 1)(\lambda_2 + 1)e^L_b + (\lambda_2 + \lambda_1(2\lambda_2 + 3) + 2)e^Z_b - 4(\lambda_1 + 1)(\lambda_2 + 1)}{2(\lambda_1 + 1)(\lambda_2 + 1)e^L_b + (\lambda_2 + \lambda_1(2\lambda_2 + 3) + 2)e^Z_b - 4(\lambda_1 + 1)(\lambda_2 + 1)} \]

Case 7: Asymmetric Mutual Foreign Envy

\[ a_L = \frac{e^Z_b((\lambda_1 + 1)(\lambda_2 + 1)e^L_a + \lambda_1 - \lambda_2) - (\lambda_1 + 1)(\lambda_2 + 1)(e^Z_a - 2)e^L_b}{3(\lambda_1 + 2)(\lambda_2 + 1)e^L_b + (\lambda_2 + \lambda_1(2\lambda_2 + 3) + 2)e^Z_b} \]

\[ a_Z = \frac{(\lambda_1 + 1)(\lambda_2 + 1)(e^L_a(-e^Z_b) + e^Z_a e^L_b + 2e^Z_b)}{3(\lambda_1 + 2)(\lambda_2 + 1)e^L_b + (\lambda_2 + \lambda_1(2\lambda_2 + 3) + 2)e^Z_b} \]

\[ b_L = \frac{e^L_b((\lambda_1 + 1)(\lambda_2 + 2) + 1)e^Z_b - 2(\lambda_1 + 1)(\lambda_2 + 1)) - e^Z_b((\lambda_1(\lambda_2 + 2) + 1)e^L_a - \lambda_1 + \lambda_2)}{3(\lambda_1 + 2)(\lambda_2 + 1)e^L_b + (\lambda_2 + \lambda_1(2\lambda_2 + 3) + 2)e^Z_b - 4(\lambda_1 + 1)(\lambda_2 + 1)} \]

\[ b_Z = \frac{-(\lambda_1 + 1)(\lambda_2 + 1)e^L_b + (\lambda_2 + \lambda_1(2\lambda_2 + 3) + 2)e^Z_b}{3(\lambda_1 + 2)(\lambda_2 + 1)e^L_b + (\lambda_2 + \lambda_1(2\lambda_2 + 3) + 2)e^Z_b - 4(\lambda_1 + 1)(\lambda_2 + 1)} \]

\[ P_a = -\frac{3(\lambda_2 + 1)(\lambda_1 + 2)(\lambda_2 + 1)e^L_b + (\lambda_2 + \lambda_1(2\lambda_2 + 3) + 2)e^Z_b - 4(\lambda_1 + 1)(\lambda_2 + 1)}{3(\lambda_1 + 2)(\lambda_2 + 1)e^L_b + (\lambda_2 + \lambda_1(2\lambda_2 + 3) + 2)e^Z_b - 4(\lambda_1 + 1)(\lambda_2 + 1)} \]
Case 8: Asymmetric Dual Home Comparison

\[ a_L = \frac{e_b^L \left( (2\lambda_2^2 + 3\lambda_2 + 1) e_b^Z - \lambda_2^2 - 4\lambda_2 - \lambda_1 (3\lambda_2 + 2) - 2 \right) - (\lambda_2 + 1) e_a^Z \left( (2\lambda_2 + 1) e_b^L + \lambda_1 - \lambda_2 \right)}{(\lambda_1 + 1) \left( e_b^Z \left( (2\lambda_2^2 + 3\lambda_2 + 1) e_b^L - \lambda_2^2 - 4\lambda_2 - 2 \right) - (\lambda_2 + 1) e_a^Z \left( (2\lambda_2 + 1) e_b^Z - \lambda_2 \right) \right)} \]

\[ a_Z = \frac{e_a^Z \left( (\lambda_1 + 1) (\lambda_2 + 1) e_b^L - \lambda_1 + \lambda_2 \right) - (\lambda_1 + 1) (\lambda_2 + 1) e_a^L (e_b^Z - 2)}{(\lambda_1 + 1) \left( e_b^L \left( (\lambda_1 + 1) (\lambda_2 + 1) e_a^L + e_b^Z \right) + (3\lambda_2 + \lambda_1 (\lambda_2 + 1) + 2) e_a^Z \right)} \]

\[ b_L = e_a^Z \left( (\lambda_1 + 1) (\lambda_2 + 1) e_a^L + e_b^Z \right) \]

\[ b_Z = \frac{e_b^L \left( (\lambda_2 + 1) e_b^Z + e_b^Z \right) - e_a^Z \left( (\lambda_1 + 1) \left( (\lambda_2 + 1) e_b^L + 2 (\lambda_2 + 1) + \lambda_1 (\lambda_2 + 2) \right) \right) e_a^Z + 3\lambda_2 + 8\lambda_2 + \lambda_1 (2\lambda_2 + 7\lambda_2 + 4) + 4}{(\lambda_2 + 1) \left( (\lambda_1 + 1) (\lambda_2 + 2) e_a^Z + (3\lambda_2 + \lambda_1 (\lambda_2 + 1) + 2) e_a^Z \right)} \]

\[ P_a = \frac{e_b^L - (\lambda_2 + 1) (3\lambda_2 + 5\lambda_2 + 2) e_a^L - (\lambda_2 + 1) (3\lambda_2 + \lambda_1 (\lambda_2 + 1) + 2) e_a^Z + 3\lambda_2 + 8\lambda_2 + \lambda_1 (2\lambda_2 + 7\lambda_2 + 4) + 4}{(\lambda_2 + 1) \left( (\lambda_1 + 1) (\lambda_2 + 2) e_a^Z + (3\lambda_2 + \lambda_1 (\lambda_2 + 1) + 2) e_a^Z \right)} \]

Case 9: Ubiquitous Comparison

\[ a_L = e_b^L \left( (\lambda_1 + 1) (2\lambda_2 + 1) e_b^Z + \lambda_2^2 + 4\lambda_2 + \lambda_1 (3\lambda_2 + 2) + 2 \right) + e_a^Z \left( (\lambda_1 + 1) (2\lambda_2 + 1) e_a^L + (\lambda_1 - \lambda_2) (\lambda_2 + 1) \right) \]

\[ a_Z = \frac{(\lambda_2 + 1) e_b^L \left( - (2\lambda_2^2 + 3\lambda_2 + 1) e_a^Z + \lambda_2^2 + 2 (\lambda_2 + 1) + \lambda_1 (3\lambda_2 + 4) \right) + (\lambda_1 + 1) e_b^Z \left( (2\lambda_2 + 1) e_a^L - \lambda_1 + \lambda_2 \right)}{(3\lambda_2 + \lambda_1 (4\lambda_2 + 3) + 2) (e_b^L + e_a^Z)} \]

\[ b_L = (\lambda_2 + 1) e_b^L \left( - (2\lambda_2^2 + 3\lambda_1 + 1) e_a^Z + \lambda_2^2 + 2 (\lambda_2 + 1) + \lambda_1 (3\lambda_2 + 4) \right) + (\lambda_1 + 1) e_b^Z \left( (2\lambda_2 + 1) e_a^L - \lambda_1 + \lambda_2 \right) \]

\[ b_Z = \frac{(\lambda_2 + 1) \left( (3\lambda_2 + 3\lambda_2 + 1) e_b^L \left( - e_a^Z \right) + (2\lambda_2^2 + 3\lambda_2 + 1) e_a^Z \right) - (\lambda_2 + 1) e_a^L (\lambda_1 - \lambda_2) e_b^Z + (\lambda_2^2 + (3\lambda_2 + 4) \lambda_2 + 2 (\lambda_2 + 1))}{(3\lambda_2 + \lambda_1 (4\lambda_2 + 3) + 2) (e_b^L + e_a^Z)} \]

\[ P_a = - \frac{(e_b^L + e_a^Z)}{(\lambda_1 e_a^L + e_a^L + \lambda_1 e_a^Z + e_a^Z) - \lambda_1 - \lambda_2 - 2} \]
Appendix B  Proofs for Welfare Analysis

B1 Welfare analysis for Case 4: Dual Home Comparison

The following demonstrates that for the case of Dual Home Comparison, for the situation of extreme endowments and equal social comparison weights across goods: $e^L_a = 1$, $e^L_b = 0$, $e^Z_b = 1$, $e^Z_a = 0$, $\lambda_1 = \lambda_2 = \lambda$, both countries are better off compared to the case of no social comparison preferences.

The utility function of country L equals $u_L = ((\lambda + 1)a_L - \lambda a_Z)b_L$. Since in equilibrium we have $a_L = \frac{2\lambda+1}{3\lambda+2}$, $b_L = \frac{\lambda+1}{3\lambda+2}$, and $a_Z = \frac{\lambda+1}{3\lambda+2}$, thus we can obtain the equilibrium utility for country L, $u_L = \frac{(\lambda+1)^3}{(3\lambda+2)^2}$.

Note that $\frac{(\lambda+1)^3}{(3\lambda+2)^2} = \frac{\lambda}{9} + \frac{5}{27} + \frac{1}{9(3\lambda+2)} + \frac{1}{27(3\lambda+2)^2}$ is increasing in $\lambda$ as the derivative with respective to $\lambda$ is $\frac{1}{9} - \frac{2}{9(3\lambda+2)^2} - \frac{1}{3(3\lambda+2)^2}$, which is always positive since $\lambda > 0$. Therefore, $u_L(\lambda) > u_L(\lambda = 0) = \frac{1}{4}$.

Similarly, the utility function of country Z equals $u_Z = a_Z((\lambda + 1)b_Z - \lambda b_L)$. Since in equilibrium we have $a_Z = \frac{\lambda+1}{3\lambda+2}$, $b_Z = \frac{2\lambda+1}{3\lambda+2}$, and $b_L = \frac{\lambda+1}{3\lambda+2}$, thus we can obtain the equilibrium utility for country Z, $u_Z = \frac{(\lambda+1)^3}{(3\lambda+2)^2}$, which is greater than $\frac{1}{4}$ as well.

B2 Welfare analysis for Case 5: Mutual Foreign Envy/Admiration

The following demonstrates that for the case of Mutual Foreign Envy/Admiration, for the situation of extreme endowments and equal social comparison weights across goods: $e^L_a = 1$, $e^L_b = 0$, $e^Z_b = 1$, $e^Z_a = 0$, $\lambda_1 = \lambda_2 = \lambda$, both countries are better off compared to the case of no social comparison preferences.

Substituting equilibrium value $a_L = \frac{\lambda+1}{3\lambda+2}$, $b_L = \frac{2\lambda+1}{3\lambda+2}$, and $b_Z = \frac{\lambda+1}{3\lambda+2}$ into the utility function of country L, which is $u_L = a_L((\lambda + 1)b_Z - \lambda b_L)$, we have $u_L = \frac{(\lambda+1)^3}{(3\lambda+2)^2}$, which is greater than $\frac{1}{4}$.

Using the similar method, we can obtain country Z’s equilibrium utility $u_Z = \frac{(\lambda+1)^3}{(3\lambda+2)^2}$, which is greater than $\frac{1}{4}$ as well.
Appendix C  Proofs of Comparisons across Cases

C1 Comparison of good a consumption in different cases

The following provides the comparison of the consumption levels of good \(a > \frac{1}{2}\) in Table 3:

With \(\frac{2\lambda+1}{3x+3a+2} - \frac{\lambda+1}{x+2}\), we obtain \(-\frac{\lambda^2}{3x^2 + 8a + 4} < 0\). With \(\frac{(\lambda+1)^2}{\lambda^2 - 4a + 2}\), we obtain \(-\frac{\lambda(\lambda+1)}{(\lambda+2)(\lambda^2 + 4a + 2)} < 0\). With \(\frac{(\lambda+1)^2}{\lambda^2 + 4a + 2} - \frac{2\lambda+1}{3x+2}\), we obtain \(\frac{\lambda(\lambda^2 - \lambda - 1)}{(3\lambda+2)(\lambda^2 + 3a + 2)}\). Therefore, we can derive \(\frac{(\lambda+1)^2}{\lambda^2 + 4a + 2} < \frac{2\lambda+1}{3x+2}\) when \(\lambda < \frac{1}{2}(\sqrt{5} + 1)\) holds. If \(\lambda > \frac{1}{2}(\sqrt{5} + 1)\), the results are \(\frac{2\lambda+1}{3x+2} < \frac{(\lambda+1)^2}{\lambda^2 + 4a + 2}\).

C2 Comparison of good b consumption in different cases

The following provides the comparison of the consumption levels of good \(b > \frac{1}{2}\) in Table 4:

It is obvious that \(\frac{2\lambda+1}{3x+3a+2}\) is smaller than \(\frac{(\lambda+1)^2}{\lambda^2 + 3a + 2}\), respectively. With \(\frac{(\lambda+1)^2}{\lambda^2 + 3a + 2} - \frac{2\lambda+1}{3x+2}\), we obtain \(\frac{\lambda(\lambda^2 - \lambda - 1)}{(3\lambda+2)(\lambda^2 - 3a - 2)}\). Therefore, we can find \(\frac{2\lambda+1}{3x+2} < \frac{\lambda(\lambda^2 - \lambda - 1)}{(3\lambda+2)(\lambda^2 - 3a - 2)}\) when \(0 < \lambda < \frac{1}{2}(\sqrt{5} + 1)\) holds. If \(\frac{1}{2}(\sqrt{5} + 1) < \lambda < \frac{1}{2}(\sqrt{17} + 3)\), the results are \(\frac{3\lambda+1}{2x+2} < \lambda + 1 < \frac{\lambda^2 + 4a + 2}{\lambda^2 + 3a + 2}\).

C3 Comparison of good a price in different cases

The following provides the comparison of the price of good \(a\) in Table 5:

It is obvious that \(\frac{3\lambda+1}{2x+2}\) has smaller value than \(\frac{\lambda(\lambda^2 - \lambda - 1)}{\lambda^2 + 3a + 2}\) and \(\lambda + 1\), respectively. With \(\lambda + 1 - \frac{\lambda^2 + 4a + 2}{\lambda^2 + 3a + 2}\), we derive \(\frac{\lambda(\lambda^2 - \lambda - 1)}{\lambda^2 + 3a + 2}\). Therefore, we find \(\frac{3\lambda+1}{2x+2} < \frac{\lambda(\lambda^2 - \lambda - 1)}{\lambda^2 + 3a + 2}\) when \(0 < \lambda < \frac{1}{2}(\sqrt{5} + 1)\) holds. If \(\frac{1}{2}(\sqrt{5} + 1) < \lambda < \frac{1}{2}(\sqrt{17} + 3)\), the results are \(\frac{3\lambda+1}{2x+2} < \lambda + 1 < \frac{\lambda^2 + 4a + 2}{\lambda^2 + 3a + 2}\).

C4 Comparison of the export and import volume in different cases

The following provides the comparison of export and import volumes in Table 6:

With \(\frac{\lambda+1}{3x+2} - \frac{2\lambda+1}{\lambda^2 + 4a + 2}\), we derive \(\frac{\lambda(\lambda^2 - \lambda - 1)}{(3\lambda+2)(\lambda^2 + 4a + 2)}\). We have \(\frac{\lambda+1}{3x+2} < \frac{2\lambda+1}{\lambda^2 + 4a + 2}\) when \(0 < \lambda < \frac{1}{2}(\sqrt{5} + 1)\) holds. If \(\lambda > \frac{1}{2}(\sqrt{5} + 1)\), the result is \(\frac{\lambda+1}{3x+2} > \frac{2\lambda+1}{\lambda^2 + 4a + 2}\). With \(\frac{2\lambda+1}{\lambda^2 + 3a + 2} - \frac{\lambda+1}{3x+2}\), we obtain \(\frac{\lambda^3}{\lambda^2 + 3a + 2}\). Therefore, we find \(\frac{2\lambda+1}{\lambda^2 + 3a + 2} > \frac{\lambda+1}{3x+2}\) when \(0 < \lambda < \frac{1}{2}(\sqrt{17} + 3)\). Based on the previous proofs of comparison of good \(a, b\), export and import volume, we find the the relationship as follows:

(1) \(\frac{\lambda+1}{3x+2} < \frac{2\lambda+1}{\lambda^2 + 4a + 2} < \frac{\lambda+1}{x^2 + 2a + 2} < \frac{\lambda^2 + 2\lambda + 1}{\lambda^2 + 3a + 2} < \frac{\lambda^2 + 2\lambda + 1}{\lambda^2 + 3a + 2} < \frac{2\lambda+1}{\lambda^2 + 3a + 2}\), if \(0 < \lambda < \frac{1}{2}(\sqrt{5} + 1)\)

(2) \(\frac{2\lambda+1}{\lambda^2 + 4a + 2} < \frac{\lambda+1}{3x+2} < \frac{2\lambda+1}{\lambda^2 + 4a + 2} < \frac{\lambda+1}{x^2 + 2a + 2} < \frac{\lambda^2 + 2\lambda + 1}{\lambda^2 + 3a + 2} < \frac{\lambda^2 + 2\lambda + 1}{\lambda^2 + 3a + 2}\), if \(\frac{1}{2}(\sqrt{5} + 1) < \lambda < \frac{1}{2}(\sqrt{17} + 3)\)
References


