

Serial and Parallel Duopoly Competition in Multi-Segment Transportation Routes*

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Abstract

We consider duopoly competition among transportation firms operating on a multi-segment route, and analyze the welfare consequences of different market structures in serial and parallel transport network structures. Travellers are concerned with both price and latency, and may opt to avoid travel in the system altogether if conditions are insufficiently attractive. Firms simultaneously set prices to maximize their own profits. We derive the traffic flow consequences of duopoly, monopoly, price-free, and socially optimal scenarios for both the serial and parallel structures, drawing comparisons regarding traffic flows, prices, and welfare. In extensions of the model, we consider firms' operating costs, more than two firms, and hybrid market structures. The results point to a higher overall social welfare desirability of parallel competition along multi-segment transportation routes. Our study delivers important policy implications regarding the optimal route structure design.

Keywords: Duopoly competition, Multi-segment journey, Transportation routes, Serial structure, Parallel structure

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1 Introduction

Many transportation plans involve more than one segment, rather than completing the travel in a single non-stop journey. When a travel route is naturally divided into segments, how does competition among transportation service suppliers determine the ideal market structure for the segments? Take for example the journey between a major city in one state to a small town in another state. A traveller may take a train or flight from the city of departure to the largest city close to the small town. From there, a bus or car ride can take the traveller to the final destination. Many commuters have a two-part journey to their workplaces each morning. Suburban residents might drive to the nearest commuter rail station, park their car there and take the rail into the city. Urban residents may take a bus followed by the subway to their place of employment.

When there is a natural interim point in the journey, is it better to have competing transportation companies supply transport over the entire route (parallel structure), or to divide the route at the interim point and let each serve as the sole supplier over their particular segment (serial structure)? Furthermore, how does the case of competition between suppliers compare to the analogous case of a monopolistic supplier (over the same routing structure), the ideal transportation arrangement implemented by a social planner, and the case of a price-free scenario? In this paper, we analyze these questions in a model in which commuters care about both time and transport price and can decide whether to travel or not, while two firms engage in price competition with one another under the specified market structure over routes.

Our findings favor the parallel competition structure from a total social welfare perspective, as well as for commuters. For the competing firms, the favorability (profit) result depends on the latency conditions of the route segments. For sufficiently favorable travel times, the serial structure provides higher profit for the firms than the parallel structure. This suggests that only when traffic flows are fast moving enough, would there be a conflict of interest between private transportation firms, and the government and commuters they sell to. However, for the vast instances of unfavorable travel flow efficiencies, the welfare of commuters, firms and social planners are aligned in favor of the parallel duopoly market structure.

We consider several extensions of our baseline model, including incorporation of firms' operating costs, generalization of the model to a larger number of firms competing over the transportation segments, and hybrid market structures over the transportation system such as duopoly-monopoly combinations. We find that our main conclusions about the welfare favorability of parallel structures are robust to these variations in the model specification. Thus, our study contributes to logistics management and government policy by providing a new answer regarding the optimal route structure design.

Our paper is closely related to a number of studies which model competition in transportation systems game theoretically via duopoly models. van der Weijde, Verhoef, and van den Berg (2013)[1]

consider the equilibrium dynamics between a two-part public transportation system with fees, and an alternative set of unpriced congestible roads. Comparing the scenarios in which the public transport provider is a monopolist versus a serial duopoly, they show that fares under the two market structures can be different, and in some cases, the monopoly fare can be higher than the duopolistic fare. van den Berg (2013)[2] considers the market structure of two transportation infrastructures which must be used in sequence by passengers. Four market structures/timings are considered: Monopoly; Duopoly with simultaneous setting of fees and capacities; Duopoly with sequential setting of capacities followed by fees; and a Stackelberg duopoly in which a leader sets capacity first, and both leader and follower set fees subsequently. The analysis finds that capacity decisions can be influential in the outcomes and social benefits of duopoly compared to monopoly. Our model shares with these previous studies in examining two-part transportation systems, but differs in the following aspects: In contrast to van der Weijde et al. (2013)[1], all routes in our model are congestible and priced. While comparing with a monopolistic provider in each case, our main focus is on the comparison of the two types of duopoly, parallel and serial. Finally, in our framework, commuters can elect to not travel if the time and price they face is sufficiently unattractive, whereas their setup focuses on inelastic demand but allows for commuters' departure timing choice. Compared to van den Berg (2013)[2], our analysis assumes Bertrand competition, and focuses on the comparison between serial and parallel duopolies in a pricing game, rather than analyzing the timing of capacity and price decisions by the firms.

In terms of a broader scope on duopoly competition, our work belongs to the operation management literature on related topics. Klastorin, Mamani and Zhou (2016)[3] show that pre-announcement of a new product can benefit both the innovator and imitator by increasing product differentiation in a duopoly market. Park, Chang and Zou (2018)[4] take the port capacity's impact on the emission control into account in a duopoly port environment. They find that the level of emissions standard established by the government is associated with the maximum reservation price of shipping operators. Wei et al. (2019)[5] study green capacity strategies in a duopoly market. The results show that firms prefer inflexible capacity over flexible capacity when a green product becomes more advantaged. In Lotfi and Sarkar's (2016)[6] work, the price competition in a duopoly market with uncertain competition depends on the numbers of available units for sale, and a symmetric Nash Equilibrium uniquely exists.

By studying competition in multi-segment routes, our study also bears relation to the literature on logistical transport routes, and in particular the literature on multi-modal transportation. Luo, Gao and Akcay (2016)[7] apply dynamic forecasting to a revenue management model, which establishes the optimal strategy in the intermodal transportation system. Zhang and Cetinkaya (2017)[8] study the practical extension of a classical dynamic inventory problem, using the limited capacity of private trucks to transport the replenishment quantity. To solve the positioning problem in unmodeled dynamics, Sun et al. (2018)[9] propose a new method to control underactuated double-

pendulum crane systems. Xia and Zhang (2016)[10] analyze the vertical differentiation between high-speed rail and air transport. They find the conditions and price consequences of cooperation and competition between the two popular transport modes in a hub-and-spoke network. Yang, Ban and Mitchell (2017)[11] model emergency evacuation procedures using multimodal transportation network while allowing for capacities, congestion and varying cooperation of evacuees. Almur, Yaman and Kara (2012)[12] consider the multi-modal hub location problem via air transport hubs and ground transport hubs, and conduct cost sensitivity analysis on the locations of the hubs in Turkey. Similarly, Zhang, Yang, Wu and Wang (2014)[13] develop an algorithm to solve the multi-modal transportation network design problem. In a bi-modal transportation network, Ghaderi and Burdett (2019)[14] consider the long-distance transport of dangerous goods by an integrated location and routing approach in order to reduce the cost and risk. Wen et al. (2019)[15] constructs a function to evaluate the utility of multiple path selections on the “One Belt and One Road” initiative. Related to Carlsson et al.’s (2016)[16] study which compares the pros and cons between decentralized and centralized logistical systems, our research is concerned with transportation systems in both serial and parallel structures. Under their model, the overall carbon footprint of the region decreases with the same level of required services.

When comparing the parallel versus serial structures that we consider in our model, one natural interpretation is the notion of substitutes and complements in transport modes. In the parallel structure, firms are providing substitute modes for consumers, while in the serial structure, firms are providing complementary transport modes. As such, our work is also related to the literature on substitutabilities and complementarity of transport modes. For example, Rus and Socorro[17] (2014) theoretically examine the conditions for investment in a complementary or rival new infrastructure, given an existing infrastructure system. In another theoretical paper, Clark, Jorgensen and Mathisen (2014)[18] study the competition among complementary service providers in a transport chain. On the empirical side, transportation complementarities and substitutability are also of substantial interest. Zhang, Graham and Wong (2018)[19] study the substitutability and complementarity between high-speed rail and air transport in Asia, finding that high speed rail and air travel serve as substitutes on medium and short distance routes, while being complementary on long distance routes in China. Hall, Palsson and Price (2018)[20] conduct an empirical analysis to find that Uber and public transit are on average complements in the United States. Studying the complements and substitutes issue in Beijing, Liu, Jiang, Yang and Zhang (2012)[21] find that public transport and cars are complementary while public transport and bicycles are substitutes.

In terms of methodology, our modeling approach follows the literature on game theoretic analysis of transport providers networks under passenger flows, as in Mazalov and Melnik (2016)[22] and Lien, Mazalov, Melnik and Zheng (2016)[23]. As a recent development, Kuang, Mazalov, Tang and Zheng (2018)[24] examine the transportation network competition problem with externalities. For analytical convenience, we also invoke the connection between a transportation network and

an electrical circuit system, which was first studied by Furber (1936) [25]. Beyond Furber (1936), the analogy of a transportation system with an electrical circuit has been further addressed in a few recent studies. Zhang and Ren (2010) [26] establish such an analogy between one-way transportation routes and disconnected circuits and summarize the similarity between traffic flow and current, travel time and voltage, latency parameters and resistance, respectively. Nagurney and Nagurney (2017) [27] investigate the Braess paradox in electric circuits and other network systems. In our paper, we generalize this analogy in three ways: (1) By Kirchhoff’s Current Law and Kirchhoff’s Voltage Law, we extend the analogy from a one-way transportation network to more complex networks; (2) We link the endogenized total traffic flow in a transportation network to the power source in a circuit; (3) We apply Thévenin’s Theorem to help us solve for the equilibrium flow in the transportation network.

The remainder of the paper is organized as follows: Section 2 begins with the model of serial structure, in which commuters need to travel from A to B and B to C, where each firm is in control of a single segment. We conduct equilibrium analyses for the duopoly case and compare the results with three other alternatives (monopolistic, price-free, and socially optimal scenarios). Section 3 introduces the parallel structure model, in which firms compete on the entire route A to C, and again equilibrium analyses are provided and comparisons across the 4 equilibrium scenarios are drawn. Section 4 introduces the comparison between the serial and parallel structures, including equilibrium traffic flows, prices, profits, consumer and social surpluses, and discusses about several extensions (firms’ operating costs, more than two firms, and hybrid market structures) that show the main findings are robust. Section 5 concludes by summarizing the results, highlighting the contributions and discussing future work. Technical details are relegated to Appendix.

2 Benchmark Model I: Serial Structure

2.1 Settings

Commuters would like to travel from A to C through an intermediate point B . There are two firms providing transport services. Firm 1 is in charge of route segment AB and sets an entrance fee p_1 . Firm 2 is in charge of route segment BC and sets an entrance fee p_2 . All commuters that are willing to travel from A to C must pay $p_1 + p_2$ as total entrance fee. Each firm sets their price strategically but independently of the other firm, in order to maximize their profit, which is the product of travel flow x and their own price, assuming zero operating costs.¹

¹For simplicity, in the benchmark models, we assume that firms’ operating costs are negligible. In our extension I (Appendix C), we allow for non-negligible operating costs and show our main results still hold.

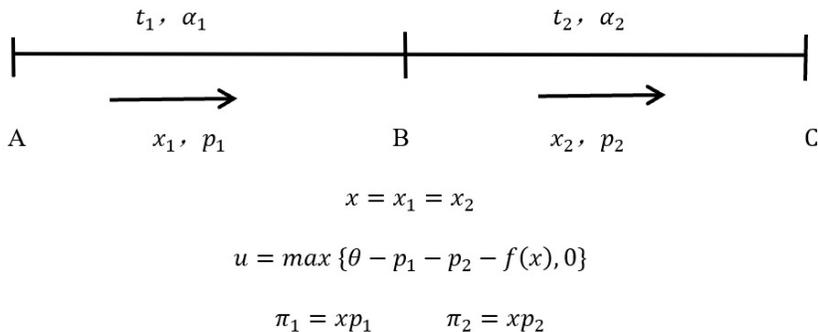


Figure 1: Serial Structure

All commuters originate from A , and the value of transport from A to C is a random variable θ that is uniformly distributed within the interval $[0, 1]$. We also assume that there is no value of transport arising from A to B only.²

Transportation networks in which location B is serving essentially as a hub for connecting routes provide real world examples which closely match our setup and analysis for the serial structure. For example, point B may be a check-point or intermediate hub that all transport modes from points A to C , are required to pass through. One example is for transportation that crosses a legal border, for which passengers are required to disembark and go through immigration or customs check. A specific example is the transportation between point A in Hong Kong SAR, and point C in Shenzhen, mainland China. In this example, point B corresponds to the border checkpoint, such as the Lo Wu connecting station on the Hong Kong/Shenzhen border where the vast majority of travelers have a final destination inside the city of Shenzhen. Outside of a border-crossing context, there are other examples of intermediate check-points that have minimal demand for transport in either direction to the check-point itself. One specific example is the Lantau Link Toll Plaza in Hong Kong, which serves as an intermediate point in transportation between most of Hong Kong to and from the Hong Kong International Airport. While there are virtually no attractions or reasons to want to visit the toll plaza itself, the station serves as a transport hub for connecting bridges and roads to and from Lantau Island. An example of such an intermediate point in the transportation system in the United States is in the journey from Penn Station, New York City to Newark Liberty International Airport. New Jersey Transit trains stop at the station Newark Airport, where passengers actually have to disembark and purchase a new ticket for the AirTrain

²By assuming that all commuters have the same origin A and the same destination C , we do not study the need for local traffic A to B or B to C . Such a simplification is made partially for modeling convenience, so that the serial structure in this section can be comparable to the parallel structure in the next section, but also partially for capturing the phenomenon of commuting for work or other specific purposes by using two transport services sequentially. The study of transportation systems with local traffic is thus beyond our current scope, and we thank an anonymous referee for pointing this out.

into the airport itself.

We normalize the size of commuters as a mass of size 1. Commuters take both price and time into consideration in their utility function. For the sake of simplicity, we assume that a commuter is willing to travel if and only if his/her value is larger than (or equal to) the total entrance fee $p_1 + p_2$ plus the travel latency $f(x)$. Therefore, the utility for a commuter is defined as

$$u(\theta, x) = \theta - p_1 - p_2 - f(x) \quad (1)$$

We normalize the utility of the outside option (deciding not to travel, or using an alternative means of travel) as 0.

We assume that the latency of each segment $i = 1, 2$ is linear with respect to total traffic load x ,

$$f_i(x) = t_i(1 + \alpha_i x) \quad (2)$$

where t_i indicates the trip time on a completely unoccupied route i . Thus, t_i can also be interpreted as the distance of a route, or representing transportation modes of different speeds. For example, a large t_i could represent a lengthy route, or alternatively a slower transport mode such as cycling. A small t_i could represent either a short route, or a fast transport mode such as high speed rail. Since the commuter's value of transport θ is in the range $[0,1]$, we focus on the reasonable situations in which $t_1 + t_2 \leq 1$. Note that if $t_1 + t_2$ is greater than 1, then no commuters will choose to travel from A to C . α_i is the other latency parameter, which determines how much traffic is slowed due to increased traffic flow. An interpretation of the parameter is the condition of the road, where low α indicates good road conditions and high α indicates poor road conditions. The settings for the benchmark model of serial structure can be illustrated by Figure 1.

In a serial structure, the total latency is hence the summation of the two route segments,

$$f(x) = f_1(x) + f_2(x) = (t_1 + t_2) + (t_1\alpha_1 + t_2\alpha_2)x$$

which still has a linear latency form. We define $t = t_1 + t_2$ and $\alpha = \frac{t_1\alpha_1 + t_2\alpha_2}{t_1 + t_2}$, then we have

$$f(x) = t(1 + \alpha x) \quad (3)$$

where t and α are sufficient for analyzing the serial structure.

In the subsections below, we will first analyze 4 different scenarios (duopoly case, social optimum, price-free, and monopoly), respectively, then conduct comparison of traffic flows among these 4 scenarios, and finally provide results on comparative statics analysis.

2.2 Duopoly Equilibrium

We first derive the demand function when the total entrance fee is p . Assuming the demand (or traffic flow) is x , the equilibrium condition requires that commuters with value greater than

or equal to $1 - x$ take the trip while commuters with value less than $1 - x$ choose outside option. Therefore, the demand is given by

$$x = \frac{1 - p - t}{1 + \alpha t} \quad (4)$$

which is solved by $u(1 - x, x) = 0$.

Hence the profits for firm 1 and firm 2 are

$$\Pi_1(p_1) = \frac{1 - p_1 - p_2 - t}{1 + \alpha t} p_1 \quad (5)$$

$$\Pi_2(p_2) = \frac{1 - p_1 - p_2 - t}{1 + \alpha t} p_2 \quad (6)$$

respectively. The profit maximization by each firm results in the following equilibrium prices

$$p_1 = p_2 = \frac{1 - t}{3} \quad (7)$$

and the equilibrium traffic flow is given by

$$x_{Duo}^S = \frac{1 - t}{3(1 + \alpha t)} \quad (8)$$

where the superscript S denotes the serial structure.

2.3 Social Optimum

When the total traffic flow is x , the social welfare, defined as the sum of consumer surplus and producer surplus (or firms' profits), is given by

$$S(x) = \int_{1-x}^1 \theta d\theta - x f(x) = \frac{2x - x^2}{2} - tx - \alpha t x^2 \quad (9)$$

where we assume that commuters with higher value have priority to take the trip. Maximization of the social welfare gives us the socially optimal traffic flow

$$x_{Opt}^S = \frac{1 - t}{1 + 2\alpha t} \quad (10)$$

2.4 Price-free User Equilibrium

Now we consider the price-free scenario, which is commonly studied in the literature. Under such a scenario without any pricing system, the commuters can travel freely on any route subject only to the latency of the route, thus the equilibrium traffic flow will be

$$x_{UE}^S = \frac{1 - t}{1 + \alpha t} \quad (11)$$

which can be easily obtained from equation (4) by letting $p = 0$.

2.5 Monopoly Equilibrium

The last scenario we consider is monopoly. If the two firms merge and become a monopolist by setting the total entrance fee as p , then the profit maximization of the monopolist gives the equilibrium price

$$p = p_1 + p_2 = \frac{1-t}{2} \quad (12)$$

and equilibrium traffic flow is

$$x_{Mon}^S = \frac{1-t}{2(1+\alpha t)} \quad (13)$$

2.6 Comparison of Traffic Flows

A comparison of the above findings regarding equilibrium traffic flows demonstrates that the following relationship holds.

Lemma 2.1. $x_{Duo}^S < x_{Mon}^S < x_{Opt}^S < x_{UE}^S = 2x_{Mon}^S = 3x_{Duo}^S$.

Lemma 2.1 shows that under the serial structure, traffic flows are smallest in the case of duopoly, followed by the case of monopoly, followed by the social optimum, and the price-free user equilibrium, respectively.

We note that intuitively, the socially optimal traffic flow is less than the price-free equilibrium in which commuters can travel without monetary cost on any route that is able to suit their needs, which points to the gap between social optimum and individual optimization, given that individuals in the user equilibrium fail to take into consideration their congestion externality on other users. Given the presence of the congestion externality, it is also intuitive that both the duopoly and monopoly pricing schemes induce less traffic flow in equilibrium on the sequential route from A to B to C.

Lower traffic flows under duopoly compared to monopoly are driven by the higher total price set by firm 1 and firm 2 collectively through their mutual best response. This feature is an application of the well-known ‘double-marginalization’ result in the comparison between two vertical monopolists and a single vertically integrated monopolist (Cournot, 1838 (1925) [28]; see also Economides, 1996 [29] and Lantz, 2009 [30] for discussion). In such scenarios, two independently operating monopolists competing on different portions of a vertical chain (in our terminology, ‘duopolists’ in the serial structure) set a higher total price, leaving the consumer (and firms’ side of the market) worse off than the case of a single monopolist operating the entire chain. Our result shows that the double-marginalization result extends to this transportation network setting, thus reducing traffic flows under duopoly by means of firms’ competition.

2.7 Comparative Statics Analysis

We are also interested in how the equilibrium traffic flows under each scenario change with variation in the latency parameters. Based on the results from previous subsections, we can derive comparative statics results with respect to parameters t_i and α_i , $i = 1, 2$ and also the composite parameters t and α . The results are summarized in the following proposition, and all proofs are in the Appendix.

Proposition 2.2. *For every scenario k , $k = Duo, Opt, UE, Mon$, for every segment i , $i = 1, 2$, we have (i) $\frac{\partial x_k^S}{\partial t} < 0$; (ii) $\frac{\partial x_k^S}{\partial \alpha} < 0$; (iii) $\frac{\partial x_k^S}{\partial t_i} < 0$; (iv) $\frac{\partial x_k^S}{\partial \alpha_i} < 0$.*

Proposition 2.2 states that the traffic flows under each structure are decreasing in the unoccupied travel time for the whole trip t , the unoccupied travel time for each segment t_i , the composite latency parameter for the whole trip α , as well as the latency parameter for each segment α_i , where $i = 1, 2$.

Recall that one interpretation for the parameter t (or t_i) is in representing the length of the route. The proposition states that in each scenario we analyzed, equilibrium traffic flows are decreasing in route length, whether for the composite route in the entire travel plan, or the individual route segments. In each scenario, the effect is driven by the direct effect of an increase in unoccupied travel time on the travel latency in travellers' utility functions. Since travel time is costly to travellers, more of them will opt out of utilizing this travel system when t is longer. In the cases of duopoly and monopoly, although t factors negatively into the optimal price, this effect is dominated by the aforementioned latency component of utility $f(x)$.

The result with respect to α (or α_i) says that when the latency of any given travel segment increases, including that of the entire travel plan, the equilibrium traffic flow decreases. The reasoning is very similar to that for the unoccupied travel time parameter t . It can be clearly seen from the expressions for equilibrium traffic flows in each scenario, that the latency parameter α appears only in the denominator. Through the adverse effect of increased latency on traveller utility, an increased volume of travellers will decide to exercise their outside option, thus reducing travel flows in the system, no matter which market structure scenario is considered.

3 Benchmark Model II: Parallel Structure

We now analyze the competition between two firms in a transportation route system of parallel structure. Without loss of generality, we focus on interior solutions in equilibrium.

3.1 Settings

Similarly to the setup in Section 2.1, commuters would like to travel from origin A to destination C . There are two firms: Firm 1 takes charge of one route from A to C and sets an entrance fee of

p_1 ; Firm 2 takes charge of an alternative route from A to C and sets an entrance fee of p_2 . Both firms set their prices strategically and independently to maximize their profits, while again facing zero operating costs. All commuters that are willing to travel from A to C must choose exactly one route among the two options and pay the associated entrance fee. A commuter is willing to travel if and only if his/her value of transportation is larger than (or equal to) the entrance fee plus the travel latency. For ease of notation, we again use t_i and α_i for latency parameters for route i ($i = 1, 2$) in the parallel structure, where t_i indicates the trip time on a completely unoccupied route i and α_i is the latency parameter representing the condition of route i .³ Since a commuter's value of transportation is no more than 1, we restrict our focus to the reasonable situations in which $\max(t_1, t_2) \leq 1$. The settings for the benchmark model of the parallel structure can be illustrated by Figure 2.

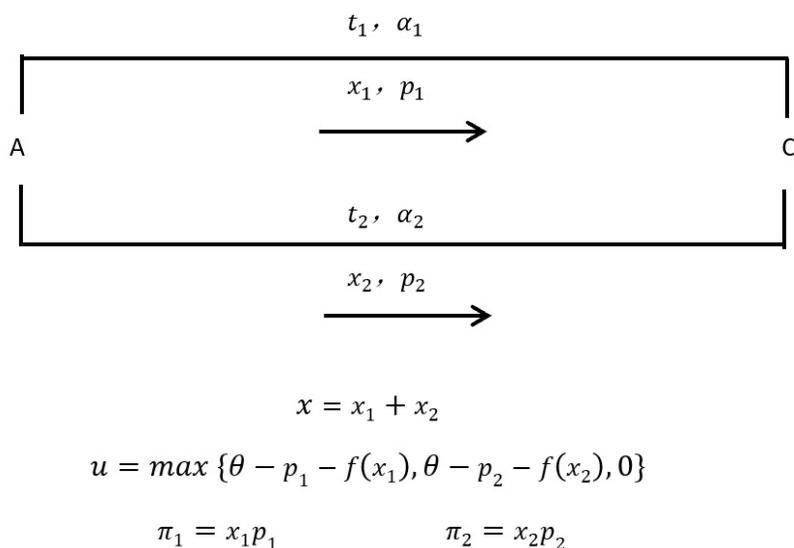


Figure 2: Parallel Structure

3.2 Duopoly Equilibrium

Suppose that in equilibrium commuters of size x_1 go through route 1 and commuters of size x_2 go through route 2. Hence, the (marginal) consumer with value $1 - x_1 - x_2$ must be indifferent between the three options: going through route 1, going through route 2, and not traveling, which

³Note that in Section 2, t_i and α_i are latency parameters for segment i in the serial structure. The parameters in the serial structure and those in the parallel structure, though represented by the same symbols, do not necessarily have the same values.

implies

$$\begin{aligned} 1 - x_1 - x_2 - p_1 - t_1(1 + \alpha_1 x_1) &= 0 \\ 1 - x_1 - x_2 - p_2 - t_2(1 + \alpha_2 x_2) &= 0 \end{aligned}$$

The above equilibrium conditions imply that the two routes for AC have the same total cost:

$$p_1 + t_1(1 + \alpha_1 x_1) = p_2 + t_2(1 + \alpha_2 x_2)$$

and we can express traffic flows as functions of firms' prices:

$$x_1 = \frac{t_2 \alpha_2 + p_2 + t_2 - (1 + t_2 \alpha_2)(p_1 + t_1)}{t_1 \alpha_1 + t_2 \alpha_2 + t_1 \alpha_1 t_2 \alpha_2} \quad (14)$$

$$x_2 = \frac{t_1 \alpha_1 + p_1 + t_1 - (1 + t_1 \alpha_1)(p_2 + t_2)}{t_1 \alpha_1 + t_2 \alpha_2 + t_1 \alpha_1 t_2 \alpha_2} \quad (15)$$

$$x = \frac{1 - \frac{t_1 \alpha_1 (p_2 + t_2)}{t_1 \alpha_1 + t_2 \alpha_2} - \frac{t_2 \alpha_2 (p_1 + t_1)}{t_1 \alpha_1 + t_2 \alpha_2}}{1 + \frac{t_1 \alpha_1 t_2 \alpha_2}{t_1 \alpha_1 + t_2 \alpha_2}} \quad (16)$$

where $x = x_1 + x_2$ denote the total flow. Hence the profits for firm 1 and firm 2 are given by the following expressions

$$\Pi_1(p_1) = \frac{t_2 \alpha_2 + p_2 + t_2 - (1 + t_2 \alpha_2)(p_1 + t_1)}{t_1 \alpha_1 + t_2 \alpha_2 + t_1 \alpha_1 t_2 \alpha_2} p_1 \quad (17)$$

$$\Pi_2(p_2) = \frac{t_1 \alpha_1 + p_1 + t_1 - (1 + t_1 \alpha_1)(p_2 + t_2)}{t_1 \alpha_1 + t_2 \alpha_2 + t_1 \alpha_1 t_2 \alpha_2} p_2 \quad (18)$$

To maximize firms' profits, the first order conditions are

$$\begin{aligned} \frac{\partial \Pi_1(p_1)}{\partial p_1} &= \frac{t_2 \alpha_2 + p_2 + t_2 - t_1 - t_1 t_2 \alpha_2 - 2(1 + t_2 \alpha_2)p_1}{t_1 \alpha_1 + t_2 \alpha_2 + t_1 \alpha_1 t_2 \alpha_2} = 0 \\ \frac{\partial \Pi_2(p_2)}{\partial p_2} &= \frac{t_1 \alpha_1 + p_1 + t_1 - t_2 - t_1 t_2 \alpha_1 - 2(1 + t_1 \alpha_1)p_2}{t_1 \alpha_1 + t_2 \alpha_2 + t_1 \alpha_1 t_2 \alpha_2} = 0 \end{aligned}$$

which jointly determine the equilibrium prices

$$p_1 = \frac{2t_2 \alpha_2 + 2t_1 \alpha_1 t_2 \alpha_2 + t_2 - t_1 + t_1 t_2 \alpha_1 - 2t_1^2 \alpha_1 - 2t_1 t_2 \alpha_2 - 2t_1^2 t_2 \alpha_1 \alpha_2 + t_1 \alpha_1}{4t_1 \alpha_1 + 4t_2 \alpha_2 + 4t_1 \alpha_1 t_2 \alpha_2 + 3} \quad (19)$$

$$p_2 = \frac{2t_1 \alpha_1 + 2t_1 \alpha_1 t_2 \alpha_2 + t_1 - t_2 + t_1 t_2 \alpha_2 - 2t_2^2 \alpha_2 - 2t_1 t_2 \alpha_1 - 2t_1 t_2^2 \alpha_1 \alpha_2 + t_2 \alpha_2}{4t_1 \alpha_1 + 4t_2 \alpha_2 + 4t_1 \alpha_1 t_2 \alpha_2 + 3} \quad (20)$$

Substituting (19) and (20) into (14)-(16), we obtain the equilibrium traffic flows for each route and for the whole system, respectively.

$$x_1 = \frac{(1 + t_1 \alpha_1)(1 + t_2 \alpha_2)(2t_2 \alpha_2 - 2\alpha_2 t_1 t_2 + t_2 - t_1) + (t_1 \alpha_1 + t_1 \alpha_1 t_2 \alpha_2)(1 - t_1)}{(t_1 \alpha_1 + t_2 \alpha_2 + t_1 \alpha_1 t_2 \alpha_2)(4t_1 \alpha_1 + 4t_2 \alpha_2 + 4t_1 \alpha_1 t_2 \alpha_2 + 3)} \quad (21)$$

$$x_2 = \frac{(1 + t_1 \alpha_1)(1 + t_2 \alpha_2)(2t_1 \alpha_1 - 2\alpha_1 t_1 t_2 + t_1 - t_2) + (t_2 \alpha_2 + t_1 \alpha_1 t_2 \alpha_2)(1 - t_2)}{(t_1 \alpha_1 + t_2 \alpha_2 + t_1 \alpha_1 t_2 \alpha_2)(4t_1 \alpha_1 + 4t_2 \alpha_2 + 4t_1 \alpha_1 t_2 \alpha_2 + 3)} \quad (22)$$

$$\begin{aligned}
x_{Duo}^P &= \frac{2(1+t_1\alpha_1)(1+t_2\alpha_2)(2t_2\alpha_2-2\alpha_2t_1t_2+t_2-t_1)}{(t_1\alpha_1+t_2\alpha_2+t_1\alpha_1t_2\alpha_2)(4t_1\alpha_1+4t_2\alpha_2+4t_1\alpha_1t_2\alpha_2+3)} \\
&+ \frac{(t_1\alpha_1+t_1\alpha_1t_2\alpha_2)(1-t_1)}{(t_1\alpha_1+t_2\alpha_2+t_1\alpha_1t_2\alpha_2)(4t_1\alpha_1+4t_2\alpha_2+4t_1\alpha_1t_2\alpha_2+3)} \\
&+ \frac{(t_2\alpha_2+t_1\alpha_1t_2\alpha_2)(1-t_2)}{(t_1\alpha_1+t_2\alpha_2+t_1\alpha_1t_2\alpha_2)(4t_1\alpha_1+4t_2\alpha_2+4t_1\alpha_1t_2\alpha_2+3)}
\end{aligned} \tag{23}$$

where the superscript P denotes parallel.

3.3 Social Optimum

When the total traffic is $x_1 + x_2$, the social welfare is given by

$$\begin{aligned}
S(x_1, x_2) &= \int_{1-x_1-x_2}^1 \theta d\theta - x_1 f_1(x_1) - x_2 f_2(x_2) \\
&= \frac{1 - (1 - x_1 - x_2)^2}{2} - t_1(1 + \alpha_1 x_1)x_1 - t_2(1 + \alpha_2 x_2)x_2
\end{aligned} \tag{24}$$

with first order conditions

$$\begin{aligned}
1 - x_1 - x_2 - t_1(1 + 2\alpha_1 x_1) &= 0 \\
1 - x_1 - x_2 - t_2(1 + 2\alpha_2 x_2) &= 0
\end{aligned}$$

By the above first order conditions, we obtain the socially optimal traffic flows for each route and for the whole system.

$$x_1 = \frac{t_2 - t_1 + 2t_2\alpha_2 - 2t_1t_2\alpha_2}{2t_1\alpha_1 + 2t_2\alpha_2 + 4t_1\alpha_1t_2\alpha_2} \tag{25}$$

$$x_2 = \frac{t_1 - t_2 + 2t_1\alpha_1 - 2t_1t_2\alpha_1}{2t_1\alpha_1 + 2t_2\alpha_2 + 4t_1\alpha_1t_2\alpha_2} \tag{26}$$

$$x_{Opt}^P = \frac{t_1\alpha_1 + t_2\alpha_2 - t_1t_2(\alpha_1 + \alpha_2)}{t_1\alpha_1 + t_2\alpha_2 + 2t_1\alpha_1t_2\alpha_2} \tag{27}$$

3.4 Price-free User Equilibrium

Under the user equilibrium without any pricing system, in which commuters are subject only to the latency of the routes, the equilibrium flows are

$$x_1 = \frac{t_2\alpha_2 + t_2 - (1 + t_2\alpha_2)t_1}{t_1\alpha_1 + t_2\alpha_2 + t_1\alpha_1t_2\alpha_2} \tag{28}$$

$$x_2 = \frac{t_1\alpha_1 + t_1 - (1 + t_1\alpha_1)t_2}{t_1\alpha_1 + t_2\alpha_2 + t_1\alpha_1t_2\alpha_2} \tag{29}$$

$$x_{UE}^P = \frac{t_1\alpha_1 + t_2\alpha_2 - t_1t_2(\alpha_1 + \alpha_2)}{t_1\alpha_1 + t_2\alpha_2 + t_1\alpha_1t_2\alpha_2} \tag{30}$$

which can be easily obtained from equations (14)-(16) by letting $p_1 = p_2 = 0$.

3.5 Monopoly Equilibrium

Suppose that the two firms merge and manage the two routes jointly under a single firm. By total profit maximization, the first order conditions with respect to prices are

$$\begin{aligned}\frac{\partial(\Pi_1 + \Pi_2)}{\partial p_1} &= \frac{t_2\alpha_2 + 2p_2 + t_2 - t_1 - t_1t_2\alpha_2 - 2(1 + t_2\alpha_2)p_1}{t_1\alpha_1 + t_2\alpha_2 + t_1\alpha_1t_2\alpha_2} = 0 \\ \frac{\partial(\Pi_1 + \Pi_2)}{\partial p_2} &= \frac{t_1\alpha_1 + 2p_1 + t_1 - t_2 - t_1t_2\alpha_1 - 2(1 + t_1\alpha_1)p_2}{t_1\alpha_1 + t_2\alpha_2 + t_1\alpha_1t_2\alpha_2} = 0\end{aligned}$$

which jointly determine the equilibrium prices

$$p_1 = \frac{1}{2}(1 - t_1) \quad (31)$$

$$p_2 = \frac{1}{2}(1 - t_2) \quad (32)$$

Substituting (31) and (32) into (14)-(16), we obtain the monopoly traffic flows for each route and for the whole system, respectively.

$$x_1 = \frac{(\alpha_2 + 1)t_2 - t_1(\alpha_2t_2 + 1)}{2(t_1\alpha_1 + t_2\alpha_2 + t_1\alpha_1t_2\alpha_2)} \quad (33)$$

$$x_2 = \frac{(\alpha_1 + 1)t_1 - t_2(\alpha_1t_1 + 1)}{2(t_1\alpha_1 + t_2\alpha_2 + t_1\alpha_1t_2\alpha_2)} \quad (34)$$

$$x_{Mon}^P = \frac{t_1\alpha_1 + t_2\alpha_2 - t_1t_2(\alpha_1 + \alpha_2)}{2(t_1\alpha_1 + t_2\alpha_2 + t_1\alpha_1t_2\alpha_2)} \quad (35)$$

3.6 Comparison of Traffic Flows

The following relationship holds with regard to the traffic flows in the parallel structure across 4 different scenarios considered:

Lemma 3.1. $x_{Mon}^P < x_{Duo}^P < x_{Opt}^P < x_{UE}^P = 2x_{Mon}^P$.

The total traffic flow ordering between the case of price-free scenario and the social planner's solution remains the same compared to the previously analyzed case of the serial structure. However, in contrast to the serial structure, the duopoly leads to a higher traffic flow than monopoly in the parallel structure setting. This is due to the standard result that competition by two firms offering identical products (the parallel segments) lowers prices, and hence raises the total traffic flow beyond that of the monopolist. The monopolist, as in other settings, under-supplies the good in question, which is traffic volume in our context. Due to the parallel transportation structure rather than the serial structure previously considered, the double-marginalization result does not apply, and the standard intuition regarding competition prevails.

3.7 Comparative Statics Analysis

In this subsection, we analyze how the equilibrium traffic flows under each scenario change in the latency parameters t_i and α_i , $i = 1, 2$. The results are summarized in the following proposition, and the proofs are provided in the Appendix.

Proposition 3.2. *For every scenario k , $k = Duo, Opt, UE, Mon$, for every route i , $i = 1, 2$, we have (i) $\frac{\partial x_k^P}{\partial t_i} < 0$; (ii) $\frac{\partial x_k^P}{\partial \alpha_i} < 0$.*

The proposition states that traffic flows in the parallel structure for any of the market structure scenarios are decreasing in the latency parameters t_i and α_i of each firm's segment. The reasoning is that an increase in the latency parameter(s) on any firm's segment corresponds to slower travel time on their segment. This serves to reduce the number of commuters who find traveling on that particular segment appealing. While some of the commuters will indeed switch to traveling on the other firm's route, the increase in latency of the other firm's route (due to the increased number of commuters now using it) will deter some of the original commuters from traveling at all in equilibrium. The result is a lower total traffic flow in the system.

4 Comparison Between Serial and Parallel Structures

We now compare the cases of serial and parallel structures described previously. For ease of comparison, we restrict our attention to the case that firm 1 and firm 2 under a given structure are identical in their price competition (by setting $\alpha_1^j = \alpha_2^j$ and $t_1^j = t_2^j$ for $j = S, P$) (a symmetry condition), which simplifies the expressions substantially. To make the comparison reasonable, we also impose another equivalence condition that the two structures under the price-free scenario should have the same equilibrium flow.⁴

4.1 Equilibrium Flow

Under the symmetry and equivalence conditions, if we assume that the latency function for the serial structure is $f(x) = t(1 + \alpha x)$, then the associated latency function for both routes in the parallel structure is $f_1(x) = f_2(x) = t(1 + 2\alpha x)$. Intuitively, one can consider the symmetric parallel structure as two sub-roads with the same road condition that were generated by dividing the original road (the serial structure) into halves. Assuming the original road has parameters t and α , since each sub-road has the same length and half width of the original road, the parameters for each sub-road should be t and 2α , accordingly. Given that the equilibrium outcomes are the same in the price-free scenario under both structures, it is easy to verify that both structures are also outcome

⁴The equivalence condition requires $t_1^S + t_2^S = \frac{t_1^P t_2^P (\alpha_1^P + \alpha_2^P)}{t_1^P \alpha_1^P + t_2^P \alpha_2^P}$ and $t_1^S \alpha_1^S + t_2^S \alpha_2^S = \frac{t_1^P \alpha_1^P t_2^P \alpha_2^P}{t_1^P \alpha_1^P + t_2^P \alpha_2^P}$, which can be obtained by comparing the two graphs in the right panel of Figure 4 in Appendix B.

equivalent (including demand and social welfare) in the social optimum and monopoly scenarios. The equilibrium traffic flows under both structures across different scenarios are summarized in the following table.

Table 1: Traffic Flows

	Serial	Parallel
User Equilibrium	$\frac{1-t}{1+\alpha t}$	
Social Optimum	$\frac{1-t}{1+2\alpha t}$	
Monopoly	$\frac{1-t}{2(1+\alpha t)}$	
Duopoly	$\frac{1-t}{3(1+\alpha t)}$	$\frac{(1-t)(1+2\alpha t)}{(1+\alpha t)(4\alpha t+1)}$

Using x_{Duo}^S to denote traffic flows under the duopoly context in the serial structure, x_{Duo}^P to denote that under the parallel structure, and $x_{Mon}^{S/P}, x_{Opt}^{S/P}, x_{UE}^{S/P}$ to denote the identical traffic flows under monopoly, social optimum and user equilibrium respectively, by Lemmas 2.1 and 3.1, we have the following proposition.

Proposition 4.1. *Under the symmetry and equivalence conditions, the equilibrium flows under different scenarios and structures satisfy the following relationship:*

$$x_{Duo}^S < x_{Mon}^{S/P} < x_{Duo}^P < x_{Opt}^{S/P} < x_{UE}^{S/P}.$$

The result in Proposition 4.1 can be roughly understood by the ordering of traffic flows in different scenarios under the serial and parallel structure, respectively (Lemmas 2.1 and 3.1). Here, with the additional assumptions of symmetry and equivalence as defined earlier, we can make a formal comparison between the traffic flow orderings across the serial and parallel structures.

The proposition states that when latency parameters are identical across segments/routes (symmetry), and equilibrium traffic flows are identical under the user equilibrium (equivalence), the traffic flows under the scenarios of social optimum and monopoly are the same across serial and parallel structures, where the relationships between monopoly, social optimum, and user equilibrium maintain their previously discussed orderings. The individual relationships between monopoly and duopoly traffic flows in the serial and parallel structures also maintain their original orderings, with serial duopoly yielding the least traffic flow, again due to the double-marginalization intuition in the vertical structure. Parallel duopoly under the standard intuition with substitutable routes yields higher traffic flows than under monopoly, although less than the socially optimal traffic flow.

4.2 Equilibrium Price and Profit

We now consider the price and profit expressions for the scenarios of monopoly and duopoly under the serial and parallel structures. By the equivalence condition, the case of monopoly is identical across the serial and parallel structures in terms of both price and profit. However, the

duopoly case results in different price and profit levels under these two structures, and thus warrants comparison. The results are summarized in first two rows of Table 2.

Table 2: Comparison Between Serial Structure and Parallel Structure in Duopoly

	Serial	Parallel
Price	$p_1 = p_2 = \frac{1-t}{3}$	$p_1 = p_2 = \frac{2\alpha t(1-t)}{1+4\alpha t}$
Profit	$\Pi_1 = \Pi_2 = \frac{(1-t)^2}{9(1+\alpha t)}$	$\Pi_1 = \Pi_2 = \frac{\alpha t(1+2\alpha t)(1-t)^2}{(1+4\alpha t)^2(1+\alpha t)}$
Consumer Surplus	$\frac{(1-t)^2}{18(1+\alpha t)^2}$	$\frac{(1-t)^2(1+2\alpha t)^2}{2(1+\alpha t)^2(1+4\alpha t)^2}$
Social Surplus	$\frac{(4\alpha t+5)(1-t)^2}{18(1+\alpha t)^2}$	$\frac{(1-t)^2(1+2\alpha t)(4\alpha^2 t^2+6\alpha t+1)}{2(1+\alpha t)^2(1+4\alpha t)^2}$

In terms of comparing the profit levels between the parallel and serial structures, the result depends on the values of parameters α and t , where $\alpha t = \frac{1}{2}$ serves as the cutoff point. A full characterization of the duopoly profit comparison is stated in the following proposition.

Proposition 4.2. *Under the symmetry and equivalence conditions, for the duopoly scenario, the relationship between profit under the serial and parallel structures depends on the parameter values. If $\alpha t > \frac{1}{2}$, the profit under the parallel structure is higher; If $\alpha t < \frac{1}{2}$, the profit under the serial structure is higher; If $\alpha t = \frac{1}{2}$, the profits under both structures are the same.*

Proposition 4.2 implies that when the route condition is poor (such that α is large) or the travel time on the unoccupied road is long (such that t is large), the profit is higher under the parallel structure than the serial structure for the duopoly scenario.

This result can be understood in terms of the comparison between competition in a parallel versus serial (or vertical) structure. In the serial structure, duopolists compete with one another over highly complementary products which based on the double-marginalization intuition, results in higher prices, lower traffic flows, and lower revenues than under a single monopolist. The adverse road condition and longer road length represented by $\alpha t > \frac{1}{2}$ enhances this negative effect on profit.

In the parallel structure, duopolists engage in competition over the identical parallel routes, which tends to lower prices and increase traffic flows. While higher latency has a negative effect on profits in both the serial and parallel scenarios, the effect serves to maintain the relative profit ordering between the two scenarios under fixed latency. However, when latency is small ($\alpha t < \frac{1}{2}$), the favorable travel condition reverses this effect substantially in favor of the serial structure. In other words, the small latency helps to attract enough travelers to the system, enabling the system to overcome the generally negative effect of serial competition on profit.

A policy implication of this result is that when privatizing transportation networks, governments can consider the predicted latency of the travel routes when determining whether the route should be divided into serial or parallel structures from the perspective of the competing transportation-providing firms. For high latency routes, which tend to be long and/or congested, the result says

that parallel competition will be more attractive to firms that are considering entering the market or bidding for contracts.

4.3 Equilibrium Surplus

The consumer surplus and social surplus can also be calculated under both serial and parallel structures in the symmetric case. When total traffic flow is x , both structures have the same social surplus functional form $\frac{2x-x^2}{2} - tx - t\alpha x^2$, and consumer surplus is simply the difference between the social surplus and firms' profits. The results for consumer surplus and social surplus under both structures between the monopoly scenario and duopoly scenario are shown in last two rows of Table 2.

Note that in last row of Table 2, the duopoly scenario generates a lower social surplus than the monopoly scenario under the serial structure, and the reverse relation holds for the parallel structure. The different efficiency results under these two structures are driven by the different equilibrium traffic flow levels between the monopoly and duopoly scenarios (See Proposition 4.1 for details). Since the social surplus function is increasing in traffic flow up to the socially optimal flow level, a lower flow in duopoly than in monopoly under the serial structure leads to a lower efficiency level in duopoly compared to the monopoly scenario. For the parallel structure, the reverse result is due to higher flow in duopoly than in monopoly.

Based on the results in bottom two rows of Table 2, a more important question we would like to ask is whether the serial structure or the parallel structure brings higher surplus to commuters and to society overall. We address this question in the following proposition.

Proposition 4.3. *Under symmetry and equivalence conditions, for the duopoly scenario, both consumer surplus and social surplus are higher under the parallel structure than under the serial structure.*

The proposition states that regardless of the route condition and the travel time, from consumers' perspective as well as from an overall social welfare perspective, the parallel structure is strictly preferred to the serial structure.

We note that this result stands in spite of the previous result in Proposition 4.2 regarding the relative profitability of parallel versus serial structures in a duopoly scenario. While the relative profitability for firms varies based on the magnitude of latency parameters, from an overall social welfare consideration, the parallel structure is strictly superior. This implies that the gain in consumer surplus as a result of the parallel structure dominates the overall social welfare consideration, even in the case that latency is low and firms profit more from the serial structure.

The policy implication here is that although firms have some circumstances in which they will prefer the serial structure over the parallel structure, travellers, and thus society as a whole, is

substantially better off under the parallel structure. A policy-maker's choice of transportation network structure might thus depend on the relative welfare weights assigned to consumers versus firms, which here we have assumed are equal which is standard in the literature. Our result also serves as a potential indicator of policy-makers' underlying preferences in the case that the serial structure is chosen. That is, such a choice may be indicative of a relatively higher welfare weight being placed on firms' profitability over consumers' well-being.

4.4 An Alternative Setup

It is worth noting that there exists an alternative route structure that is also comparable to the serial and parallel structures we have studied. In such a structure, there are two routes from A to C , with intermediate point B and B' , respectively. By symmetry, we assume that the latency function for each route segment AB , BC , AB' or $B'C$ is $\frac{t}{2}(1 + 2\alpha x)$. Commuters make the travel decision (from A to C) to maximize their own utilities. We consider two scenarios:

Scenario 1: Firm 1 is in charge of segments AB and AB' by setting entrance fees p_1 and p'_1 , and firm 2 is in charge of segments BC and $B'C$ by setting entrance fees p_2 and p'_2 , respectively.

Scenario 2: Firm 1 is in charge of segments AB and BC , and firm 2 is in charge of segments AB' and $B'C$.

Given each segment's latency function $\frac{t}{2}(1 + 2\alpha x)$, it is easy to know that the latency functions for ABC and $AB'C$ are the same, which is $f_i(x) = t(1 + 2\alpha x)$. Since in Scenario 2, Firms 1 and 2 are indeed local monopolies for routes ABC and $AB'C$, respectively, and given that the latency functions for the two routes coincide with those in the parallel structure we studied, we can immediately obtain the result that Scenario 2 is outcome-equivalent to the parallel duopoly case.

Below, we will show that Scenario 1 is outcome-equivalent to the serial duopoly case. The key step is to prove that the equilibrium price for each segment is $\frac{1-t}{3}$ in Scenario 1.

By the symmetry condition, a symmetric equilibrium exists and suppose that the equilibrium price for each segment is p^* . Then the equilibrium condition requires that with all other segments' price levels at p^* , the best response for any given segment is p^* as well.

Without loss of generality, assume that AB , BC and AB' all set their price level at p and segment $B'C$ sets the price level at q . Then we can write out the expression for the profit of Firm 2, as follows

$$\Pi_2(p, q) = x_1(p, q)p + x_2(p, q)q$$

The first order condition requires that,

$$\frac{\partial \Pi_2(p, q)}{\partial q} = \frac{\partial x_1(p, q)}{\partial q}p + x_2 + \frac{\partial x_2(p, q)}{\partial q}q = 0$$

Given the pricing strategies, we can solve for the equilibrium traffic flows,

$$\begin{aligned} x_1(p, q) &= \frac{2\alpha t(1-t) - (4\alpha t + 1)p + q}{4\alpha t(1 + \alpha t)} \\ x_2(p, q) &= \frac{2\alpha t(1-t) - (2\alpha t - 1)p - (2\alpha t + 1)q}{4\alpha t(1 + \alpha t)} \end{aligned}$$

Then by the symmetry condition and the equilibrium condition for marginal commuters, we obtain $p^* = \frac{1-t}{3}$ and $x_1 = x_2 = \frac{1-t}{6(1+\alpha t)}$. Note that $p^* = p_{Duopoly}^S$ and $x_1 + x_2 = \frac{1-t}{3(1+\alpha t)} = x_{Duopoly}^S$, so the outcome-equivalence result between Scenario 1 and the serial duopoly is established.

Based on the equivalence results between Scenario 1 and serial duopoly, between Scenario 1 and parallel duopoly, and the comparison result between serial and parallel duopoly, we can immediately show that letting each firm operate a separate entire route (Scenario 2) is more socially desirable than letting firms compete within each route (Scenario 1). These results are thus insightful for policy making regarding the optimal route structure design. There is a social welfare advantage in building different entire transportation routes based on different transport modes. For example, in an ideal transportation network, the government may design the route structure such that different transport modes (subway, buses and taxis) can all directly take commuters to their destinations without making transfers to other transport modes.

4.5 Discussion

Operating Cost Up to this point, we have assumed that firms' operating costs are negligible, and thus that firms are essentially revenue maximizing. We can relax this assumption and allow the firms to have operating costs that could differ based upon some exogenous route characteristics. For example, operating costs may be a function of route length and/or route condition which are both exogenous parameters in our model, $C_1(t_1, \alpha_1), C_2(t_2, \alpha_2)$. Such a framework applies well to situations in which maintenance is primarily a function of distance traveled, which can often be the case for subway systems, buses, high speed rail, and airplane transport. We provide the detailed analysis in Appendix C and all of our main results regarding the comparison of the serial and parallel structures are robust after introducing the operating cost.

More than Two Firms A natural question one may raise is how our analysis regarding duopoly under both serial and parallel structures can be extended to the general case of more than two firms. Our analysis shows that in terms of consumer surplus and social surplus, the strict dominance of the parallel structure over the serial structure is still valid for any symmetric oligopoly case. Furthermore, in terms of firm profits a similar conclusion as the duopoly result holds with a generalized cutoff condition $\alpha t = \frac{1}{n}$, which reduces to $\alpha t = \frac{1}{2}$ for $n = 2$. The rigorous analysis of all the results about more than two firms is provided in Appendix D for readers with interest.

More Complex Market and Route Structures The serial structure and parallel structure are simplifications of real world market structures and route structures. The framework of analysis introduced in this paper naturally applies to more complex structures between the travel segments A to B and B to C . For example, the equilibrium can be fully characterized in a serial duopoly-monopoly hybrid structure, where there is parallel duopoly competition over the segment A to B , while the segment B to C is solely operated by one of the two firms. Readers can refer to Appendix E for the detailed analysis.

5 Conclusion

Travel routes consisting of more than one segment are common for many commuters and travellers. A relevant transportation policy question is how to manage such routes in terms of the market structure. Our analysis makes progress in understanding this issue in a modeling framework of a multi-segment transportation system of either serial or parallel structure of transport provider competition.

The results show that in terms of traffic flows, neither monopoly nor duopoly competition can reach the socially optimal traffic flows. However if maximal traffic flows are a desired policy target, monopoly is the better performing market structure under the serial structure, while duopoly is preferable under the parallel structure.

It may be the case that transportation policy-makers are less concerned about traffic volumes compared to traditional economic notions of social welfare. We demonstrate that if two firms are symmetric in price competition, commuters under utility that incorporates price and time costs additively, are always better off under parallel competition than serial competition. Indeed when considering social welfare as a whole, parallel duopoly also comes out ahead of the serial duopoly scenario. Therefore, the government may have good reason to support the parallel structure over the serial structure, if the focus is on consumer and/or overall social welfare. The only source of disparity in the comparison of duopoly structures is under some circumstances, from the perspective of the firms. Firms' profits can be higher under the serial duopoly if the route is sufficiently unburdened in terms of travel times or route conditions. The opposing directions of firm profits and social welfare in such cases could present a conflict of interest between firms in the market and governments or commuters for some relatively less-traveled routes.

In terms of the welfare advantages of the parallel structure, through extensions of the model we find that this result is robust when taking into account firms' operating costs that depend on exogenous route features. The welfare advantage of the parallel structure is also robust to a generalization of an n -firm oligopoly competing over several parallel or serial segments. We also consider hybrid market structures of the baseline model, in which either both monopoly and duopoly market structures are present in the transport system, or both serial and parallel features

are present.

Our study is the first to our knowledge to provide a systematic analysis comparing different market structures (monopoly, duopoly, price-free) against the socially optimal traffic flow for serial and parallel structures in the framework of a multi-part transportation system with traveller choice of transportation options. The results show that parallel duopoly competition among transportation providers is preferred over serial duopoly, regardless of the latency parameters of the system. Furthermore, we identify a conflict of interest between the supply side of the market and social welfare for low latency parameter values, in that firms prefer the serial structure, while both consumers and the social planner prefer the parallel structure. These insights may be useful from a policy perspective when governments are making decisions about the ideal transportation network structure when privatizing transport systems. The results also have potential industrial applications for firms seeking to outsource components of their supply chain or transport chain to other companies, for example, while being simultaneously concerned about the impact on the value added for their customers.

We can see several directions for future research. For the welfare analysis, our model has assumed symmetric roles of the duopolists, while in practice the duopolists could be heterogeneous in terms of the features of the route they are managing. Future work could evaluate welfare under differing assumptions about the symmetry of the competing firms. In addition, all firms in our model have been profit maximizing in their objectives. A reality is that in many transport networks, some segments of the route are run by fully or partially state-owned organizations whose objectives may differ from those of private firms. A possible extension is to incorporate a mixed oligopoly structure (as for example in Li, Lien and Zheng, 2019[31]) into the transportation system using the framework studied here. Another note is that our model is general and does not attempt to make policy recommendations based on any actual statistics or data. Actual parameters could potentially be applied to the model to obtain quantitative welfare and surplus estimates. Finally, future work may apply a similar framework to more complex transportation networks than the ones we consider here.

Appendix

A Proofs

A.1 Proof of Proposition 2.2

First, we derive the results of comparative statics analysis with respect to parameters t and α .

$$\frac{\partial x_{Duo}^S}{\partial t} = \frac{\partial \frac{1-t}{3(1+\alpha t)}}{\partial t} = -\frac{\alpha+1}{3(\alpha t+1)^2} < 0; \quad \frac{\partial x_{Opt}^S}{\partial t} = \frac{\partial \frac{1-t}{1+2\alpha t}}{\partial t} = -\frac{2\alpha+1}{(2\alpha t+1)^2} < 0;$$

$$\frac{\partial x_{UE}^S}{\partial t} = \frac{\partial \frac{1-t}{1+\alpha t}}{\partial t} = -\frac{\alpha+1}{(\alpha t+1)^2} < 0; \quad \frac{\partial x_{Mon}^S}{\partial t} = \frac{\partial \frac{1-t}{2(1+\alpha t)}}{\partial t} = -\frac{\alpha+1}{2(\alpha t+1)^2} < 0;$$

$$\frac{\partial x_{Duo}^S}{\partial \alpha} = \frac{\partial \frac{1-t}{3(1+\alpha t)}}{\partial \alpha} = -\frac{t(1-t)}{3(\alpha t+1)^2} < 0; \quad \frac{\partial x_{Opt}^S}{\partial \alpha} = \frac{\partial \frac{1-t}{1+2\alpha t}}{\partial \alpha} = -\frac{2t(1-t)}{(2\alpha t+1)^2} < 0;$$

$$\frac{\partial x_{UE}^S}{\partial \alpha} = \frac{\partial \frac{1-t}{1+\alpha t}}{\partial \alpha} = -\frac{t(1-t)}{(\alpha t+1)^2} < 0; \quad \frac{\partial x_{Mon}^S}{\partial \alpha} = \frac{\partial \frac{1-t}{2(1+\alpha t)}}{\partial \alpha} = -\frac{t(1-t)}{2(\alpha t+1)^2} < 0.$$

Then, we derive the results of comparative statics analysis with respect to parameters t_i and α_i , $i = 1, 2$

$$\frac{\partial x_{Duo}^S}{\partial t_i} = \frac{\partial \frac{1-t}{3(1+\alpha t)}}{\partial t_i} = -\frac{\alpha_1 t_1 + \alpha_2 t_2 + 1 + \alpha_i(1-t_1-t_2)}{3(\alpha_1 t_1 + \alpha_2 t_2 + 1)^2} < 0;$$

$$\frac{\partial x_{Opt}^S}{\partial t_i} = \frac{\partial \frac{1-t}{1+2\alpha t}}{\partial t_i} = -\frac{2\alpha_1 t_1 + 2\alpha_2 t_2 + 1 + 2\alpha_i(1-t_1-t_2)}{(2\alpha_1 t_1 + 2\alpha_2 t_2 + 1)^2} < 0;$$

$$\frac{\partial x_{UE}^S}{\partial t_i} = \frac{\partial \frac{1-t}{1+\alpha t}}{\partial t_i} = -\frac{\alpha_1 t_1 + \alpha_2 t_2 + 1 + \alpha_i(1-t_1-t_2)}{(\alpha_1 t_1 + \alpha_2 t_2 + 1)^2} < 0;$$

$$\frac{\partial x_{Mon}^S}{\partial t_i} = \frac{\partial \frac{1-t}{2(1+\alpha t)}}{\partial t_i} = -\frac{\alpha_1 t_1 + \alpha_2 t_2 + 1 + \alpha_i(1-t_1-t_2)}{2(\alpha_1 t_1 + \alpha_2 t_2 + 1)^2} < 0;$$

$$\frac{\partial x_{Duo}^S}{\partial \alpha_i} = \frac{\partial \frac{1-t}{3(1+\alpha t)}}{\partial \alpha_i} = -\frac{t_i(1-t_1-t_2)}{3(\alpha_1 t_1 + \alpha_2 t_2 + 1)^2} < 0; \quad \frac{\partial x_{Opt}^S}{\partial \alpha_i} = \frac{\partial \frac{1-t}{1+2\alpha t}}{\partial \alpha_i} = -\frac{2t_i(1-t_1-t_2)}{(2\alpha_1 t_1 + 2\alpha_2 t_2 + 1)^2} < 0;$$

$$\frac{\partial x_{UE}^S}{\partial \alpha_i} = \frac{\partial \frac{1-t}{1+\alpha t}}{\partial \alpha_i} = -\frac{t_i(1-t_1-t_2)}{(\alpha_1 t_1 + \alpha_2 t_2 + 1)^2} < 0; \quad \frac{\partial x_{Mon}^S}{\partial \alpha_i} = \frac{\partial \frac{1-t}{2(1+\alpha t)}}{\partial \alpha_i} = -\frac{t_i(1-t_1-t_2)}{2(\alpha_1 t_1 + \alpha_2 t_2 + 1)^2} < 0.$$

A.2 Proof of Proposition 3.2

First, we derive the comparative statics results with respect to parameters t_i , $i = 1, 2$ for the Duopoly scenario.

$$\begin{aligned}
& \frac{\partial x_{Du0}^P}{\partial t_1} \text{ (ignoring the denominator)} \\
&= -\alpha_2 t_2^2 (2\alpha_2(\alpha_2 t_2 + 1)(4\alpha_2 t_2 + 3) + \alpha_1(\alpha_2(2t_2(2\alpha_2((2\alpha_2 + 1)t_2 + 4) + 3) + 9) + 3)) \\
&\quad - 2\alpha_1 \alpha_2 t_1 t_2 (\alpha_2 t_2 + 1)(8(\alpha_1 + 1)\alpha_2^2 t_2^2 + 2\alpha_2 t_2(2\alpha_1(t_2 + 3) + 5) + 2\alpha_1(t_2 + 3) + 3) \\
&\quad - \alpha_1^2 t_1^2 (\alpha_2 t_2 + 1)^2 (8(\alpha_1 + 1)\alpha_2^2 t_2^2 + 2\alpha_2 t_2(2\alpha_1(t_2 + 2) + 3) - 2\alpha_1(t_2 - 3) + 3) < 0 \\
& \frac{\partial x_{Du0}^P}{\partial t_2} \text{ (ignoring the denominator)} \\
&= -4\alpha_1^3 (\alpha_2 + 2\alpha_1(\alpha_2 + 1)) t_1^4 (\alpha_2 t_2 + 1)^2 \\
&\quad - 2\alpha_1^2 t_1^3 (\alpha_2 t_2 + 1)(3\alpha_2(\alpha_2 t_2 + 1) + \alpha_1(\alpha_2((12\alpha_2 + 11)t_2 + 8) + 7)) \\
&\quad - \alpha_1 t_1^2 (\alpha_2(4\alpha_2 t_2 + 3) + \alpha_1(\alpha_2(t_2(\alpha_2((30\alpha_2 + 23)t_2 + 36) + 26) + 9) + 6)) \\
&\quad - 2\alpha_2 t_2 t_1 (\alpha_1(2\alpha_2((5\alpha_2 + 3)t_2 + 3) + 3) - \alpha_2^2 t_2) - 3\alpha_2^2 (2\alpha_2 + 1) t_2^2 < 0 \\
& \frac{\partial x_{Du0}^P}{\partial \alpha_1} \\
&= \frac{\alpha_2 t_2 (t_2(\alpha_2(t_1 - 1) - 1) + t_1)}{(\alpha_1 t_1 + \alpha_2 t_2(\alpha_1 t_1 + 1))^2} + \frac{2(2\alpha_2 t_2 + 3)(t_2(2\alpha_2(t_1 - 1) + 1) + 2t_1 - 3)}{(4\alpha_1 t_1 + 4\alpha_2 t_2(\alpha_1 t_1 + 1) + 3)^2} \\
&= -\frac{\alpha_2 t_2 ((\alpha_2 + 1)t_2 - t_1(\alpha_2 t_2 + 1))}{(\alpha_1 t_1 + \alpha_2 t_2(\alpha_1 t_1 + 1))^2} - \frac{2(2\alpha_2 t_2 + 3)(3(1 - t_2) + 2((\alpha_2 + 1)t_2 - t_1(\alpha_2 t_2 + 1)))}{(4\alpha_1 t_1 + 4\alpha_2 t_2(\alpha_1 t_1 + 1) + 3)^2} < 0
\end{aligned}$$

where the last inequality holds by the fact that $t_2 < 1$ and $(\alpha_2 + 1)t_2 - t_1(\alpha_2 t_2 + 1) > 0$ where the latter is implied by equation (33).

$$\begin{aligned}
& \frac{\partial x_{Du0}^P}{\partial \alpha_2} \\
&= \frac{\alpha_1 t_1 (t_1(\alpha_1(t_2 - 1) - 1) + t_2)}{(\alpha_2 t_2 + \alpha_1 t_1(\alpha_2 t_2 + 1))^2} + \frac{2(2\alpha_1 t_1 + 3)(t_1(2\alpha_1(t_2 - 1) + 1) + 2t_2 - 3)}{(4\alpha_2 t_2 + 4\alpha_1 t_1(\alpha_2 t_2 + 1) + 3)^2} \\
&= -\frac{\alpha_1 t_1 ((\alpha_1 + 1)t_1 - t_2(\alpha_1 t_1 + 1))}{(\alpha_2 t_2 + \alpha_1 t_1(\alpha_2 t_2 + 1))^2} - \frac{2(2\alpha_1 t_1 + 3)(3(1 - t_1) + 2((\alpha_1 + 1)t_1 - t_2(\alpha_1 t_1 + 1)))}{(4\alpha_2 t_2 + 4\alpha_1 t_1(\alpha_2 t_2 + 1) + 3)^2} < 0
\end{aligned}$$

where the last inequality holds by the fact that $t_1 < 1$ and $(\alpha_1 + 1)t_1 - t_2(\alpha_1 t_1 + 1) > 0$ where the latter is implied by equation (34).

Second, we show the results from the Social Optimum scenario. For $\frac{\partial x_{Opt}^P}{\partial t_i} < 0$ ($i = 1, 2$), the proof is by contradiction. Suppose there exist t_i , α_i ($i = 1, 2$) and i^* such that x_{Opt}^P increases when t_{i^*} increases. Note that the first order conditions imply that the equilibrium travel time for both routes should decrease since $f_i(x_i) = 1 - x_{Opt}^P$ for $i = 1, 2$. Since $f_i(x_i)$ is increasing in x_i for $i = 1, 2$, a decrease in f_{-i^*} implies a decrease in x_{-i^*} . Since $x_{Opt}^P = x_{i^*} + x_{-i^*}$, this must mean x_{i^*} increases. The increase in x_{i^*} together with the increase in t_{i^*} determines the increase in $f_{i^*}(x_{i^*})$

based on the fact that f_{i^*} is increasing in x_{i^*} and t_{i^*} . However, the increase in f_{i^*} contradicts with the decrease in f_{-i^*} in equilibrium. We can prove $\frac{\partial x_{Opt}^P}{\partial \alpha_i} < 0$ ($i = 1, 2$) by applying the same logic.

Using an analogous reasoning approach, we can also easily prove the results for the Price-free scenario ($\frac{\partial x_{UE}^P}{\partial t_i} < 0$ and $\frac{\partial x_{UE}^P}{\partial \alpha_i} < 0$) and for the Monopoly scenario ($\frac{\partial x_{Mon}^P}{\partial \alpha_i} < 0$) where one extra step relying on the fact that equilibrium price is independent of α_i is needed.

Last, to show $\frac{\partial x_{Mon}^P}{\partial t_i} < 0$ for the Monopoly scenario, note

$$\begin{aligned} \frac{\partial x_{Mon}^P}{\partial t_i} &= \frac{(\alpha_i - t_{-i}(\alpha_1 + \alpha_2))(t_1\alpha_1 + t_2\alpha_2 + t_1\alpha_2t_2\alpha_2) - (\alpha_i + t_{-i}\alpha_1\alpha_2)(t_1\alpha_1 + t_2\alpha_2 - t_1t_2(\alpha_1 + \alpha_2))}{2(t_1\alpha_1 + t_2\alpha_2 + t_1\alpha_2t_2\alpha_2)^2} \\ &= \frac{(-t_{-i}t_1\alpha_1 - t_{-i}t_2\alpha_2 + \alpha_it_1t_2)(\alpha_1 + \alpha_2 + \alpha_1\alpha_2)}{2(t_1\alpha_1 + t_2\alpha_2 + t_1\alpha_2t_2\alpha_2)^2} \\ &= \frac{-t_{-i}^2\alpha_{-i}(\alpha_1 + \alpha_2 + \alpha_1\alpha_2)}{2(t_1\alpha_1 + t_2\alpha_2 + t_1\alpha_2t_2\alpha_2)^2} < 0 \end{aligned}$$

B Circuits and Thévenin's Theorem

B.1 Analogy between the Circuit System and the Transportation System

We utilize some established results from physics to help us compare the equilibrium outcomes under the serial and parallel structures. We provide two figures to describe the analogy between a circuit system and a transportation system. The detailed explanations are offered below.

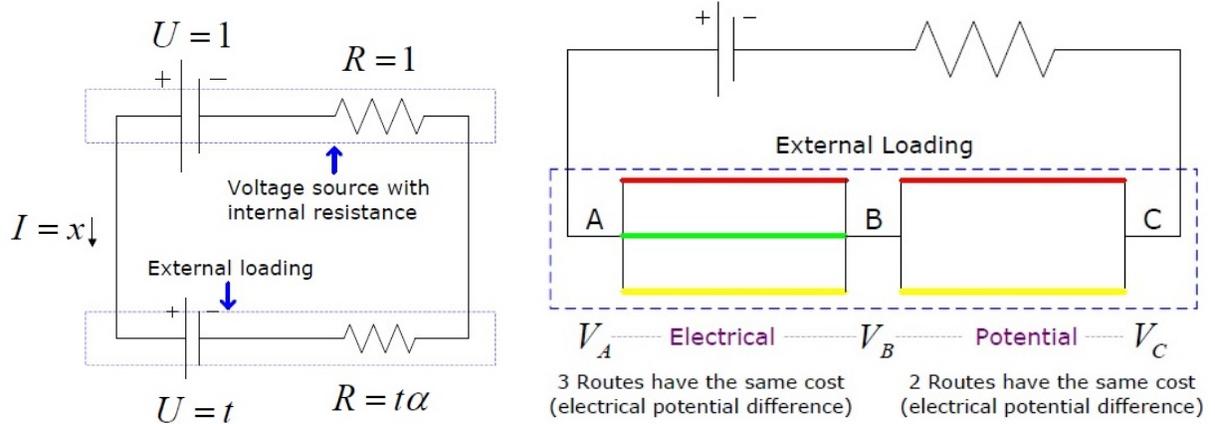


Figure 3: Analogy between the Circuit System and the Transportation System

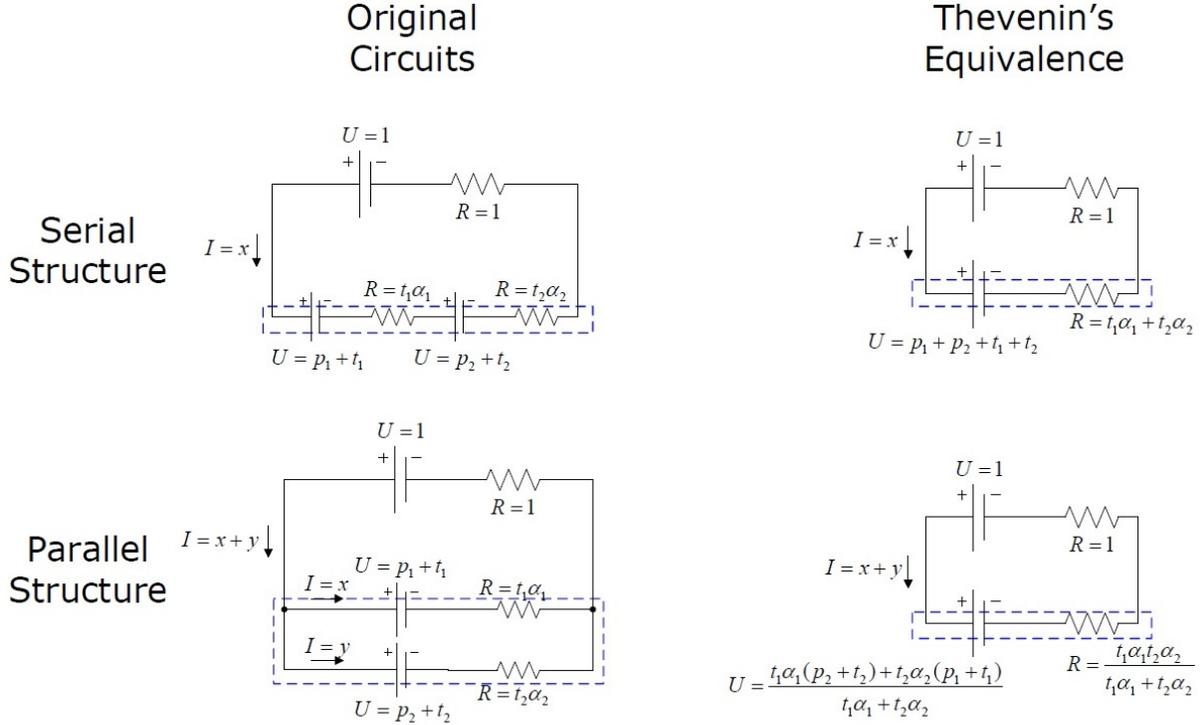


Figure 4: Thévenin's Theorem

We can interpret the expression of flow x (see equation (4) for example) as follows. Assume at is the resistance for the route with latency $f(x) = t(1 + \alpha x)$, then the conductance is the reciprocal of the resistance. Thus, the equivalent conductance of the parallel structure takes the summation of the two routes, while the equivalent resistance of the serial structure takes the summation of the two segments. Hence, the denominator in the expression of x can be expressed as 1 plus total resistance. We define the idle cost for the route with latency $f(x) = t(1 + \alpha x)$ and entrance fee p as $p + t$. Idle cost takes the conductance-weighted average when connected in the parallel structure and takes the summation when connected in the serial structure. Therefore, the numerator in the expression of x for the parallel structure should equal 1 minus total conductance-weighted idle cost. Note that the following relationship is satisfied in equilibrium:

$$\text{Flow} = 1 - \text{IdleCost} - \text{FlowCost}$$

where $\text{FlowCost} = \text{Flow} \times \text{Resistance}$. Equivalently, we have

$$\text{Flow} = \frac{1 - \text{IdleCost}}{1 + \text{Resistance}} \quad (36)$$

This analogy is based on one key assumption: the latency for any specific route in the transportation network has a linear form, $f(x) = t(1 + \alpha x)$, where x denotes the traffic flow through the route.

The analogy can be described as follows:

1. Regard the traffic flow in the transportation network as the current in one circuit;
2. Regard the cost (including latency and entrance fee) in one specific route AB as the electrical potential difference between two nodes in the circuit;⁵
3. Regard the total cost from A to C (through some mediators) as the electrical potential difference between A and C ;⁶
4. Regard the heterogeneous consumer as a $1V$ voltage source with 1Ω internal resistance.⁷

B.2 Thévenin's Theorem

Thévenin's theorem holds that:⁸

- Any linear electrical network with voltage and current sources and resistances only can be replaced at terminals A-B by an equivalent voltage source V in series connection with an equivalent resistance R .
- The equivalent voltage V is the voltage obtained at terminals A-B of the network with terminals A-B open circuited.
- The equivalent resistance R is the resistance that the circuit between terminals A and B would have, if all ideal voltage sources in the circuit were replaced by a short circuit and all ideal current sources were replaced by an open circuit.

In circuit theory terms, the theorem allows for any one-port network with any complex structure to be reduced to a single voltage source and a single impedance. The Thévenin's equivalences of serial and parallel structures are thus special case application of the general theorem.

C Extension I: Operating Cost

In this extension we allow the firms to have operating costs that depend on exogenous route characteristics, $C_1(t_1, \alpha_1), C_2(t_2, \alpha_2)$.

⁵Then the linear latency function of the route can be regarded as a wire connecting A and B that includes a $U = p + t$ voltage source in series connection with a resistance of $R = t\alpha$.

⁶When traffic network is in equilibrium, multiple routes from A to C must have the same cost, which coincides with Kirchhoff's law in circuit.

⁷When traffic flow is x , it means that consumer with valuation $1 - x$ has zero utility. This zero utility condition coincides with Kirchhoff's law and the corresponding external voltage should be $1 - x$. This external voltage source is equivalent to a $1V$ source with internal resistance 1Ω .

⁸From Wikipedia, https://en.wikipedia.org/wiki/Th%C3%A9venin%27s_theorem

C.1 Serial Structure

We first examine the case of the serial structure. Given entrance fees p_1, p_2 , the traffic flow is given by $x = \frac{1-p_1-p_2-t_1-t_2}{1+t_1\alpha_1+t_2\alpha_2}$.

Hence, the profits for each firm are given by

$$\begin{aligned}\Pi_1(p_1) &= \frac{1-p_1-p_2-t_1-t_2}{1+t_1\alpha_1+t_2\alpha_2}(p_1-C_1) \\ \Pi_2(p_2) &= \frac{1-p_1-p_2-t_1-t_2}{1+t_1\alpha_1+t_2\alpha_2}(p_2-C_2)\end{aligned}$$

with first order conditions

$$\begin{aligned}\frac{\partial \Pi_1(p_1)}{\partial p_1} &= \frac{1-2p_1-p_2-t_1-t_2+C_1}{1+t_1\alpha_1+t_2\alpha_2} = 0 \\ \frac{\partial \Pi_2(p_2)}{\partial p_2} &= \frac{1-p_1-2p_2-t_1-t_2+C_2}{1+t_1\alpha_1+t_2\alpha_2} = 0\end{aligned}$$

yielding equilibrium prices

$$p_1 = p_1^* + \frac{2C_1 - C_2}{3} \quad (37)$$

$$p_2 = p_2^* + \frac{2C_2 - C_1}{3} \quad (38)$$

where $p_1^* = p_2^* = \frac{1-t_1-t_2}{3}$ denotes the previously derived equilibrium prices without any operating costs (equation (7)). Hence, depending on the relative magnitudes of costs among the two firms, the equilibrium price with operating costs may be greater than or less than the equilibrium price without operating costs. Note that at least one firm's price under operating costs will exceed its price without operating costs.

The equilibrium traffic flow is

$$x = \frac{1-t_1-t_2-(C_1+C_2)}{3(1+t_1\alpha_1+t_2\alpha_2)} \quad (39)$$

which is indeed lower than the equilibrium traffic flow without operating costs.

C.2 Parallel Structure

For the case of the parallel structure, the profits for each firm are given by

$$\begin{aligned}\Pi_1(p_1) &= \frac{t_2\alpha_2 + p_2 + t_2 - (1+t_2\alpha_2)(p_1+t_1)}{t_1\alpha_1 + t_2\alpha_2 + t_1\alpha_1 t_2\alpha_2}(p_1 - C_1) \\ \Pi_2(p_2) &= \frac{t_1\alpha_1 + p_1 + t_1 - (1+t_1\alpha_1)(p_2+t_2)}{t_1\alpha_1 + t_2\alpha_2 + t_1\alpha_1 t_2\alpha_2}(p_2 - C_2)\end{aligned}$$

The first order conditions (ignoring the common denominators) give us

$$\begin{aligned}\frac{\partial \Pi_1(p_1)}{\partial p_1} &= t_2\alpha_2 + p_2 + t_2 - t_1 - t_1 t_2\alpha_2 - 2(1+t_2\alpha_2)p_1 + (1+t_2\alpha_2)C_1 = 0 \\ \frac{\partial \Pi_2(p_2)}{\partial p_2} &= t_1\alpha_1 + p_1 + t_1 - t_2 - t_1 t_2\alpha_1 - 2(1+t_1\alpha_1)p_2 + (1+t_1\alpha_1)C_2 = 0\end{aligned}$$

with the pricing solution

$$p_1 = p_1^* + \frac{(1 + t_2\alpha_2)(C_1 + 2C_2 + 2t_1\alpha_1C_2)}{4t_1\alpha_1 + 4t_2\alpha_2 + 4t_1\alpha_1t_2\alpha_2 + 3} \quad (40)$$

$$p_2 = p_2^* + \frac{(1 + t_1\alpha_1)(2C_1 + C_2 + 2t_2\alpha_2C_1)}{4t_1\alpha_1 + 4t_2\alpha_2 + 4t_1\alpha_1t_2\alpha_2 + 3} \quad (41)$$

where p_1^*, p_2^* denote equilibrium prices without any operating costs as derived previously (equations (19) and (20)). Therefore, equilibrium prices are strictly higher when introducing operating costs. By contrast, equilibrium prices in the serial structure may be higher or lower than the equilibrium prices without operating costs, depending on the relative magnitudes of firms' operating costs.

Since total traffic flow is expressed as

$$\frac{1 - \frac{t_1\alpha_1(p_2+t_2)}{t_1\alpha_1+t_2\alpha_2} - \frac{t_2\alpha_2(p_1+t_1)}{t_1\alpha_1+t_2\alpha_2}}{1 + \frac{t_1\alpha_1t_2\alpha_2}{t_1\alpha_1+t_2\alpha_2}}$$

the higher prices under operating costs lead to a lower equilibrium traffic flow.

C.3 Comparison with No Operating Costs

Comparing the analysis with operation costs to the baseline case without operational costs, we observe that operational costs reduce the equilibrium traffic flow for both the parallel and serial structures. In terms of pricing, the result depends on the transportation structure. In the case of the parallel structure, prices are higher for both firms due to the operating cost. For the case of the serial structure, the price for any given firm may be either higher or lower than the situation without operating costs, where the result depends on the relative costs between the two firms. However, at least one of the firms (and potentially both) implements a higher price than in the setup without operating costs, thus reducing the equilibrium traffic flow.

C.4 Comparison Between Serial and Parallel Structures

We now compare the cases of serial and parallel structures with operating costs. As previously, we restrict our attention to the symmetric case of Firm 1 and Firm 2 in their price competition and impose the equivalence condition, both of which simplify the expressions substantially. In addition, we assume that each firm incurs the same operating cost, $C_1 = C_2 = C$.

C.4.1 Equilibrium Flow

Under the symmetry and equivalence conditions, the latency function for the serial structure is $f(x) = t(1 + \alpha x)$, while the associated latency function for both roads in the parallel structure is $f(x) = t(1 + 2\alpha x)$. Both structures are outcome equivalent in the price-free equilibrium as well as in the social optimum scenario.

When considering the scenario of monopoly, we need to set the total operating cost of the system equal for each case for comparison purposes. Assume the unit operating cost for the monopoly is C , then when analyzing the duopoly context, the corresponding costs should be $C_1 = C_2 = \frac{C}{2}$ for each road in the serial structure, and $C_1 = C_2 = C$ for each road in the parallel structure. With such symmetry and equivalence conditions on operating costs, the scenario of monopoly yields identical traffic flows under the contexts of serial and parallel structures.

For the case of duopoly, the traffic flow is higher under the parallel structure than the serial structure. The equilibrium traffic flows are summarized in the following table and their relationships are stated in Proposition C.1.

Table 3: Traffic Flow

	Serial	Parallel
User Equilibrium		$\frac{1-t-C}{1+\alpha t}$
Social Optimum		$\frac{1-t-C}{1+2\alpha t}$
Monopoly		$\frac{1-t-C}{2(1+\alpha t)}$
Duopoly	$\frac{1-t-C}{3(1+\alpha t)}$	$\frac{(1-t-C)(1+2\alpha t)}{(1+\alpha t)(4\alpha t+1)}$

Proposition C.1. *Under symmetry and equivalence conditions with operating costs, the equilibrium flows under different scenarios and different structures have the following relationship:*

$$x_{Duo}^S < x_{Mon}^{S/P} < x_{Duo}^P < x_{Opt}^{S/P} < x_{UE}^{S/P}.$$

C.4.2 Equilibrium Price and Profit

In terms of pricing, the monopolist's price for each route in the case of the parallel structure is equivalent to the monopolist's total price over the two segments in the serial structure. For the case of duopoly, the parallel structure has a higher equilibrium price for a sufficiently high operational cost, the cutoff being a function of α and t . Note that our previous condition, $\alpha t > \frac{1}{2}$ in the case of no operational cost, is sufficient to satisfy this condition for any positive C .

Table 4: Comparison Between Serial Structure and Parallel Structure in Duopoly with Cost

	Serial	Parallel
Price	$p_1 = p_2 = \frac{1-t}{3} + \frac{C}{6}$	$p_1 = p_2 = \frac{2\alpha t(1-t)}{1+4\alpha t} + \frac{(1+2\alpha t)C}{1+4\alpha t}$
Profit	$\Pi_1 = \Pi_2 = \frac{(1-t-C)^2}{9(1+\alpha t)}$	$\Pi_1 = \Pi_2 = \frac{\alpha t(1+2\alpha t)(1-t-C)^2}{(1+4\alpha t)^2(1+\alpha t)}$
Consumer Surplus	$\frac{(1-t-C)^2}{18(1+\alpha t)^2}$	$\frac{(1-t-C)^2(1+2\alpha t)^2}{2(1+\alpha t)^2(1+4\alpha t)^2}$
Social Surplus	$\frac{(4\alpha t+5)(1-t-C)^2}{18(1+\alpha t)^2}$	$\frac{(1-t-C)^2(1+2\alpha t)(4\alpha^2 t^2+6\alpha t+1)}{2(1+\alpha t)^2(1+4\alpha t)^2}$

The monopolist's profit is the same whether in the serial context or the parallel context. Duopoly profit equals half of equilibrium flow multiplied by the duopoly price. When including operating

costs, the $(1 - t)^2$ terms in the numerator of the profit expression without operating cost, change to $(1 - t - C)^2$. Therefore, the profit comparison between structures is again defined by the cutoff $\alpha t = \frac{1}{2}$, and is fully characterized in Proposition C.2.

Proposition C.2. *Under symmetry and equivalence conditions with operating costs, for the duopoly scenario, the relationship between profits under serial and parallel structures depends on the parameter values. If $\alpha t > \frac{1}{2}$, the profit under the parallel structure is higher; If $\alpha t < \frac{1}{2}$, the profit under the serial structure is higher; If $\alpha t = \frac{1}{2}$, the profits under both structures are the same.*

C.4.3 Equilibrium Surplus

Similarly, when including operating costs, the $(1 - t)^2$ terms in the numerator of the surplus expressions without operating cost, change to $(1 - t - C)^2$, as shown in last two rows of Table 4. Thus, both consumer surplus and social surplus are higher under the parallel structure, and such a result is described in Proposition C.3.

Proposition C.3. *Under symmetry and equivalence conditions with operating costs, for the duopoly scenario, both consumer surplus and social surplus are higher under the parallel structure than under the serial structure.*

D Extension II: More than Two Firms

We now allow for the possibility that there are n competitors in charge of n route segments under the cases of serial and parallel structures. For ease of comparison, we restrict our attention to the case that the n firms are symmetric in their price competition, which simplifies the expressions substantially. The analysis for the serial structure is a straightforward generalization of that in Section 2 by defining $t = \sum_{k=1}^n t_k$ and $\alpha = \frac{\sum_{k=1}^n t_k \alpha_k}{\sum_{k=1}^n t_k}$, and is hence skipped. In what follows, we first conduct analysis for the parallel structure, then compare equilibrium outcomes between two structures, and finally summarize all the results with interpretations and discussions.

D.1 Analysis for Parallel Structure

Since n routes are identical, without loss of generality, we analyze the symmetric equilibrium in which all firms set the same price $p_i = p^*$. The equilibrium requires the following: if all other $n - 1$ firms set the price at p^* , it is optimal for the last firm to set the price at p^* . Let $\Pi(x, y)$ denote the profit when the last firm sets the price at x while other firms set the price at y . Therefore, the following condition holds,

$$\frac{\partial \Pi(p, p^*)}{\partial p} \Big|_{p=p^*} = 0$$

We first calculate the flow of each route when the last firm sets the price at p while other firms set the price at p^* . Assume the flow of the last route equals x while the flow of all other routes are identical and equal to \bar{x} . Then, the total flow equals $(n-1)\bar{x} + x$. The commuter with value $1 - (n-1)\bar{x} - x$ is indifferent among three options: (1) not traveling, (2) choosing the last route and (3) choosing other routes. Mathematically, the following two equations will determine the flow parameters x and \bar{x} given p and p^* ,

$$\begin{aligned} p + t(1 + n\alpha x) &= p^* + t(1 + n\alpha\bar{x}) \\ 1 - (n-1)\bar{x} - x &= p^* + t(1 + n\alpha\bar{x}) \end{aligned}$$

Rearranging the first equation, we get

$$x = \bar{x} + \frac{p^* - p}{n\alpha t}$$

Substituting x in the second equation, we obtain

$$\begin{aligned} \bar{x}(p, p^*) &= \frac{1}{n(1 + \alpha t)} \left(1 - p^* - t + \frac{p^* - p}{n\alpha t} \right) \\ x(p, p^*) &= \frac{1}{n(1 + \alpha t)} \left(1 - p^* - t + \frac{p^* - p}{n\alpha t} \right) + \frac{p^* - p}{n\alpha t} \end{aligned}$$

The profit for the last firm is defined by

$$\Pi(p, p^*) = px(p, p^*) = \frac{p(1 - p^* - t + \frac{p^* - p}{n\alpha t})}{n(1 + \alpha t)} + \frac{p^*p - p^2}{n\alpha t}$$

with first order condition

$$\frac{1 - p^* - t + \frac{p^* - 2p}{n\alpha t}}{n(1 + \alpha t)} + \frac{p^* - 2p}{n\alpha t} = 0$$

where $p = p^*$.

Price We solve for p^* and get

$$p^* = \frac{n\alpha t(1 - t)}{2n\alpha t + n - 1} \quad (42)$$

Flow The equilibrium flow of each route is thus

$$x_i^* = \frac{1 - p^* - t}{n(1 + \alpha t)} = \frac{(1 - t)(n\alpha t + n - 1)}{n(1 + \alpha t)(2n\alpha t + n - 1)} \quad (43)$$

and total flow is

$$X = \frac{(1 - t)(n\alpha t + n - 1)}{(1 + \alpha t)(2n\alpha t + n - 1)} \quad (44)$$

Profit The profit for each firm is hence

$$\Pi_i = p^* x_i^* = \frac{\alpha t(1 - t)^2(n\alpha t + n - 1)}{(1 + \alpha t)(2n\alpha t + n - 1)^2} \quad (45)$$

Consumer Surplus We first calculate the average surplus for the consumer, which is exactly $\frac{X}{2}$. Therefore, the consumer surplus is defined by

$$\frac{X^2}{2} = \frac{(1-t)^2(n\alpha t + n - 1)^2}{2(1+\alpha t)^2(2n\alpha t + n - 1)^2} \quad (46)$$

Social Surplus The total social surplus is the summation of firms' profits and consumer surplus,

$$\begin{aligned} \text{Social Surplus} &= \text{Total Profit} + \text{Consumer Surplus} \\ &= n \times \text{Firm Profit} + \text{Consumer Surplus} \\ &= n \frac{\alpha t(1-t)^2(n\alpha t + n - 1)}{(1+\alpha t)(2n\alpha t + n - 1)^2} + \frac{(1-t)^2(n\alpha t + n - 1)^2}{2(1+\alpha t)^2(2n\alpha t + n - 1)^2} \\ &= \frac{(1-t)^2(n\alpha t + n - 1)^2}{2(1+\alpha t)^2(2n\alpha t + n - 1)^2} [2(1+\alpha t)(n\alpha t) + (n\alpha t + n - 1)] \\ &= \frac{(1-t)^2(n\alpha t + n - 1)^2(2n\alpha^2 t^2 + 3n\alpha t + n - 1)}{2(1+\alpha t)^2(2n\alpha t + n - 1)^2} \end{aligned} \quad (47)$$

D.2 Comparison Between Serial and Parallel Structures

D.2.1 Total Flow

$$\frac{X_{Oli}^P}{X_{Oli}^S} = \frac{(n+1)(n\alpha t + n - 1)}{(2n\alpha t + n - 1)} > \frac{n+1}{2} > 1$$

Hence, $X_{Oli}^P > X_{Oli}^S$.

D.2.2 Profit

$$\begin{aligned} \frac{\Pi_{Oli}^P}{\Pi_{Oli}^S} - 1 &= \frac{(n+1)^2 t \alpha (n\alpha t + n - 1) - (2n\alpha t + n - 1)^2}{(2n\alpha t + n - 1)^2} \\ &= \frac{(n-1)^2 n (\alpha t)^2 + (n-1)^3 (\alpha t) - (n-1)^2}{(2n\alpha t + n - 1)^2} \\ &= \frac{(n-1)^2 (n\alpha t - 1)(\alpha t + 1)}{(2n\alpha t + n - 1)^2} \end{aligned}$$

Hence, $\Pi_{Oli}^P > \Pi_{Oli}^S$ if and only if $\alpha t > \frac{1}{n}$.

D.2.3 Consumer Surplus

$$\frac{CS_{Oli}^P}{CS_{Oli}^S} = \frac{(n+1)^2 (n\alpha t + n - 1)^2}{(2n\alpha t + n - 1)^2} > \frac{(n+1)^2}{4} > 1$$

Hence, $CS_{Oli}^P > CS_{Oli}^S$.

D.2.4 Total Surplus

When $n \geq 3$,

$$\begin{aligned} \frac{TS_{Oli}^P}{TS_{Oli}^S} &= \frac{(n+1)^2(n\alpha t + n - 1)(2n\alpha^2 t^2 + 3n\alpha t + n - 1)}{(2n\alpha t + n - 1)^2(2n\alpha t + 2n + 1)} \\ &> \frac{(n+1)^2(2n\alpha^2 t^2 + 3n\alpha t + n - 1)}{2(2n\alpha t + n - 1)(2n\alpha t + 2n + 1)} \\ &= \frac{2n(n+1)^2\alpha^2 t^2 + 3n(n+1)^2\alpha t + (n+1)^2(n-1)}{8n(\alpha t)^2 + 12n^2\alpha t + (4n+2)(n-1)} \end{aligned}$$

Note that $2n(n+1)^2 > 8n$, $3n(n+1)^2 > 12n^2$, and $(n+1)^2(n-1) > (4n+2)(n-1)$. Hence, $TS_{Oli}^P > TS_{Oli}^S$.

For the special case when $n = 2$,

$$\frac{TS_{Oli}^P}{TS_{Oli}^S} = \frac{9(2\alpha t + 1)(4\alpha^2 t^2 + 6\alpha t + 1)}{(4\alpha t + 1)^2(4\alpha t + 5)} = \frac{72(\alpha t)^3 + 144(\alpha t)^2 + 72(\alpha t) + 9}{64(\alpha t)^3 + 112(\alpha t)^2 + 44(\alpha t) + 5} > 1$$

D.3 Results and Discussion

D.3.1 Equilibrium Flow

Assume the latency function for the serial structure is $f(x) = t(1 + \alpha x)$, then the associated latency function for all n routes in the parallel structure is $f_i(x) = t(1 + n\alpha x)$. Similar to the previous section, the parallel and serial structures are outcome equivalent in the price-free equilibrium, social optimum and monopoly. The following lemma characterizes the equilibrium.

Lemma D.1. *For n routes in the parallel structure with latency function $f_i(x) = t(1 + n\alpha x)$, (1) Equilibrium traffic flow for each route is $\frac{(1-t)(n\alpha t + n - 1)}{n(1+\alpha t)(2n\alpha t + n - 1)}$. (2) Equilibrium prices are identical and equal $\frac{(1-t)n\alpha t}{2n\alpha t + n - 1}$.*

If all other routes set the equilibrium price, then it is optimal for a route to set the equilibrium price as well. The equilibrium traffic flows are summarized in the following table.

Table 5: Traffic Flows

	Serial	Parallel
User Equilibrium		$\frac{1-t}{1+\alpha t}$
Social Optimum		$\frac{1-t}{1+2\alpha t}$
Monopoly		$\frac{1-t}{2(1+\alpha t)}$
Oligopoly	$\frac{1-t}{(n+1)(1+\alpha t)}$	$\frac{(1-t)(n\alpha t + n - 1)}{(1+\alpha t)(2n\alpha t + n - 1)}$

Using the same notations for equilibrium flows as in Section 4.1, we have the following proposition.

Proposition D.2. *Under symmetry and equivalence conditions, the equilibrium traffic flows under different scenarios and different structures have the following relationship:*

$$x_{Oli}^S < x_{Mon}^{S/P} < x_{Oli}^P < x_{Opt}^{S/P} < x_{UE}^{S/P}.$$

As the number of firms n grows, the traffic flow of the oligopoly market increases under the parallel structure while it decreases under the serial structure. When the number of firms approaches infinity, the traffic flow in the oligopoly market converges to the socially optimal outcome under the parallel structure while it converges to zero under the serial structure.

D.3.2 Equilibrium Price and Profit

Similarly, for equilibrium prices and profits, our main results from the case of two firms in Section 4.2 can be generalized to the n -firm case. The expressions for price and profit under monopoly and oligopoly in the serial and parallel structures are provided in first two rows in Table 6.

Table 6: Comparison Between Serial Structure and Parallel Structure in Oligopoly

	Serial	Parallel
Price	$p_i = \frac{1-t}{n+1}$	$p_i = \frac{n\alpha t(1-t)}{2n\alpha t+n-1}$
Profit	$\Pi_i = \frac{(1-t)^2}{(n+1)^2(1+\alpha t)}$	$\Pi_i = \frac{\alpha t(n\alpha t+n-1)(1-t)^2}{(2n\alpha t+n-1)^2(1+\alpha t)}$
Consumer Surplus	$\frac{(1-t)^2}{2(n+1)^2(1+\alpha t)^2}$	$\frac{(1-t)^2(n\alpha t+n-1)^2}{2(1+\alpha t)^2(2n\alpha t+n-1)^2}$
Social Surplus	$\frac{(2n\alpha t+2n+1)(1-t)^2}{2(n+1)^2(1+\alpha t)^2}$	$\frac{(1-t)^2(n\alpha t+n-1)(2n\alpha^2 t^2+3n\alpha t+n-1)}{2(1+\alpha t)^2(2n\alpha t+n-1)^2}$

Proposition D.3. *Under symmetry and equivalence conditions, for the oligopoly scenario, the relationship between oligopoly profits under serial and parallel structures depends on the parameter values. If $\alpha t > \frac{1}{n}$, the profit under the parallel structure is higher; If $\alpha t < \frac{1}{n}$, the profit under the serial structure is higher; If $\alpha t = \frac{1}{n}$, the profit under both structures are the same.*

The above proposition implies that when the route condition is poor (such that α is large) or the ideal travel time is long (such that t is large), the oligopolist's profit is higher under the parallel structure. Additionally, the threshold is decreasing with n , implying that the oligopolist's profit is more likely to be higher under the parallel structure as the number of competitors grows.

D.3.3 Equilibrium Surplus

The consumer surplus and social surplus can also be calculated under both serial and parallel structures in the symmetric case, shown in last two rows in Table 6, respectively.

Our analysis shows that the result that the parallel structure dominates the serial structure based on welfare criteria in the case of duopoly still holds when there are more than two firms. We

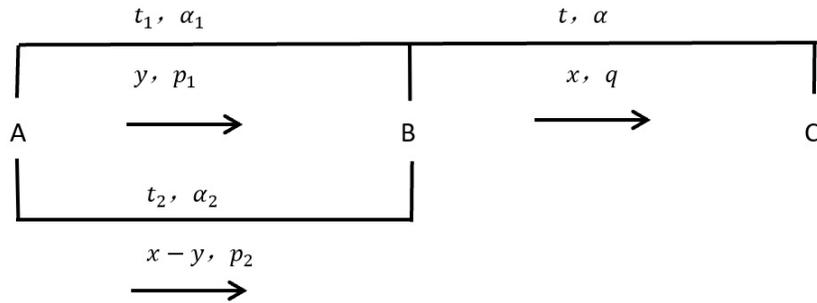
summarize this result in the following proposition.

Proposition D.4. *Under symmetry and equivalence conditions, for the oligopoly scenario, both consumer surplus and social surplus are higher under the parallel structure than under the serial structure.*

The above proposition indicates that the welfare comparison result that the parallel structure is strictly preferred to the serial structure from a consumer and social welfare perspective, is indeed robust for the case of oligopoly.

E Extension III: Hybrid Market Structures

In our benchmark model, we have considered situations where a single market structure (duopoly, monopoly) applies to the entire transportation structure, including serial duopoly, serial monopoly, parallel duopoly, and parallel monopoly. In this extension we consider a hybrid structure, for which there is duopoly competition over the segment A to B , while the segment B to C is solely operated by one of the two firms. Since commuters decide whether to take the trip or not based on their valuation of the complete travel path A to C , this analysis is identical to the reverse case in which A to B is operated by a monopolist, while B to C has duopoly competition. Without loss of generality, we analysis the former case, that is, duopoly over AB and monopoly over BC .



$$u = \max \{ \theta - p_1 - q - f(y) - f(x), \theta - p_2 - q - f(x - y) - f(x), 0 \}$$

$$\pi_1 = yp_1 + xq$$

$$\pi_2 = (x - y)p_2$$

Figure 5: Duopoly in AB and Monopoly in BC

Two firms compete for route AB by setting an entrance fee of p_1, p_2 respectively, and firm 1 also takes charge of the route BC by setting an entrance fee of q . The setting for this hybrid structure can be illustrated by Figure 5.

In equilibrium, suppose the total traffic flow is denoted by x while y of them choose firm 1 over the A to B segment. Thus, the (marginal) consumer with valuation $1 - x$ must be indifferent

between three options: (1) not traveling, (2) traveling through firm 1 over AB , and (3) traveling through firm 2 over AB . This equilibrium condition implies the following two equations.

$$\begin{aligned} 1 - x - p_1 - q - t_1(1 + \alpha_1 y) - t(1 + \alpha x) &= 0 \\ 1 - x - p_2 - q - t_2(1 + \alpha_2(x - y)) - t(1 + \alpha x) &= 0 \end{aligned}$$

Expressing the flow variables x and y in terms of price variables p_1 , p_2 , and q , we obtain

$$x = \frac{1 - (q + t) - \frac{t_1 \alpha_1}{t_1 \alpha_1 + t_2 \alpha_2} (p_2 + t_2) - \frac{t_2 \alpha_2}{t_1 \alpha_1 + t_2 \alpha_2} (p_1 + t_1)}{1 + t\alpha + \frac{t_1 \alpha_1 t_2 \alpha_2}{t_1 \alpha_1 + t_2 \alpha_2}} \quad (48)$$

$$y = \frac{t_2 \alpha_2}{t_1 \alpha_2 + t_2 \alpha_2} x + \frac{p_2 + t_2 - p_1 - t_1}{t_1 \alpha_1 + t_2 \alpha_2} \quad (49)$$

$$x - y = \frac{t_1 \alpha_1}{t_1 \alpha_2 + t_2 \alpha_2} x + \frac{p_1 + t_1 - p_2 - t_2}{t_1 \alpha_1 + t_2 \alpha_2} \quad (50)$$

Note that the profit functions for the two firms are given by

$$\Pi_1 = yp_1 + xq \quad (51)$$

$$\Pi_2 = (x - y)p_2 \quad (52)$$

Substituting (48)-(50) into (51) and (52), we have the following first order conditions by profit maximization.

$$\begin{aligned} \frac{\partial \Pi_1}{\partial p_1} = 0 &\Rightarrow (-2p_1 + p_2 - t_1 + t_2)(\alpha t + 1) - \alpha_2 t_2 (2p_1 + 2q + t + t_1 - 1) = 0 \\ \frac{\partial \Pi_1}{\partial q} = 0 &\Rightarrow \alpha_1 (-t_1)(p_2 + 2q + t + t_2 - 1) - \alpha_2 t_2 (2p_1 + 2q + t + t_1 - 1) = 0 \\ \frac{\partial \Pi_2}{\partial p_2} = 0 &\Rightarrow (p_1 - 2p_2 + t_1 - t_2)(\alpha t + 1) - \alpha_1 t_1 (2p_2 + q + t + t_2 - 1) = 0 \end{aligned}$$

Equilibrium prices are given by the following expressions

$$p_1 = -\frac{2(t_1 - t_2)(\alpha t + 1) + \alpha_1 t_1 (t + 3t_1 - 2t_2 - 1)}{6(\alpha t + \alpha_1 t_1 + 1)} \quad (53)$$

$$p_2 = \frac{(t_1 - t_2)(\alpha t + 1) - \alpha_1 t_1 (t + t_2 - 1)}{3(\alpha t + \alpha_1 t_1 + 1)} \quad (54)$$

$$q = -\frac{(3t + t_1 + 2t_2 - 3)(\alpha t + 1) + 2\alpha_1 t_1 (t + t_2 - 1)}{6(\alpha t + \alpha_1 t_1 + 1)} \quad (55)$$

Substituting (53)-(55) into (48) and (49), we obtain the equilibrium traffic flows, where $x^* - y^*$ denotes the traffic flow on Firm 2's route over the AB segment.

$$x^* = \frac{\alpha_1 t_1 ((3t + t_1 + 2t_2 - 3)(-\alpha t + 1)) - 2\alpha_1 t_1 (t + t_2 - 1) - 3\alpha_2 (t + t_1 - 1)t_2 (\alpha t + \alpha_1 t_1 + 1)}{6(\alpha t + \alpha_1 t_1 + 1)(\alpha_2 t_2 (\alpha t + 1) + \alpha_1 t_1 (\alpha t + \alpha_2 t_2 + 1))} \quad (56)$$

$$y^* = \frac{(\alpha t + 1)(-2(t_1 - t_2)(\alpha t + 1) - \alpha_1 t_1 (t + 3t_1 - 2t_2 - 1)) - 3\alpha_2 (t + t_1 - 1)t_2 (\alpha t + \alpha_1 t_1 + 1)}{6(\alpha t + \alpha_1 t_1 + 1)(\alpha_2 t_2 (\alpha t + 1) + \alpha_1 t_1 (\alpha t + \alpha_2 t_2 + 1))} \quad (57)$$

$$x^* - y^* = \frac{(t_1 - t_2)(\alpha t + 1) - \alpha_1 t_1(t + t_2 - 1)}{3\alpha_2 t_2(\alpha t + 1) + 3\alpha_1 t_1(\alpha t + \alpha_2 t_2 + 1)} \quad (58)$$

Given the above closed-form equilibrium solutions, it is indeed possible to conduct welfare comparisons and comparative statics analysis as we have done in our previous model setups. Since our main objective for this section is merely to provide the analytical framework for this market structure, such extended analysis is beyond the scope of this current paper.

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